Twenty Questions for Calculus Reformers

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There is little doubt that calculus is the central subject in the mathematics curriculum. Every student with serious aspirations for a career in science, engineering, or business takes a crack at it. The mathematics curriculum of the high schools is focused on calculus as a target, and most mathematics-based courses in college use it as a common foundation. Each year about 500,000 students in the United States study calculus, about half the number who will eventually graduate from college. Calculus is big business with far reaching consequences for students, for colleges and universities, for the mathematical community, and for our nation.

The occasion of a conference on the role of calculus brings to mind a host of questions that need to be addressed. In advance of the Tulane conference, I posed 20 questions that I hoped would help stimulate discussion of important issues. At the conference few questions were answered, but more were added. So these 20 questions have now grown to 28!

1 *Should fewer students study calculus?* Perhaps society would be better served if more students were introduced instead to statistics, matrix algebra, discrete mathematics, or computer science. Does the mystique of calculus as the unique gateway to mathematics obscure the comparable value of collateral subjects?

2. *Is calculus an appropriate filter for the professions?* The majority of students who study calculus do so en route to careers in business, medicine, and law where calculus *per se* will almost never be required. Is calculus really the best means of assuring analytical skills for such students?

3. *Will computer science dethrone calculus?* For the first three quarters of this century, calculus has been *the* gateway to all significant college-level mathematics. Now that computer scientists urge discrete mathematics rather than calculus for freshman-level study, students progressing through typical university science and engineering curricula can no longer be assumed to be literate in calculus. If knowledge of calculus is not assumed in advanced courses, will the motivation to study it remain strong?
4. *Do students really learn the major ideas of calculus?* Physicists employ calculus as the language of science; philosophers talk of it as the rosetta stone of the scientific age; historians see it as the culmination of a millenium of investigation. But all students usually see (and are examined on) are calculations of derivatives, integrals, differential equations, and series. Is this because the average U.S. calculus student is not intellectually ready to understand the central ideas of calculus?
5. *Has calculus become a cookbook course?* In typical texts, exposition is subservient to template examples and carefully planned exercises. Routine exercises dominate tests,

most being entirely within the capabilities of symbolic algebra programs. Do students really learn the fundamental structure of calculus from courses dominated by calculation?

6. *Does calculus focus excessively on closed-form formulas?* Calculus in action deals with functions as they are—from laboratory instruments, from graphs, and from messy formulas. Calculus in courses deals with shadow functions—sterile, simple, artificial constructs designed to make the exercises work out well. Will students for whom the main clue to an error is a messy answer ever be able to make reliable use of mathematics in the real world?
7. *Should calculus students learn to use or to imitate computers?* Symbolic mathematics packages (MACSYMA, SMP, MAPLE, MUMATH, etc.) will soon be so widely available that algebraic work will be done as arithmetic is now—at the push of a button. How soon will this change the way in which students use their homework time in a typical calculus course?
8. *What new topics are essential for calculus in a computer age?* Asymptotic analysis, spline calculations, numerical analysis, and scientific computation have changed the way calculus is used in the scientific and engineering communities. Should the content of university calculus reflect these changes?
9. *Which topics in calculus are no longer essential?* Calculus is now over-filled, because topics are always added but rarely removed. Many topics remain only because of tradition or because they are useful for some advanced course—not necessarily because they are crucial to the course itself. Are integration techniques any more useful than the square root algorithm?
10. *Do engineers still need the traditional calculus?* The perception, whether justified or not, that physics and engineering students must be taught the full "engineering calculus" in its traditional form impedes curricular reform now being demanded by computer scientists. Is it possible that even students in the physical sciences could benefit from a new blend of discrete and continuous mathematics in the first two years?
11. *Should calculus be a laboratory course?* Computer graphics now make possible a visual presentation of many of the dynamic phenomena studied in calculus. This unprecedented capability suggests wonderful pedagogical possibilities. Can we afford to provide every calculus student with access to a powerful workstation for calculus learning? Can we afford not to?
12. *Is there any reason to teach high school calculus?* About ten years ago MAA and NCTM adopted a joint statement urging high schools to refrain from offering calculus unless it can be taught at a university level—staffed by a qualified instructor, and enrolled by qualified and capable students. Despite this position paper, the majority of high school calculus courses are still thin introductions, inadequate to provide advanced placement into college courses. Shouldn't the only calculus taught in high school be university calculus?
13. *Why do U.S. students perform so poorly on international tests?* On the 1982 International Mathematics Assessment, only 30% of U.S. high school calculus students could select the correct answer out of five choices for the integral of a linear function presented in graphical form. On the whole, the top 2% of our students performed only at the median of the top 10% of the students in other countries. On the Graduate Record Examination, which is largely based on advanced calculus, foreign-educated students

average one standard deviation higher than U.S. educated students. Are U.S. students less able, or less well prepared?

14. *Is there any value to precalculus remedial programs?* Much of the effort in remedial work is devoted to repeating early parts of algebra that are needed for the study of calculus, a fantasy that is totally beyond the grasp of most students in these courses. Would exploratory data analysis or elementary programming be better for these students, even though it unhitches them from the calculus bandwagon?
15. *Why do calculus books weigh so much?* The economics of publishing compels authors of calculus textbooks to add every topic that anyone might want so that no one can reject the book just because some particular item is omitted. The result is an encyclopaedic compendium of techniques, examples, exercises and problems that more resembles an overgrown workbook than an intellectually stimulating introduction to a magnificent subject. Would the health of calculus be improved if it were put on a diet?
16. *Can one design a good calculus course from a survey?* Publishers now are fond of surveying calculus instructors to determine the precise blend of topics for their next calculus book. Responses to these surveys never represent a random selection, and are rarely informed by active users of calculus. Can this process possibly produce progress, or is it doomed to reflect the *status quo?*
17. *Is calculus a good course to train the mind?* Many students take calculus not for its content but for its reputation of rigor: for many it is a modern equivalent of Latin or Greek—a means to train the mind. Professional schools repeatedly use calculus as a filter, to identify students who have the right stuff. Are these perceptions warranted by the results?
18. *Can calculus courses convey cultural literacy?* Calculus is one of the great intellectual achievements of mankind, with a distinguished history of theory and applications. Few texts and few teachers can communicate much of that cultural impact in a course crammed with techniques and theory. Do courses that fail to impart the cultural significance of calculus do justice to the aims of liberal education?
19. *Does calculus contribute to scientific literacy?* Since for the majority of educated citizens calculus is the climax of their mathematical studies, one might hope that it would leave them with a good appreciation of the role mathematics plays in society. Is calculus really the best college mathematics course to prepare educated citizens to function thoughtfully in the world of the 21st century?
20. *What will calculus be like in the year 2000?* The topical outline of calculus courses today is little changed from the early textbooks around 1700. Stability of fundamental ideas speaks to the enduring value of the subject. But the world for which we are preparing our students is profoundly different than it was three centuries ago. Will calculus be able to adapt to these differences?

These were the original 20 questions, posed in advance to those who attended the conference. The ensuing discussion of the many papers produced more questions than answers, and led me to posit these additional queries:

21. *Do students ever read their calculus books?* The shape of university calculus is largely determined by massive mainstream calculus compendiums which students are required

to purchase by virtue of university-wide adoption. But do students actually read these books? Don't most students learn instead by doing problems and by discussing mathematics with other students? Would these gargantuan books sell if they were not required?

22. *Should precalculus be a prerequisite for calculus?* Too often students enter calculus without having completed a thorough four-year preparatory course of high school mathematics. The result is frustration, failure, and wasted time for both student and teacher. Shouldn't placement standards be enforced so that only those who are proficient in precalculus would enroll in calculus?
23. *Is teaching calculus most like teaching a foreign language?* It used to be assumed by administrators and teachers alike that mathematics instruction required the same intensive one-on-one interaction that takes place in foreign language classes. Isn't this still true? Wouldn't more learning take place if calculus were taught like beginning intensive French?
24. *Should the student-faculty ratio for calculus be limited?* Calculus is taught in many different class structures, from 10-student classes to 500-student lectures. There are examples of success and examples of failure at virtually all class sizes. But is there any alternative to regular, detailed feedback on student problem-solving efforts? Is there a minimum instructional effort, however it is packaged, that is necessary to insure success in teaching calculus?
25. *Do student evaluations favor calculation-based courses?* The decline in student interest in mathematics roughly parallels the increasing standardization of calculus texts, course structures, and student evaluations—all in the direction of mimicry mathematics. Is it possible that student evaluations have become an evolutionary force favoring calculation-dominated courses?
26. *Are there enough qualified calculus teachers?* To maintain quality of content and excitement of purpose, calculus teachers should be active users of the subject. But to insure good teaching, they must be interested in their students and dedicated to good pedagogy. Are there enough teachers who meet both standards? Are there any?
27. *Who will be the calculus teachers in the year 2000?* Increasing demand for mathematically trained persons coupled with a rising college population in the late 1990's will push demand for calculus to record levels. But retirements from post-war faculty will also be high, and supply of new faculty—recruited from the presently depleted classes of high school and college mathematics majors—will be very low. Will calculus teaching be left to those without advanced training in mathematics—or perhaps to computers?
28. *Should calculus be taught only by experienced teachers?* Consistent reports from different institutions suggest that calculus too often suffers from poor teaching: inexperienced and often inarticulate teachers, excessive failure rates, disillusioned client departments, counterproductive departmental policies all point to a climate of neglect in which the most teaching is done by those with the least experience. For the good of mathematics, and for the good of the nation, shouldn't we make sure that only the very best teachers teach calculus?