Math is a Four Letter Word

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In 1957 several dozen mathematics educators met in Washington, D.C. to establish priorities for mathematics instruction in the post-Sputnik era. Their recommendations helped launch programs that led to major curriculum improvements in college mathematics, as well as to what the public now disparagingly calls "New Math". Last May, two decades later, a similar group convened in Alexandria, Va. to outline issues in mathematics education for the 1980's. This time the focus was on the newer fields of statistics and computing, which have, in the words of one participant, replaced mathematics as the handmaiden of science. Speaker after speaker, representing high school, junior college, four-year college and university mathematics, reiterated the need for a unified approach to mathematical science in which mathematics, statistics, computer science and operations research are taught together at the elementary level. While this message was supported by strong intellectual rationale-coherence, logical unity, applicability- it was also clearly motivated by students voting with their feet in favor of newer, applicable areas of the mathematical sciences. "New Math" was neither praised nor buried at the Alexandria meeting; it was, rather, replaced with the "Modern Math" of applied discrete mathematics. The problems facing mathematics education in the 1980's are enormous: new fields, new technology, open admissions, declining exam scores, dwindling enrollments, tenure limits, student pragmatism, outcry over basic skills. Together these issues form a real crisis, one that has already hit hard in certain institutions. For this reason the Mathematical Association of America (M.A.A.), with financial support from the Sloan Foundation, organized "PRIME-80," a three-day conference on April 29-May 1, during which representatives of various groups debated "Prospects in Mathematics Education in the 1980's."

The conference opened with a series of background briefings on many topics, ranging from the reasons for the decline in college entrance scores to the emerging role of statistics and computer science in higher education. Subsequent task force meetings discussed possible reactions to the various problems, and eventually the entire assembly endorsed a score of general recommendations that M.A.A. officials hope will both guide their work and influence public policy towards the mathematical sciences in the next ten years. Two issues from the secondary schools set the tone for much of the discussion: the continuing decline in basic skills and the increasing public demand for accountability from those who run the schools.

M.A.A. President Henry Alder reported that those few high schools whose records have run counter to national trends of declining scholastic aptitude (SAT) scores were those in which a special effort was made to keep students in advanced academic subjects. This evidence supported one of the conference's major recommendations, that all college-bound students should complete at least three years of high school mathematics. Accountability, the second of these two public issues, is a two-way street. Society pressures schools to provide necessary skills, while schools, to be faithful to their mission, must insure that students learn enough to function in contemporary society. Because every citizen now constantly encounters arguments based on quantitative data, inferences rooted in statistical tests, and applications of electronic computers, the conferees felt that it was now more urgent than ever to stress the important role of mathematical science in general education. Although no precise recommendation on this point could be made by such a very general commission, the conferees clearly believed that it would be wise for all undergraduate institutions to insure that students study some mathematical science. The beginning years of college pose a special challenge because of the enormous range of entry level of college freshmen: even though many students enter college having completed a full year of calculus, the fastest growing subject in freshman college mathematics is grade school arithmetic! Moreover, virtually all college mathematics programs are oriented towards calculus as the single port of entry into higher mathematics. Thus students who enter college with poor preparation (including as much as 92% of the female students) must retake prerequisite subjects for a long time before reaching anything they perceive as of significant value. This problem of delayed gratification, argues Henry Pollack of Bell Labs, results in reduced motivation and unnecessary mathematical dropouts. Several conferees urged that the college mathematics curriculum be diversified to provide multiple entries through finite mathematics, statistics, or computer science, as well as through the traditional algebra-calculus sequence. William Lucas of Cornell University suggested the possibility of even M.S. work in mathematical science with very little (if any) analysis; such a program would concentrate on applications of finite mathematics, e.g., graph theory, analysis of algorithms, minimal path problems, computation and statistics. The conferees reported that it was "extremely important" that an alternative track be designed at the freshman-sophomore level, "mindful of possible re-entry into the calculus sequence without unreasonable difficulty."

Perhaps the most controversial issue discussed at PRIME-80 was a detailed yet preliminary proposal by a curriculum committee headed by Alan Tucker of The State University of New York at Stony Brook. This committee, established by the Committee on the Undergraduate Program in Mathematics (C.U.P.M.), is preparing recommendations for an undergraduate curriculum in applied mathematical science. Because of the many societal pressures for greater emphasis on applied mathematics, the Tucker committee is inclined to recommend its program as the primary curriculum for those schools that can afford only one track, and as an alternative to the traditional mathematics major in those institutions .where several tracks can be provided. Nearly fifteen years ago C.U.P.M. proposed a general college curriculum that set standards for minimal preparation of undergraduate mathematics majors: this curriculum included 13 courses, climaxing in abstract algebra, real analysis, geometry/topology, and complex analysis. The Tucker committee's current model includes 14-16 courses, with 2-3 each in computer science, statistics and numerical mathematics. It climaxes in a "practicum" in which students gain experience in realistic mathematical modelling in a non-textbook atmosphere. Traditional upper division courses (e.g., abstract algebra, real analysis, topology, geometry) are replaced with courses in advanced linear algebra and applied analysis. These recommendations, which were debated but not explicitly endorsed by the Alexandria conferees, are rooted in recent enrollment trends.

Mathematics majors are declining in numbers, even though the demand for mathematically trained people is increasing. Moreover, 60% of undergraduate mathematics enrollments are now in applied areas, with the remaining 40% split evenly between required and elective courses. Only those students preparing for careers as high school teachers are continuing to take the traditional courses because they are required to by certification regulations. This situation, according to Tucker, is doubly dangerous: not only does it mean that future high school teachers may be ill-prepared to cope with their students' demands for new applications, but the commitment to teaching disqualifies them from other mathematics-related jobs that now universally require majors with an applied concentration. Much of the PRIME-80 discussion was dominated by a concern for employment.

I. Edward Block, Executive Director of the Society for Industrial and Applied Mathematics, reported that employers frequently have a low regard for mathematics majors because they have no real mastery of technique, they have no experience with real world problems, and they cannot communicate their results intelligibly to their employers. In industry, according to one experienced observer, "math has become a four letter word." Because of this, the Tucker committee argued strongly for development of an intense, practical experience in each undergraduate mathematics program, where students would come to grips with messy problems without preexisting answers. Programs such as the Mathematics Clinic at the Claremont Colleges (cf. Amer. Math. Monthly 84 (1977) 648-650) were frequently cited as impressive (albeit hard to duplicate) examples. To be effective, such courses must require participants to make substantive written and oral reports in appropriate nontechnical terminology. Block reported that the types of mathematical training used in industry are enormously varied, yet they all have in common the need for clear communication.

Because of the extensive nature of the curriculum changes under discussion, the conferees addressed many of their recommendations to the need for retraining of teachers at all levels. Those experienced in applied mathematics repeatedly cited attitude and value changes as more critical than technical expertise. Diverting the mathematics teacher's attention from proving theorems to creating models, from solving textbook exercises to analysing realistic problems, is a more important and more difficult re-educational task than, say, instruction in the rudiments of computer science or design of experiments. Accordingly, the conference strongly urged colleges and industries to work out cooperative arrangements whereby faculty members teaching applied mathematics could be given an opportunity to participate in a situation where mathematics is really being used rather than merely being taught. In addition, the conference will recommend to all the professional mathematics societies a substantially increased effort to establish summer institutes, short courses and similar opportunities to help faculty who wish to re-orient their interests in more applicable directions. The constant stress on application did not go unchallenged however.

Many at the conference expressed concern that if mathematics is taught solely in terms of applications and solutions to problems it may lack the coherence and structure which gives it meaning. Moreover, since problems and applications change more rapidly than theory, a student who learns how to solve only today's problems may not have an education that will serve him throughout his lifetime. Mathematics is the science of structure, so fundamental structures must in some way remain at the core of mathematics education, "for they endure throughout all changes in applications." There remains serious disagreement concerning the means by which these various objectives can be reached. Undoubtedly different schools will choose different strategies, some emphasizing applications over fundamental structures, others doing the reverse. Most PRIME-80 conferees were acutely aware of the value of diversity in higher education, and were careful to avoid recommendations that would appear incompatible with this diversity. But they did agree on certain basic principles, and hope that departments will transform these principles at their own institutions into programs of mathematical science that more closely reflect the demands of science and society in the coming decade.

Further information on PRIME-80, including a complete set of the conference recommendations, can be obtained by writing to Al Willcox, Executive Director, Mathematical Association of America, 1529 Eighteenth Street, N. W., Washington, D.C. 20036.

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