International Statistical Review, **67** (1999), 250–252. Invited discussion of a paper by Wild and Pfannkuch.

Discussion: What Shall We Teach Beginners?

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Wild and Pfannkuch provide much food for thought in their extensive and valuable discussion of statistical thinking (Section 2) and of variation and randomness (Section 3). To keep my discussion within bounds, I shall raise questions related to just one issue: *What in the way of statistical thinking ought we to attempt to teach beginners in a first statistics course*?

Statistical thinking for beginners

I fully agree that "development of a theoretical structure with which to make sense of reality" is an important goal of instruction, and that "the thinking and problem solving performance of most people can be improved by suitable structured frameworks." I most emphatically agree that *mathematical* structure is inadequate (even when it exists and can be grasped by students) as a basis for understanding and doing statistics in practice. Thus the working teacher should ask what kinds of structures she can offer students as frameworks for statistical thinking. Wild and Pfannkuch offer help.

Not, however, help specifically intended for teachers. Their discussion of "the thought processes involved in statistical problem solving" is quite complex. This surely reflects the actual complexity of these processes, but the resulting sets of cycles and epicycles bear a daunting resemblance to Ptolemaic astronomy. Hence my most important overall comment: *Beginning students need a more selective introduction to statistical thinking.* Beginners often lack intellectual maturity; they often lack the context knowledge needed for full statistical problem-solving; we know that beginners do not find elementary statistics an easy subject, so that we must be hesitant to add to their cognitive load. I find that industrial statisticians, whose teaching deals with mature and motivated company employees, often have quite unrealistic notions of the extent and sophistication of content that typical undergraduates can assimilate.

A fair response to Wild and Pfannkuch's mild criticism of the ASA/MAA committee's brief discussion of statistical thinking is therefore that the committee's statement appears in the explicit context of a first course in statistics and so is quite appropriately "only a subset" of what statisticians understand about statistical thinking. I believe that, in fact, the ASA/MAA committee has a good grasp of the elements that are most important in teaching beginners. Their longer discussion (Cobb, 1992) is very much worth reading.

Let me offer some personal opinions on the kinds of statistical thinking that any first course ought to provoke in students. First, important as they are, issues of measurement and of problem formulation require substantial grounding in the details of a specific context. Beginners can be expected to deal with only elementary levels of these aspects of statistical thinking. That is, I think current practice shows good judgment in generally assuming that variables are satisfactorily measured and that the problem posed is indeed one that clinicians or ecologists or civil engineers find worth investigating.

Second, we can start by mapping more detailed structures for the "Data, Analysis, Conclusions" portion of the investigative cycle, that is, for conceptual content currently central to elementary instruction. Here is an example of such a structure:

> When you first examine a set of data, (1) begin by graphing the data and interpreting what you see; (2) look for overall patterns and for striking deviations from those patterns, and seek explanations in the problem context; (3) based on examination of the data, choose appropriate numerical descriptions of specific aspects; (4) if the overall pattern is sufficiently regular, seek a compact mathematical model for that pattern.

This "suitable structured framework" for thinking supports yet more detailed frameworks in more specific settings. Wild and Pfannkuch rightly emphasize that "subcycles are initiated within major cycles." Students learn in each setting what standard graphs may be helpful, what typical overall patterns to look for, what numerical measures and mathematical models may be appropriate. For example, faced with data on a single quantitative variable, we can expect students to choose wisely among at least histograms and stemplots, to look for the shape, center, and spread of the displayed distribution, to weigh the five-number summary against $\overline{x}$ and s as a description of center and spread, and to consider standard density curves as possible compact models. Structures such as these are specific enough to guide beginners, yet general enough to be genuinely helpful. They are not, of course, simply recipes on the order of the algebraic recipes that filled our first courses before the blossoming of technology. In particular, data always have a context, and students must learn by (rather simple) examples and experience to, as Wild and Pfannkuch nicely put it, pursue a synthesis of context knowledge and statistical knowledge.

I hope it is clear that this discussion does not indicate disagreement with Wild and Pfannkuch's principles. I simply want to illustrate the work that teachers must do to make explicit the aspects of statistical thinking that we will emphasize in helping beginners learn. I also want to emphasize the need to be selective by reminding readers how much explicit content lies behind the structures.

Finally, though, I agree with Wild and Pfannkuch's implicit judgment that our current instruction is too narrow. We have done well to place much more emphasis than in the past on the design of data production and on exploratory analysis of data, and to situate formal inference more solidly in a setting shaped by design and exploration. Yet, as technology continues to automate the details, we must continue to ask what broader intellectual skills our students should carry away from a modern introduction to the science of data. I make some preliminary comments in Moore (1998), whose title "Statistics among the liberal arts" suggests the status I think statistical thinking deserves.

Randomness and variation for beginners

Wild and Pfannkuch offer an excellent and thoughtful discussion of variation, randomness, and causation. These are all issues that we must address in teaching beginners, and the discussion here should be helpful to any teacher. I want in particular to endorse their discussion of "Modelling Variation" with its emphasis that the "random" portion of statistical models is our way of describing unexplained individual variation and that "We have no idea whether this variation really is random." We would be wise, I think, to continue to reduce the place of formal probability in teaching statistical practice to beginners. I find that "unexplained individual variation" is clearer to students (because more concrete) than "random variation." Elementary inference procedures often assume that this variation is roughly described by a normal distribution. Normal distributions are not primarily "probability distributions" in this setting, but simply compact descriptions of the overall pattern of some sets of data.

What, no Bayes?

No doubt others will make the point that, for all the thought and empirical investigation behind it, the framework offered by Wild and Pfannkuch is itself "only a subset" of statistical thinking. Omission of the Bayesian paradigm for organizing statistical problems is striking, for example. That omission will bring no complaints from me, as I think (Moore, 1997) there are compelling reasons to avoid Bayesian inference when we teach beginners. Given the broader aims of this article and the prominence of Bayes methods in current research literature, however, it would be helpful if Wild and Pfannkuch commented on where these ideas fit. Did the statisticians they interviewed show no traces of formal Bayesian thinking?

Conclusion

I am sure that readers will agree that Wild and Pfannkuch have made a stimulating contribution to our continuing reexamination of the nature of statistical thinking. I hope that we will continue to reexamine our teaching in the light of this and other discussions.

Additional references

1. Cobb, G. (1992). Teaching statistics. In *Heeding the Call for Change: Suggestions for Curricular Action*, Ed. L. A. Steen, 3–43. Washington, D.C.: Mathematical Association of America.

2. Moore, D. S. (1997). Bayes for beginners? Some reasons to hesitate. *American Statistician* **51**, 254–261.

3. Moore, D. S. (1998). Statistics among the liberal arts. *J. Amer. Statist. Assoc.* **93**, 1253–1259.