NUMERACY, STATISTICS, AND MATHEMATICS

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Statistics and Numeracy

The goal of this section is to describe some connections between numeracy (or quantitative literacy (QL)) and statistics and to demonstrate that statistical thinking is fundamentally different from mathematical thinking, even though mathematics is an essential tool for statistics. These observations support the conclusion that statistics can serve as one important (perhaps essential) vehicle for effecting improvements in the teaching of quantitative literacy in mathematical science departments and beyond.

Of the many definitions of numeracy, one that will serve well for current purposes is from the 1982 study in the UK called *Mathematics Counts*:

> "an 'at homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical demands of everyday life."

Statistics has a similar goal of providing skills and habits of mind that allow individuals to understand and use quantitative information for improvement of life and work, and to do so with confidence rather than fear. That the tie between statistics and QL goes deeper than goals, however, can be demonstrated by looking at the expressions of QL formulated by Steen in *Mathematics and Democracy: The Case for Quantitative Literacy.* These include:

- Understanding comparative magnitudes of risk
- Understanding that unusual events (such as cancer clusters) can easily occur by chance alone
- Analyzing economic and demographic data to support or oppose policy proposals
- Appreciating common sources of bias in surveys
- Understanding how small samples can accurately predict public opinion
- Understanding how assumptions influence the behavior of mathematical models and how to use models to make decisions

All are statistical in nature and require statistical thinking for full appreciation.

Here are a few examples of the kinds of questions that may be posed in a curriculum that covers modern approaches to data analysis. They are essentially questions about quantitative literacy.

Q1. The following table gives the times each girl has recorded for seven 100 meter runs this year. Only one girl may compete in the upcoming tournament. Which girl would you select for the tournament and why?

RACE #	1	2	3	4	5	6	7
SUZIE	15.2	14.8	15.0	14.7	14.3	14.5	14.5
TANISHA	15.8	15.7	15.4	15.0	14.8	14.6	14.5
DARA	15.6	15.5	14.8	15.1	14.5	14.7	14.5

[Source: NCTM Assessment Sampler (draft)]

Comment: This open ended question explores ways to use real (or realistic, at least) data to answer an important practical question. It requires statistical thinking, as there are a number of plausible answers; no one of which is obviously "best" under all considerations. It is the statistical ideas that clearly stand out, as the mathematics to be used here is elementary.

Q2. Certain questions on a Teenage Attitudes and Practices Survey dealt with smoking and stress. The teenagers answering the questions are classified as NS = Never smoked, EX = Experimented with smoking, FS = Former smoker, and CS = Current smoker. The data are reported in the form of percentages rather than frequencies. Two examples are the following:

Do you believe cigarette smoking helps reduce stress?

	NS	**EX**	**FS**	**CS**
Yes	12.0	18.7	29.8	46.5
No	84.9	78.5	68.9	51.7
Don't know	3.0	2.5	1.6	1.6

Do you believe almost all doctors are strongly against smoking?

	NS	**EX**	**FS**	**CS**
Yes	80.1	78.8	80.1	80.5
No	17.3	18.8	17.3	16.7
Don't know	2.5	2.3	2.6	2.6

(a) How were these percentages calculated? What do they mean? Are these percentages joint, marginal, or conditional?

(b) Do the opinions on whether or not cigarette smoking helps reduce stress appear to be associated with the smoking status of the person responding? Write a paragraph justifying your answer.

(c) Do the opinions on physicians being against smoking appear to be associated with the smoking status of the person responding? Write a paragraph justifying your answer.

Comment: Of course the answers here require some basic knowledge of common statistical terms (which are almost self-explanatory here), but the statistical idea of association between variables is essential to numeracy.

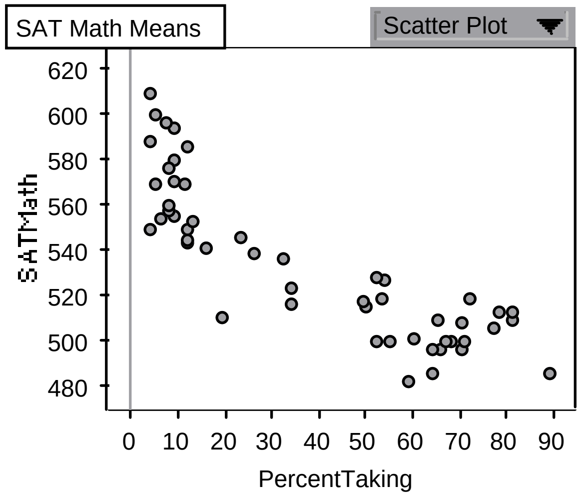
Q3. The data and plot below show the mean SAT math scores for the year 2000 versus the percentage of high school seniors taking the test for each of the states.

(a) Describe the relationship between the two variables.

(b) Explain why the pattern you see here makes sense from a practical point of view.

State	SAT-M	% Taking the SAT
Alabama	555	9
Alaska	515	50
Arizona	523	34
Arkansas	554	6
California	518	49
Colorado	537	32
Connecticut	509	81
Delaware	496	66
D.C.	486	89
Florida	500	55
Georgia	486	64
Hawaii	519	53
Idaho	541	16
Illinois	586	12
Indiana	501	60
Iowa	600	5
Kansas	580	9
Kentucky	550	12
Louisiana	558	8
Maine	500	68
Maryland	509	65
Massachusetts	513	78
Michigan	569	11
Minnesota	594	9
Mississippi	549	4
Missouri	577	8
Montana	546	23
Nebraska	571	9
Nevada	517	34
New Hampshire	519	72
New Jersey	513	81
New Mexico	543	12
New York	506	77
North Carolina	496	64
North Dakota	609	4
Ohio	539	26
Oklahoma	560	8
Oregon	527	54
Pennsylvania	497	70
Rhode Island	500	71
South Carolina	482	59
South Dakota	588	4
Tennessee	553	13
Texas	500	52
Utah	569	5
Vermont	508	70
Virginia	500	67
Washington	528	52
West Virginia	511	19
Wisconsin	597	7
Wyoming	545	12
National	514	44

Average SAT math scores by state versus percent of high school graduates who took the test [Source:College Board, 2000, www.collegeboard.org/press/senior00/html/table3.html.]



Comment: This is a study of association between quantitative variables. The discussion of pattern can go much deeper than "it goes downhill" to issues of linear trend versus curvature, clusters and gaps, and usual versus unusual data points.

Statistical Thinking: Where's the Data?

As the world of the information age becomes more quantitative, there is little question that the ability of people to deal with numerical issues of practical consequence is shrinking. Over the past twenty five years, statistics educators have developed innovative methods and materials to introduce the teaching of basic statistical concepts and statistical thinking into the K-12 mathematics curriculum and to improve such teaching at the undergraduate level. Data analysis, the corner stone of modern statistics, is heavily emphasized in these efforts, and this allows them to connect quite naturally, and quite effectively, to the current emphasis on quantitative literacy. The term *data analysis* covers the entire process of collecting, organizing, summarizing, and interpreting data. This is the heart of the discipline of statistics, and is in evidence whenever quantitative information is used in determining a course of action. The term *statistical thinking* may involve something less than a full understanding of data analysis but does include the ability to look critically at data and the processes underlying its collection, and to think stochastically about whether or not results might have a causal basis other than pure chance.

Statistics is also about numbers, but numbers in context (called data). Statistics is about variables and cases, distribution and variation, purposeful design of studies (covering data production, data analysis and data interpretation) and the role of randomness in design of studies and the interpretation of results. Statistics is about trying to understand, measure and describe real-world processes, from the relatively simple process of the waiting line at the school cafeteria to the complex processes of winning an election, developing a new drug, or building an automobile. The real value of statistical methodology lies in the usefulness of that methodology in solving the problem of interest, not in any optimality properties that can be proved theoretically. Although statistical procedures are often classified in terms of theoretical properties that are useful in comparing one procedure to another, their ultimate standing in the world of statistical data analysis depends upon their utility. John Tukey, perhaps the leading proponent of modern data analysis in the 20th century, captures this idea quite succinctly.

> "Statistics is a science in my opinion, and it is no more a branch of mathematics than are physics, chemistry and economics; for if its methods fail the test of experience - not the test of logic - they are discarded." (Tukey, 1962)

Statistics is a science that does make heavy use of mathematics, as do physics, chemistry and economics, and does so to the degree that it deserves to be called a mathematical science. In the K-12 curriculum, statistics is well on its way to establishing a permanent home within the mathematics curriculum primarily because many mathematics educators and mathematicians are receptive to this idea. (A case in point: the impetus for the Advanced Placement Statistics course came from the AP Calculus Committee.) Statisticians and others have argued that statistics should be part of the social sciences or business because these disciplines are major users of the tools of statistics. But there is no ground swell of emotion or energy among those groups to find a place for a well-rounded statistics curriculum within a social sciences or business setting. Mathematics and statistics are married for the moment; the goal now should be to make the marriage a happy one and to make it supportive of an expanded effort in improving numeracy.

Mathematical Thinking: Where's the Proof?

Mathematics is about numbers and their operations, generalizations and abstractions; it is about spatial configurations and their measurement, transformation and abstractions. "Numbers and shapes; arithmetic and geometry: these everyone recognizes as the essential foundation of mathematics." (Steen, 2001, p.68) Mathematics is about logical reasoning, patterns and optimization. It is about proof. It is about abstraction. Of course there are broader views of the subject, as seen in the following quote.

> "We may adopt a broader – partly sociological, partly epistemological – perspective and perceive mathematics as a field possessing a five-fold nature: as a pure, fundamental science; as an applied science; as a system of tools for societal and technological practice; as an educational subject; and as a field of aesthetics. … If this is how we see mathematics, then the mastery of mathematics goes far beyond the ability to operate within the theoretical edifice of purely mathematical topics." (Mogens Niss as quoted in Steen, 2004, p. 35)

This broader view could well accommodate modern statistics. But this view appears to be far from what is taught as mathematics in most K-12 programs in the United States, although it may be consistent with what some standards-based mathematics curricula hope to be moving toward.

One illustration of the difference in emphasis between statistics and mathematics comes about by looking at the titles of recent articles appearing in educational publications from each side. These have recently appeared in the *Mathematics Teacher*, a journal for secondary school mathematics published by the National Council of Teachers of Mathematics (NCTM):

- Farey Sums and Understanding Ratios
- Circumscribable Quadrilaterals: A Journey in Honors Geometry
- Centroid of a Polygon-Three Views
- Geometric Approaches to Quadratic Equations from Other Times and Places

These have recently appeared in *STATS*, a journal for students published by the American Statistical Association (ASA):

- Unseen, Unfelt, and Understated: The Dangers Posed to Children by Arsenic-Treated Lumber in Playgrounds
- The Statistics of Hurricanes
- What is a Sigma and Why Do I Need Six of Them? (statistics in quality improvement)

The difference is self-evident. Both types of articles are important in the broader educational scheme, but the second type is closer to QL and can foster the transition from the first type to QL.

Returning to the notion of posing questions, one sees another clear distinction in statistical versus mathematical thinking when mathematics educators (comfortable in the latter but shaky in the former) pose questions on data analysis. Here are a few real examples.

Q4. The students in Mr. Kirby's class voted for their favorite book of the past 3 months. The three books that they read were:

- Babe, the Gallant Pig
- Sarah, Plain and Tall
- Stone Fox

Here are some clues about how the vote came out.

a. 34 students voted
b. The winner got the most votes, but got fewer than half the votes
c. There was a two-way tie for second place.

In the space below, make a chart of graph that shows voting results that fit all three clues.
[Source: Balanced Assessment]

Comment: This is not a data analysis question, even though it does ask something about data. It is a question about mathematical reasoning (properties of numbers), not about statistical reasoning (producing and using numbers in context, data, to answer a practical question of interest).

Q5. According to The World Almanac 2000, page 987 the number of hits each New York Yankee player had for the1999 season were as follows: 219, 202, 176, 170, 69, 128, 156, 51, 117, 93, 50, 48. Using these values it was determined that the mean is 123.25, the median is 122.5, the range is 171 and the interquartile range (IQR) is 113.

Subsequent to these calculations it was learned that a transcription error had been made and the third lowest number should have been 61 not 51. Which of these four measures would be affected most?

a. Mean
b. Median
c. Range
d. IQR

[Source: NCTM Assessment Sampler (draft)]

Comment: This is a question about data, but there is little, if any, statistical reasoning involved. The question calls for manipulation of the data to calculate the four statistical summaries under two different scenarios. That requires knowledge of arithmetic (an important mathematical skill), but requires no statistical reasoning. The only connection to statistics is knowledge of how the four summary statistics are calculated. The result is not related to the context and is not even important to a contextual description or use of the data. Even worse, a student who does understand the context could be distracted by the fact that mean number of hits per player is not a statistic that would interest any baseball fan. A more subtle statistical point is that a small change in the mean may be of much greater practical significance than a larger change in the IQR. A well designed question about the effect of outliers on measures of center and spread, and how an outlier might change an interpretation of the data, would be a good question to ask, but this one does not achieve that goal.

Q6. The table below shows the daily attendance at two movie theaters for 5 days and the mean (average) and the median attendance.

	Theater A	**Theater B**
Day 1	**100**	**72**
Day 2	**87**	**97**
Day 3	**90**	**70**
Day 4	**10**	**71**
Day 5	**91**	**100**
Mean	**75.6**	**82**
Median	**90**	**72**

a. Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater A? Justify your answer.

b. Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater B? Justify your answer.

[Source: NAEP Grade 12 Sample Question]

Comment: A question that looks like it has the makings of a good statistics question is sabotaged by the required answer; the answer to part (a) is required to be “median” and the answer to part (b) is required to be “mean”. This is a result of what could be sound statistical reasoning turned into an overly simplistic algorithm: when an outlier is present use the median as the measure of center, when there are no outliers use the mean. Appropriate statistical measures cannot be assigned to the data until the context is completed with a question that needs to be answered with these data. Why were the data collected in the first place? If the question is to compare the two theaters, then one cannot use the median for one and the mean for the other. If these are the same days (paired data) then the daily differences in attendance might be the appropriate measures to analyze. Shouldn't the 10 be investigated? Perhaps it was a mistake. In short, this turns out to be neither a good statistics question nor a good mathematics question, but it could become a good statistics question with a little more context and a little more leeway on the correct answer.

Q7. A drawer contains 28 pens; some white, some blue, some red, and some gray. If the probability of selecting a blue pen is 2/7, how many blue pens are in the drawer?
[Source: TIMMS, 1994, K-7]

Comment: What does this have to do with understanding probabilistic reasoning as a guide to practical decision making?

Teacher Education

Everyone agrees that a key ingredient to the improvement of numeracy is the strengthening of teacher education. Although not specifically addressing QL, the *Mathematics Education of Teachers* report of CBMS makes recommendations on teacher education in the mathematical sciences that could immeasurably improve the atmosphere for a sustained effort in QL. MAA's PMET project is carrying these ideas forward in a practical way. ASA has two initiatives along similar lines, one to improve the pre-service statistics education of teachers and another to develop detailed guidelines on the content and pedagogy of a K-12 statistics strand. Other plans need to be put forth to engage the disciplines that make use of QL skills in a broad-based, multi-disciplinary teacher education effort.

Conclusion

This article opened with a quote from *Mathematics Counts*; it will close with another.

> "Although statistics is commonly taught within mathematics courses, it should not be regarded solely as part of mathematics. Statistics is not just a set of techniques, it is an attitude of mind in approaching data. In particular it acknowledges the fact of uncertainty and variability in data collection. It enables people to make decisions in the face of uncertainty." *Mathematics Counts,* p 234

Statistical thinking is a habit of mind, as is much of QL. Data analysis does work as a mechanism to change habits of mind toward improving quantitative skills. A few quotes from teachers who use this approach may help make the argument.

- The students pay attention to the [data analysis] material. … using data to figure out what a car costs is something high school students pay attention to. Teaching students to write an equation of a line and to discuss the meaning of the slope has never been this easy! Actually I'm not sure that students ever understood this concept well before.

- The [data analysis] materials allow for the students to construct knowledge based on their experiences, and these materials provide activities and experiences to guide the students to good concept based skills. The students understand what and why they are doing things.

- Almost all of the students were amazed by the fact that some of the mathematical concepts that they study … are actually used in such data analysis situations. I must also say that I find it very exciting to engage in these topics as well!

Over the past 20 years some success has been achieved in using data analysis to improve quantitative reasoning skills of students, but much remains to be done if we are to have a quantitatively literate society in the 21st century. Statistics is now generally thought of as part of the mathematical sciences, but mathematics and statistics follow two quite different paths of reasoning. If these paths are made clear, however, these two arms of the mathematical sciences, one ancient and well established, the other new and still a bit of an outsider, can complement each other in ways that will strengthen the case for quantitative literacy.

References

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Steen, Lynn. *Achieving Quantitative Literacy*. MAA Notes #62. Washington D.C.: Mathematics Association of America, 2004.

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WWW Links

International Association for Statistics Education (IASE);
Statistics Education Research Journal (SERJ):
http://www.stat.auckland.ac.nz/~iase/

Journal of Statistics Education: http://www.amstat.org/publications/jse

Statistical Literacy, Augsburg College: http://www.statlit.org/

ARTIST Project: http://www.gen.umn.edu/artist/

David Lane, Rice University: http://www.ruf.rice.edu/~lane/