Mathematics: Our Invisible Culture

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The occasion of the Tome Centennial compels one to compare the present with the past. Today mathematics is abstract yet powerful, pervasive yet invisible, and predominantly American. One hundred years ago it was less abstract and therefore less powerful, less pervasive but perhaps more visible, and exclusively European. The largely invisible rise of American mathematics from nineteenth century impotence to twentieth century leadership is an extraordinary accomplishment, a quiet revolution as impressive as those of quantum physics and molecular biology, yet largely hidden from public view.

The evidence for the revolution is all around us. High speed computers, fuel-efficient airplanes, world-wide communications are visible technological products of contemporary mathematics. Less visible but more fundamental are deep insights such as gauge field theory, nonlinear dynamics, and computational complexity that provide unifying foundations for modern science. We live in a "minds-on" world created by the abstract theories of contemporary mathematics--yet hardly anyone really knows that mathematics even exists.

A well-known American mathematician reported an incident--all too typical--in which a new acquaintance asked his wife what her husband did. "He is a mathematician," she said. The acquaintance, confused, responded after a slight pause: "Well, what *else* does he do?" Surely, she thought, no one can keep busy just "being" a mathematician.

Tomorrow one of the Tome Symposia is devoted to C. P. Snow's classic definition of the two cultures--humanistic and scientific. One can, I believe, extend Snow's metaphor from two cultures to three: humanists are to scientists as scientists are to mathematicians. Educated laymen feel obliged to know something of Shakespeare and Mozart, of Eliot and Picasso; they are comfortable with only the most superficial (and usually distorted) acquaintance with the works of Darwin and Einstein, not to mention Pauling and Feynman. But they have not even heard of Gauss and von Neumann, or Hilbert and Poincare.

The only mathematics visible to our culture is the mathematics of the past--the completed masterpieces of Euclid and Newton, of geometry and calculus. Everyone knows that *this* mathematics exists, and that it is centuries old. No one knows that modern mathematics exists, much less that it is being continually created. Mathematics is indeed our invisible culture.

A Profile of the Past

In the nineteenth century mathematics existed only in Europe, but it was actively pursued in every country of Europe. In France and Germany Augustin Cauchy and Karl Weierstrass articulated the grand theories of what we now call "classical analysis", the mature form of Newtonian calculus. Joseph Fourier launched an analytic investigation of heat that has led to the vast modern enterprise of harmonic analysis, and James Clerk Maxwell used the mature calculus to spell out the laws of electrodynamics.

Karl Frederich Gauss in Germany, and Nicolai Lobachevski in Russia shocked the literate world with the discovery of geometries that do not obey the laws of Euclid, while Niels Henrik Abel in Norway and Evastois Galois in France used primitive ideas of group theory to help resolve all the famous unsolved problems left over from the Greek classics. The nineteenth century ended on a grand climax, with the German mathematician David Hilbert, whom many believed to be the greatest mathematician of the time, outlining for an International Congress in Paris the major problems that nineteenth century mathematics was passing onto the twentieth century. By now, nearly all of Hilbert's problems have been solved--but neither the problems nor their solutions are visible to the typical educated person.

In contrast to Europe, during the nineteenth century mathematics in America meant little more than arithmetic. Up until the Civil War, colleges like Princeton, Williams, and Yale required of entering students only "vulgar" arithmetic--the arithmetic of fractions, not decimals. By the last quarter of the nineteenth century, when most of the Midwestern land-grant institutions and liberal arts colleges were being founded, the leading institutions in the East recommended "algebra through simple equations" and "two books" of Euclid for admission.

The undergraduate curriculum reflected the entrance requirements: in 1890 Princeton required of mathematics students two years of coursework in geometry, trigonometry, analytical geometry, navigation, and mensuration, leaving calculus as an upper division elective along with a few other advanced courses. Not until the 1870's when the eminent English mathematician J. J. Sylvester came to Johns Hopkins did the United States have *any* person with training equivalent to a Ph. D. in mathematics. (Sylvester's emigration was instigated by anti-Jewish laws that prevented him from accepting an appointment at Oxford or Cambridge--an uncanny foretoken of the history of American mathematics which was later to be immeasurably enriched by hundreds of European mathematicians, mostly Jewish, fleeing Nazi persecution.)

The state of American mathematics in 1885 reflected the pragmatic needs of a new nation. Post-revolutionary attitudes stressed isolation from Europe, and rewarded productive, action-oriented, nation-building work. Teaching loads were typically 20 hours per week or more; 70% of those teaching mathematics taught as well a wide variety of other subjects--not just related sciences either. Knowledge for its own sake, the liberal education that Cardinal Newman described as "sufficient for itself, apart from every external and ulterior object" was not valued in nineteenth century America. Mathematics in that era was a utilitarian enterprise for a pre-scientific society.

The Roots of a Revolution

Today we are enveloped in (and may be consumed by) a society dominated by science and technology. Yet our roots are still pre-scientific. Contemporary Americans know mathematics primarily through their own school experiences, and through their children's experiences. Consequently, they view the "basic" mathematics of arithmetic, algebra, and elementary geometry from the same perspective as did their nineteenth century forefathers--as a pragmatic necessity to prepare for a productive life.

The "new math" introduced into the schools two decades ago did not fit this perspective. Abstract sets--the foundation of European mathematics-- replaced the concrete numbers of everyday arithmetic, yet liberal learning did not replace pragmatism as the guiding principle of American education. American culture rejected the "new math" as surely as the human body rejects foreign tissue.

It is both fitting and ironic that the centennial we celebrate today is also the centennial of Georg Cantor's monumental theory of infinite sets. In an intensive burst of creative energy, rare among scientists of any era, Cantor developed between 1879 and 1884 virtually the entire content of his revolutionary theory of sets. Hilbert called Cantor's work a paradise, while others called it bizarre. Yet no one today disputes its character or significance: the entire edifice of twentieth century mathematics rests on the foundation laid down by Cantor.

Controversy is no stranger to Cantor's theory of sets. Cantor's contemporaries such as Kronecker and Brouwer rejected many parts of it, complaining about the "monsters" that it had let loose in the pristine garden of mathematics. Yet the judgment of a century's research is that set theory made possible the abstractions of twentieth century mathematics, which in turn have yielded enormous applications in science and technology. It is ironic that this key tool of modern mathematics, born in Europe just a hundred years ago, became in America the defaced symbol of the "new math" movement--a vivid reminder of our anti-intellectual nineteenth century roots in American pragmatism.

A New Look at New Math

Cantor's work grew out of very pragmatic stuff--the attempt by Joseph Fourier to understand the transmission of heat by decomposing general mathematical functions into sums of sine and cosine waves. This technique, now known as Fourier analysis, serves today as the basis for communication engineering, quantum physics, and numerous parts of abstract mathematics. But for Cantor it posed a strong intellectual puzzle: at some points, it failed to work. He chose to study these points of failure, or as the mathematician puts it, points of non-convergence.

As Alice following the White Rabbit into wonderland, Cantor emerged from this tiny entrance into a world of unimaginable behavior--of different sizes of infinity, of sets and functions that defied intuition. Instead of being smooth and predictable, Cantor's creatures were twisted and chaotic. In a prelude of the recent "new math" debate, scientists and mathematicians took sides in a vigorous argument about whether these new ideas were of any practical use, or were just figments of a distorted imagination.

I don't know if the Greek exploration of conic sections--of parabolas, ellipses and hyperbolas--generated controversy about utility, but we all know now how Kepler and Newton found in these ideas the apt language to express the laws of planetary motion. I do know that the introduction by Gauss and Lobachevski of non-Euclidean geometries was highly controversial--so much so that Gauss was unwilling to make his discovery public. Yet in less than a century the warped spaces of these geometries proved to be the appropriate mathematics for Einstein's theory of relativity.

So too with Cantor's sets. At first only mathematicians appreciated them. They were useful--indeed, essential--as the foundation of modern mathematics. The logical architecture of all contemporary mathematics and computer science rests on the theory of sets. Most mathematicians and computer scientists, however, do not work on the foundations: they work in the upper stories, adding new rooms and refining the plumbing. They take set theory for granted, as a convenient language that helps support their work.

In recent years, however, mathematics has turned its attention from the study of order to the study of chaos. Whereas the classical mathematics of Newton explains the orderly, predictable world that Fourier explored, it is unable to explain the turbulence that occurs when air passes over an airplane wing, or when blood flows through a heart valve. Gradually, in the course of the last two decades, mathematicians realized what Cantor had seen dimly a century before: that ordered systems can create chaotic behavior, and that full understanding of complex systems requires a thorough understanding of the bizarre as well as the normal.

Benoit Mandelbrot has been the leader in this new type of mathematics, known as the theory of "fractals". Mandelbrot realized that one way to understand the strange behavior of some of the sets that Cantor and his successors had discovered was to view them as living in a fractional dimension--whence "fractals". The indefinite wiggliness of a rocky coastline or of the membranes of the lung are natural phenomena best modeled by fractional dimensions: the coastline is more than a bent one-dimensional line, but far less than a two-dimensional region, whereas the lung membranes are two-dimensional surfaces that fill the three-dimensional lung. The mathematical theory of fractals, which has led to stunning computer-generated pictures of artificial worlds, is a fulfillment of the initiative begun by Cantor in exploring the exceptional sets that originally existed only in his mind's eye.

From Abstraction to Application

Computers also began as only a gleam in the mind's eye--of Gottfried Leibniz and George Boole, of Alan Turing and Emil Post. Most persons today think of computers when they think of mathematics, often for inappropriate reasons. Nevertheless, the innards of a computer are really nothing more than the actualization in rapid electronic switches of basic logical principles articulated also in the nineteenth century--by the English mathematician George Boole.

Like Cantor, Boole studied sets. But whereas Cantor pursued the bizarre and exceptional properties of sets, Boole concentrated on similarities, and regularities--on what mathematicians call isomorphisms. He showed that basic operations on sets such as intersection and union could be mirrored in algebra, with addition and multiplication serving as the basic operations. From this it became clear that set membership can be coded most easily in binary arithmetic--1 represents inclusion, 0 exclusion. Hence binary numbers joined Boolean algebra as devices for calculating with the abstract structures of sets.

Uncannily, the abstraction of Boole turned out to be precisely the tool required to model computer circuits. Now beginning students of computer science learn "masking" as a way of mixing electrical signals to reflect algebraic operations, and "shifting" as a mechanism for multiplying in binary arithmetic. As Cantor's sets provided the intellectual foundation for modern mathematics, Boole's algebras provided the electronic representation that enabled machines to embody this mathematics.

Nowadays every school child learns a bit of the theories of Boole and Cantor. Theories originally perceived as exotic and fanciful are indeed the foundation of our computer age. But too often we seem to miss the fundamental lesson: today's abstractions become tomorrow's applications.

Heart valves and computers are part of the fallout of abstract set theory. Other applications abound in modern science and industry: satellite communications use abstract number theory; robotics uses algebraic geometry; particle physics relies on group theory; oil refining uses projective geometry. The list could go on and on. The great lesson that mathematics has to contribute to liberal education is that the most abstract ideas are the most powerful and the most abstract thinkers the most versatile.

Mathematics in the Curriculum

That theory can be pragmatic runs against the grain of our nineteenth century roots, and against the prevailing spirit of American education even today, 100 years later. Idealists cling to the classical traditions of liberal education--to awaken one's intellect, to develop one's imagination, and to marshal one's spirit. But students in overwhelming numbers choose pragmatism: business administration and engineering are more popular on today's campuses than all the liberal arts and sciences combined.

Because mathematics can be practical, it is generously supported in public opinion of educational expectations. But what the public wants is the mathematics of the checkbook: how to balance it, and how to fill it. The abstract mathematics that is rooted in the theories of Cantor and Boole, although essential for any sophisticated practical application, is not in the public view: *this* mathematics is part of our invisible culture.

The fact that not even educated persons know about deep mathematics since the age of Newton renders current discussion of the role of mathematics in liberal education particularly incoherent. Large numbers of students study mathematics to advanced levels for its direct practical benefit--it helps them pass chemistry, which will help them get into medical school. Still others substitute computer science for mathematics, believing naively that one can master these machines without learning their language. Those majoring in the humanities and arts frequently take mathematics only under duress, suffering through a required "Mathematics for Liberal Arts" course that provides a mix of games and trivial applications selected not so much for their centrality to mathematics as for their lack of prerequisite knowledge. And in some institutions students are now required to pass a mathematical or quantitative literacy examination whose level usually approximates the syllabus of a good junior high school mathematics course.

This chaos is not all the fault of higher education. In no other subject is the background of entering freshman so diverse. On the one hand, about a hundred thousand students enter college each year with some calculus under their belt; the very best of these students--a few dozen in the whole country-- are prepared, as freshmen, to begin graduate work in mathematics.

On the other hand, each year a hundred thousand workbooks on arithmetic "for college students" are published and sold in the United States--covering the standard mathematics syllabus of elementary school. In some respects we are right where we began a century ago, trying in vain to require knowledge of "vulgar arithmetic" for entrance to college.

Overall, the prevailing role of mathematics in American higher education is little changed from nineteenth century America. Students study mathematics for its immediate practical benefit, not as part of the scholarly and cultural heritage of the liberal arts and sciences. This attitude--which we as teachers of mathematics exploit because it keeps our classes full--maintains the veil of secrecy that has insulated the educated man from even passing acquaintance with modern mathematics.

The Arcane Art of Mathematics

Jerome Bruner said somewhere that an educated man must not be dazzled by the myth that advanced knowledge is the result of wizardry. Unfortunately, mathematicians and educators conspire to maintain this myth for mathematics. Students don't want to understand why it works: they only want to know how to use it. Mathematicians don't want to make the effort to explain their field to laymen; they are content to do their research and talk to other experts. The result, predictably, is a public perception of mathematics as inexplicable and arcane.

In alchemy, the arcane represented a profound secret of nature. Indeed, in this age, most profound secrets of nature are expressed in mathematical terms. Because the alchemists always associated great mystery with the arcane, it soon came to symbolize as well an elixir, a type of marvelous remedy. The same thing has happened in this age: many scientists, especially social scientists, find that the best remedy for an ailing theory is a mysterious dose of numbers and statistics. Mathematics provides for soft science what one mathematician described as "mystification, intimidation and an impression of precision and profundity." Mathematics is the elixir of the scientific age.

Despite the significance of the mathematical sciences in our technological society, the distance between the research frontier and public understanding is probably greater in mathematics than in any other field of human endeavor. In virtually all other areas of science, the educated public is aware in a rudimentary fashion of major twentieth century contributions: most people have at least a vague understanding of black holes, genetic engineering, and microprocessors, even though they neither understand nor care to understand such things in detail.

In contrast, public vocabulary concerning mathematics is quite primitive: it is not a decade, not a century, but a millennium out of date. Explaining what is actually happening in contemporary mathematical science to the average layman is like explaining artificial satellites to a citizen of the Roman Empire who believed that the earth was flat.

The typical public attitude towards mathematics is an anomalous mixture of disinterest and awe. Although the average citizen speaks in wondering tones about his genius nephew who scored 800 on his mathematical aptitude test, he appears proud of his own ignorance of things mathematical: "I never did understand percentages." Even well-educated people who wouldn't dare admit in public that they have never heard of Keynesian economics will brag of their lack of understanding of statistics or calculus. By and large non-mathematicians do not value mathematical knowledge enough to regret their ignorance of it. For the most part, the average citizen is content to leave its arcane workings to an inner sanctum of wizards.

Mathematical Literacy

Ten years ago the astronomer Benjamin Shen identified three aspects of literacy in science--practical, civic, and cultural. These levels apply as well to mathematical literacy and provide a convenient framework for understanding the challenges we face.

Practical literacy is that knowledge that can be put to immediate use in improving basic living standards. The ability to compare loans, to figure unit prices, to manipulate household measurements, and to estimate the effects of various rates of inflation brings immediate real benefit.

Civic literacy involves more sophisticated concepts, namely those that would enhance public understanding of legislative issues. Major public debates on nuclear deterrence, economic policy, and public health frequently center on scientific issues. Inferences drawn from data, projections concerning future behavior, and interactions among variables in complex systems involve issues with essentially mathematical content. A public afraid or unable to reason with figures is unable to discriminate between rational and reckless claims in the technological arena.

Cultural literacy, the most sophisticated of these three levels, involves the role of science or mathematics as a major intellectual achievement. Because cultural literacy lacks an immediate, practical purpose, its appeal will be limited largely to a small subset of the intellectual community. When one considers that the total readership of cultural magazines such as *Harpers* and *Scientific American* is about one-half of one percent of the U.S. population, a cultural approach to mathematical literacy will hardly contribute much to general public understanding of esoteric research. Yet, to be honest, this is the only level on which the arcane research of twentieth century mathematics can truly be appreciated--as an invaluable and profound contribution to the heritage of human culture.

Liberal Education

Liberal education should be one of society's most effective tools for promoting cultural literacy--whether about art or literature, science or mathematics. It is certainly not the only such tool, but it is the most important one since it provides for most people the first significant exposure to our cultural traditions. When successful, liberal education renders the arcane plain, and reduces wizardry to understanding. Unfortunately, especially when dealing with the mathematical sciences, it too often does just the opposite.

Hardly anyone today would claim that liberal education is particularly effective in promoting mathematical literacy on any of Shen's three levels. Instead of emphasizing major current issues and research frontiers, texts for those courses whose central purpose is mathematical literacy, (that is, survey courses for humanities students) focus on elementary and trite topics designed to illustrate the precision of definition, theorem, and proof. Perhaps the only thing worse than texts on mathematical literacy are those on computer literacy: whereas the former dwell excessively on the arcane and eccentric, the latter dwell primarily on the ephemeral and inconsequential.

Too often these days the general public views computer literacy as the appropriate modern substitute for mathematical knowledge. Unfortunately, this often leads students to superficial courses that emphasize vocabulary and experiences over concepts and principles. The advocates of computer literacy conjure images of an electronic society dominated by the information industries. Their slogan of "literacy" echoes traditional educational values, conferring the aura but not the logic of legitimacy.

Typical courses in computer literacy, however, are filled with ephemeral details whose intellectual life will barely survive the students' school years. These courses contain neither a Shakespeare nor a Newton, neither a Faulkner nor a Darwin; they convey no fundamental principles nor enduring truths. Such courses do not leave students well prepared for a lifetime of work in the information age.

What is Mathematics?

My concern, however, is not so much with computer literacy as with mathematical literacy. It is common now to include under the umbrella of mathematics (or mathematical sciences) such diverse quantitative and theoretical disciplines as logic and statistics, mathematical economics and theoretical physics--in addition, of course, to the traditional core of mathematics as defined by the school subject of that name. The term "mathematical sciences" is in some sense a more accurate term, juxtaposing as it does the implied tension of *a priori* reasoning characteristic of mathematics with the *a posteriori* reasoning that characterizes empirical science.

The mathematical sciences are a diverse and very loosely knit collection of pure and applied disciplines united only by a special focus on abstract structure. They include:

- *Analysis*, the representation of continuous change growing out of calculus that forms the fundamental tool of classical engineering and mathematical physics;
- *Statistics*, the theoretical basis for medical research, environmental studies, and political polls;
- *Mathematical Logic*, the theoretical basis of computer science, as well as the foundation of mathematical truth;
- *Operations Research*, the application of mathematical techniques to problems of industrial and economic optimization;
- *Group Theory*, the abstract representation of symmetry, now used to model the structure of crystals and to organize the fundamental constituents of matter;
- *Computer Science*, the study of algorithms, programming languages, and data structures;
- *Graph Theory*, the representation of relationships required for computer design, information networks, and transportation systems;
- *Topology*, the abstract study of geometric form, now used to explore the geometry of the universe, the evolution of living things, and the dynamics of the economy.

The point of this list is not completeness but variety. Mathematics today is more than just algebra, calculus and Euclidean geometry. The mathematical sciences are a vast, sprawling complex of subjects united more by research methodology than by common content. Although their influence on society is frequently hidden from public view, the mathematical sciences have shaped our world in fundamental ways and continue to exert profound yet indirect influence in virtually every aspect of our daily lives.

Just as two centuries ago, before Darwin, the educated public believed that forms of life were static, so today the educated public assumes that the forms of mathematics are static, laid down by Euclid, Newton and Einstein. Students learning mathematics from contemporary school and college textbooks are like the pupils of Linnaeus, the great eighteenth century Swedish botanist: they see a static, pre-Darwinian discipline that is neither growing nor evolving. Learning mathematics for most students is an exercise in classification and memorization, in labeling notations, definitions, theorems, and techniques that are laid out in textbooks as so much flora in a wondrous if somewhat abstract Platonic universe.

Students rarely realize that mathematics continually evolves. Notations change; conjectures emerge; theorems are proved; counterexamples are discovered. Indeed, the passion for intellectual order combined with the pressure of new problems-- especially those posed by the computer--force researchers to continually create new mathematics and archive old theories.

Here, for example, is a brief list of some major accomplishments of mathematics in the last decade. For non-specialists it will appear largely as a list of names and topics without special significance. These advances in the mathematical sciences are as important and as profound as the comparable discoveries of the same decade in cosmology and genetic engineering, but they have been largely invisible to the educated person:

1975 Effective solution of the Radon transform, leading to the computerized axial tomography (CAT) scanners.

1976 Solution to the Four Color Problem, the first major theorem of mathematics to be proved with essential aid of a computer.

1977 Beginning exploration of fractals, the world of intermediate dimension that yields fantastic graphic representations of iterative processes.

1978 Solution of the famous Serre conjectures in algebraic number theory.

1979 Discovery by a Russian mathematician of a new "ellipsoid" algorithm for solving linear programming problems.

1980 Completion of the century-long quest for a complete classification of the finite simple groups.

1981 Proof of the four-dimensional Poincare conjecture, which led to the discovery of new models for space-time not isomorphic to the ordinary one.

1982 Implementation of "public key" cryptography schemes based on one-way algorithms in number theory and combinatorics.

1983 Proof of an old conjecture by Mordell concerning the number of integer solutions to polynomial equations.

1984 Solution of the Bieberbach conjecture concerning the rate of growth of coefficients in analytic functions.

1985 Development of a new method based on elliptic curves for factoring vary large numbers.

The symbiosis of computer science and mathematics provides powerful evidence that mathematics is not just a static discipline, but is continually created in response to challenges both internal and external. Students today, even beginning students, can learn things that were simply not known twenty years ago. We must not only teach new mathematics and new computer science, but we must teach as well the fact that this mathematics and computer science is new. That's a very important lesson for students to learn.

Focus on the Significant

Learning that mathematics is alive is not, however, sufficient. We must also prepare our students to live with mathematics, and with those who use it. The issues of greatest mathematical significance in liberal education are those surrounding the question of wizardry: how can we insure that educated men and women are not left defenseless by the thousands of mathematical wizards who control the mechanisms of modern science and society? There are many ways to do this, none of them particularly easy:

- Emphasize fundamental understanding of important principles from various parts of mathematics--the central limit theorem of statistics, the fundamental theorem of calculus, the duality principle of linear programming. Deep ideas have lasting importance and resonate with related results in other fields.
- Stress important links between mathematics and other disciplines of liberal learning. The mathematical sciences are used not only in physics and engineering, but also in such

diverse fields as linguistics and biology, economics and cognitive science, politics and art.

- Focus on big issues, to separate the forest from the trees. Such contemporary endeavors as artificial intelligence and cognitive science use mathematical principles to promote innovative research in the nature of mind and the capabilities of cognition. Interdisciplinary fields such as these are attractive to students and often employ significant mathematical components as abstract bridges that connect different scientific islands.
- Build mathematical models into the teaching of science. To present science without mathematics is to present results without reasoning, conclusions without evidence. Doing this fails to communicate the natural symbiosis between the scientific method and mathematical modeling, and distorts in the student's mind the nature of scientific inquiry.
- Examine ethical issues involved in the applications of mathematics. Who is responsible when decisions that affect social and military policy are based on computer projections that in turn have been developed by programmers and mathematicians who introduce hidden assumptions and unwitting errors? How can democracy function if the bases of facts and decision are veiled by the secrecy of technical mathematics?
- Stress the complexity of complex systems, showing how order can create disorder (turbulence), and vice versa (stochastic processes). The study of complexity has emerged as one of the major interdisciplinary themes of scientific research, but to be more than fluff it must be rooted in solid mathematics.

We should, in short, emphasize in liberal education the human and humane parts of mathematics. As computers gain greater facility in doing routine calculations, major parts of what students usually learn from the traditional mathematics curriculum will in the future be done by computer programs. So there is simultaneously less need for individuals to carry out routine tasks, yet increased need for individuals of global vision who understand the behavior of complex systems.

It is wrong but all too easy to focus a major part of undergraduate mathematics on topics that only teach students to do what computers can do better. It is insufficient if not deceitful to focus instead on how to use computer systems and interpret their results, since too often this practice substitutes one set of wizards (programmers) for another (mathematicians).

For the vast majority of students interested in fields other than the mathematical sciences we must develop a new approach that is adequate to educate a generation of leaders who will live and work in an environment dominated by computers. Our students need the qualities of

Literacy--to communicate with the wizards

Confidence--to engage difficult technical issues

Skepticism--to ask the right questions

Persistence--to insist on appropriate answers

Judgment--to select what is right

I wish I knew how to provide these qualities, but I don't. Perhaps in tomorrow's workshops you can make a start on this agenda.