Review of
The Evolution With Age of Probabilitistic, Intuititively Based, Misconceptions

Milo Schield, Augsburg College

Review of "The Evolution With Age of Probabilistic, Intuitively Based, Misconceptions" by Efraim Fischbein and Ditza Schnarch. Tel Aviv University School of Education.

Background: In studying students understanding of infinity, Fischbein noted a decrease in the proportion of misconceptions as age increased.
Finding: As students age, the proportion of misconceptions concerning statistics and probability may decrease, stay constant or even increase.

Intuitively based misconceptions are a result of
1. we have limited experience
2. we tend to explain associations in terms of causation
3. we tend to focus on easily available data
4. we need time developmentally for our operational abilities to mature

"We assumed that the impact of logical constraints on intuitions may increase, with age. If intuitions present a certain evolution with age, one may expect that the strength and frequency of intuitively based misconceptions should diminish as the subject grows older. As we shall see further on, the above assumption was found to be too simplistic when compared with the reality of experimental findings."

Subjects: Five groups in grade 5, 7, 9, 11 and college. 20 students per group except 18 in college. The college students were all prospective teachers but none had any training in probability.

Instrument: Seven probability problems -- all of them related to well-known probabilistic misconceptions. These seven problems investigated seven misconceptions:

1. Representativeness: Question 1
   "In a lotto game, one has to choose 6 numbers from a total of 40. Vered has chosen: 1,2,3,4,5,6. Ruth has chosen: 39, 1, 17, 33, 8, 27. Who has the better chance to win?"
   A. Vered B. Ruth C. Verred and Ruth have the same chance to win

   Results: No one felt that Vered had the better chance to win. The proportion of those who thought Ruth had the better chance to win declined steadily from 70%(grade 5) to 22% in college.

   Student's explanation: A random set of numbers has a better chance of winning that a non-random set.
   Evaluator's reply: all sets of numbers are really random.

2. Negative and Positive recency Effects Question 2
   When tossing a coin, one gets one of two possible outcomes: Either heads or tails. Ronni flipped a coin three times and in all three cases "heads" came up. Ronni intends to flip the coin again. Is the chance to again get "heads" the fourth time, smaller, equal or bigger compared with the chance of getting "tails"?"

   Results: The correct answer (equal) increased steadily with age from 40% (5^{th} grade) to 94% (college). The most common misconception (negative recency effect or gambler's fallacy) started at 35% and decreased to 0 by age. Surprisingly, the positive recency effect was almost absent.

   Student's explanation: If justifying negative recency, "As the number of heads increases, the chances of tails also increases since the likelihood of heads and tails are equal". In justifying positive recency, "Since Ronni got heads three times successively, his direction of launching is heads".

3. Compound and Simple Events Question 3.
"Suppose one rolls simultaneously two die. Which of the following outcomes has a bigger chance (encircle the answer you consider to be correct)?:
A. To get the couple 5-6
B. To get the couple 6-6
C. Both outcomes have the same chance

Results: No one selected "A". 10% to 16% selected "other answers". The proportion selecting "C" remained steady with age at about 75%.

Evaluator's reply: This is the only stable misconception identified by us in the present research. The correct evaluation seems to require a change of perspective which does not emerge spontaneously on the basis of the natural intellectual development.

4. The conjunction fallacy: Question 5
"Don dreams of becoming a doctor. He likes to help people. When he was in high school, he volunteered for the "Red Cross" organization. He accomplished his studies with high performances and served in the army as a medical attendant. After ending his army service, Dan registered at the University. (Encircle the answer which seems to you to be more likely):
A. Dan is a student of the medical school
B. Dan is a student.

Results: The response for "A" was steady around 70% - 85% from 5th grade to 9th grade and then dropped to around 40% (11th grade and college).

Evaluator's reply: The probability of P(A) must be greater than P(A $\cap$ B)

5. A. Sample size neglection fallacy Question 5a
"In a certain town, there are two hospitals, a small one in which there are, on the average, about 15 births a day and a big one in which there are, on the average, about 45 births a day. The likelihood of giving birth to a boy is about 50%. In the small hospital one has kept a record of the days in which the number of boys born was greater than 9, which represents more than 60% of the total of births in the respective hospital. In the big hospital, one has kept a record of the days in which there were born more than 27 boys which represented more than 60% of the births. Which of the two hospitals recorded more days that boys were born. Choose and encircle the correct answer:
A. In the big hospital, more days were recorded where more than 60% boys were born.
B. In the small hospital, more days were recorded where more than 60% boys were born.
C. The number of days where more than 60% boys were born was equal in the two hospitals.

Results: No one selected "B". 'Other' answers ranged from 70% (5th grade) to 11% (college). One example of an 'other' answer was "In the big hospital, more days were recorded..". Among 5th and 7th graders, answers "A" and "C" were roughly equal while among older students (9th, 11th and college), roughly 80% selected "C" (main misconception).

Evaluator's reply: The growth in misconception (C) is due to a growth in a correct understanding involving ratios: 9/15 = 27/45".

5b Same as 5a but in simpler terms
By tossing a coin three times, the likelihood of getting heads at least twice is a (bigger than), b (equal to) or c (smaller than) the likelihood of getting, at least, 200 heads when tossing the coin 300 times.

Results: Answer 'B' (equal) increased with age from 30% (5th grade) to 75% (10th grade) but then dropped to 44% (college). Other answers decreased with age from 30% (5th grade) to none (college). The correct answer (A. Bigger) decreased with age from 35% (5th grade) to 5% (11th grade) but then increased to 50% (college).

Author's evaluation/explanation: None given

6. Heuristic of Availability Question 6

"The number of possibilities when choosing a committee composed of two members among 10 candidates is smaller, equal or bigger than the number of possibilities when choosing a committee composed of 8 members from the same 10 candidates."

Correct Answer: Equal number

Results: The correct answer ranged from 0% (5th grade) to a high of 15% (11th grade) with all other groups at 5%. Those choosing other answers were 70% in both 5th and 7th grades and 20% in 9th grade. Among the remaining answers, the main misconception was bigger in smaller than larger. The percentage choosing 'bigger' was 10% and 20% among 5th and 7th graders, but jumped to 65%, 85% and 75% among the three older ages.

7. The Falk Phenomena Question 7 – The axis of time

Yoav and Galit receive each a box containing two white marbles and two black marbles.

7A. Yoav extracts a marble from his box and finds out that it is a white one. Without replacing the first extracted marble, he extracts a second marble. The likelihood that this second marble is also white is smaller, equal or bigger compared to the likelihood that is a black marble?

7B. Galit extracts a marble from her box and puts is aside without looking at it. She then extracts a second marble and sees that it is white. Is the likelihood that the first extracted marble is white smaller than, equal to, or bigger than the likelihood that it is black?

Correct answers: Smaller in both cases since they are logically equivalent even though on the first extraction the result would have been equal (without knowing anything else). Context dominates history.

Results: Answers were grouped into 3 categories: 1. Correct answers to both questions. Group 2: Smaller from 7A, but equal for 7B (the most expected misconception). Group 3: Gave equal for both questions. Category 1 was in the range of 30 to 50% with no strong age relation. Category 2 increased from 5% (5th grade) to 70% (11th grade) and then dropped to 40% (college). Category 3 decreased from 25% to nothing. If categories 2 and 3 are grouped, then the main misconception increased from 30% to 70% of respondents from 5th grade to 11th grade and then dropped to 40% among college students.

Evaluator's reply: Students are mixing causality and logic. To say "the first extraction could not have been influenced by the second" is true casually – but not logically. Probability is logically contextual.

Alternate reply (Milo): Evaluator's reply seems excellent.