Math is Music; Statistics is Literature

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Math is Music; Statistics is Literature – or Why are there no six year old Novelists?[1]

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Almost thirty years ago, something happened that made Introductory Statistics harder to teach. Students didn't suddenly become less teachable, nor did professors forget their craft. It was then that we began to switch from teaching Statistics as a Mathematics course to teaching the art and craft of Statistics as its own discipline. When Statistics was viewed as a sub-specialty of Mathematics, students were taught to manipulate formulas and calculate the "correct" answer to rote exercises. Life for the teacher, both as instructor and grader was easy.

That started changing in the early 1980's. The video series *Against All Odds* appeared and David Moore and George McCabe published *Introduction to the Practice of Statistics*. Since then two pioneering committees —one for the MAA and ASA, and a second, that produced the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, officially adopted by the ASA—have pushed us all to change our teaching. And a new generation of texts has appeared following the advice of these reports—and challenging Statistics teachers to use this new approach.

But why is it more difficult to teach this way? And why is it so important that we do so?

By comparison, let's look at mathematics. Much of the beauty of mathematics stems from its axiomatic structure and logical development. That same structure facilitates--in fact dictates--the order in which the material is taught. It also ensures that the course is self-contained, so there are no surprises. But modern statistics courses are not like that. That can frustrate students who were expecting a math class. As a student of one of us once wrote on the course evaluation form, "This course should be more like a math course, with everything you need laid out beforehand."

Mathematics has a long history of prodigies and geniuses, with many of the most famous luminaries showing their genius at remarkably early ages. We've all heard at least one

[1] This paper is based on several talks given by the authors at USCOTS.

version of the famous story of young Carl Friedrich Gauss. A web search finds over 100 different retellings of the story, but an article in *American Scientist*[2] identifies a version actually recounted at Gauss' funeral. In that version, Gauss, aged 7 and youngest in the class, summed the numbers from 1 to 100 in seconds, wrote the answer on his slate and then threw it down on the table mumbling "there it lies" in the local dialect. It was perhaps an hour later that the teacher discovered that his answer was, in fact, the only correct one in the room.

Prodigies in math can develop at remarkably early ages because math creates its own self-consistent and isolated world. Pascal had worked out the first twenty-three propositions of Euclid by age 12 when his parents, who wanted him to concentrate on religion, finally relented and presented him with a copy of Euclid's Elements. Galois wrote down the essentials of what later became Galois theory the night before a fateful duel when he was 20, or so the legend has it. In the modern era, Norbert Weiner entered Tufts at age 11, Charles Pfefferman of Princeton was, at 22, the youngest full professor in American history, and Ruth Lawrence of Hebrew University passed her A-levels in pure math at age 9 and became the youngest student ever to enroll at Oxford two years later.

Of course mathematics isn't the only field that shows prodigies. Mozart, Schumann, and Mendelssohn, among others, were young musical prodigies. Even though his music matured, it is remarkable that some of the music Mozart wrote at age 5 is still in the repertoire. Chess prodigies continue to appear. Sergey Karjakin is the youngest grandmaster ever at 12 years 7 months. The infamous, late Bobby Fischer, who was youngest in 1958 when he became a grand master at 15 years, 6 months and 1 day, is now only 19th on that list.

But there are only a few fields that develop prodigies, and all seem to be self-contained. For example, as Thomas Dulack observed, "There are no child prodigies in literature."[3] Although one might argue that William Cullen Bryant, Thomas Chatterton, H.P. Lovecraft or Mattie Stepanek qualify as literary prodigies, that list doesn't have quite the

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[2] Brian Hayes. "Gauss's Day of Reckoning". *American Scientist*,May-June 2006,Volume 94, Number 3, Page: 200

[3] http://advance.uconn.edu/2006/060424/06042412.htm

same panache as the others we've cited. It's no easier to find prodigies in art, poetry, philosophy, or other endeavors that require life experience.

What does any of this have to do with statistics and how can it help us understand why introductory statistics is so hard to teach? The challenge for the student (and teacher) of introductory statistics is that, like literature and art, navigating through and making sense of it requires not just rules and axioms, but life experience and "common sense." Although working with elementary statistics requires some mathematical skills, we ask so much more of the intro stats student than is required by, for example, a first Calculus course. A student in calc I is not asked to comment on whether a question makes sense, whether the assumptions are satisfied (is the reservoir from which the water is pouring *really* a cone?), to evaluate the consequences of the result, or to write a sentence or two to communicate the answer to others. But that's exactly what the modern intro stats course demands.

The challenge we face is that, unlike calc I, we have a wide variety of skills to teach, and *most of them require judgment* in addition to mathematical manipulation. Judgment is best taught by example and experience, which takes time. But we're supposed to produce a student capable of these skills in one term. It would be challenging enough to teach the definitions, formulas, and skills in the standard first course. To convey in addition, the grounds for sound judgment is even more difficult. It should be no wonder that the first course in statistics is widely acknowledged to be one of the most difficult courses to teach in the university.

It is not merely that we hope to teach judgment to Sophomores; we are actually asking our students to change the way they reason about the real world. We call the skills they must acquire the seven unnatural acts of statistical thinking:[4]

1. Think Critically. Challenge the data's credentials, look for biases and lurking variables.
2. Be Skeptical. Question authority and the current theory. (Well, OK, Sophomores do find this natural.)

[4] P.F.Velleman, 2003, "Thinking with Data; Seven Unnatural Acts and Ten 400-year-old Aphorisms" Keynote address to the Beyond the Formula conference, Rochester, NY.

3. Think about variation rather than about center.
4. Focus on what we *don't* know. For example, a confidence interval exhibits how much we don't know about the parameter.
5. Perfect the Process. Our best conclusion is often a refined question, but that means a student can't memorize the "answer."
6. Think about conditional probabilities and rare events. Humans just don't do this well. Ask any gambler. But without this the student can't understand a P-value.
7. Embrace vague concepts. Symmetry, Center, Outlier, Linear… the list of concepts fundamental to Statistics but left without firm definitions is quite long. What diligent student wanting to learn the "right answer" wouldn't be dismayed?

How can we help students navigate through these woods? We don't have definitive answers to the question, in spite of our over 50 years (combined – not each) of teaching introductory statistics. But we'd like to identify some themes that might help us as a community to start a conversation about some of the challenges.

We can help students by giving them a structure for problem solving that incorporates the requirement that they exercise their judgment. In our books we've recommended that students follow the steps that W.E. Deming created over 50 years ago in his advice to industry: Plan, Do, Check, Act. We've substituted Communicate for Act to underscore the importance of communicating to others the results we see. Students must learn to communicate their results in plain language and not only in statistical jargon.

As the GAISE report emphasized, we must place more emphasis on the Plan, and Communicate steps.. The emphasis of the traditional mathematical course, on the Do step can be largely replaced by relying on technology for the calculations and graphics.

In teaching students to think through the problem, plan their attack on it, and communicate results, we bring students face-to-face with their real-world knowledge and experience—with the literature side of their maturing intellect. We owe them an acknowledgement that we've done this. It isn't fair to emphasize the simplicity of the calculations or to just provide a bunch of definitions in little boxes. No Comp Lit or Philosophy teacher would do that, and neither should we.

What guidance should we offer? First, we can note that the judgment often called for in statistics is one that invites students to state their personal views. (After all, *they* are the ones who must be 95% confident in their interval.) But we can offer guidance for their judgments; they must be guided by the ethical goal of discovering, describing, modeling, and understanding truth about the world.[5]

Second, we can remind students that their Introductory Statistics course is related to every other course they may study. The reason they are taking Statistics (or perhaps, the reason that it's *required*) is that they are accumulating the kind of knowledge about the real world that will help them write literature and read philosophy, and *that* kind of knowledge makes them qualified to make statistical judgments. Of course, by asking students to call upon what they've learned in other courses we are encouraging them to solidify their knowledge from those courses.

Third, we must actually require students to demonstrate all of the steps of a statistical analysis, from problem formulation, to communicating the results, to making real world recommendations on what they find. Unfortunately, homework and exam problems that carry these requirements are harder to write and harder to grade. Training teaching assistants to reliably grade these efforts can be problematic. Moreover, many statistics instructors are not trained in statistics and they too can find this approach challenging. But the results of teaching a modern course rewards both the student and teacher in spite of its challenges.

We should also face outward to the academic community. There is a widespread impression that introductory Statistics can be taught – or even less plausible, can be *learned*—in a single term. Any objective consideration of the breadth and depth of the concepts and methods covered shows this to be absurdly optimistic. Yet few academic programs require more than one course, and many of those that require two are cutting back. We need to argue as a discipline that an introductory Statistics course must cover more than an introduction to inference for means if it is to teach the reasoning of

[5] "Truth, Damn Truth, and Statistics," (July, 2008), *Journal of Statistics Education*, Volume 16, Number 2, http://www.amstat.org/publications/jse/v16n2/velleman.html

Statistics—and that teaching that reasoning must be its goal (and not just teaching definitions and formulas.) But a more complete course that covers techniques that require more than rudimentary sophistication such as inference for regression and multiple regression is unlikely to have time to teach judgment, planning, and communication. It will most likely be pared down to a collection of equations and rules.

As a community we need to make it clear that the subject of Statistics deserves both more respect and more time, not because it covers so many methods but because it should teach the foundations of reasoning when we have data. Part of the argument might be that, unlike students in subjects that exhibit prodigies, our students must summon their real-world knowledge to learn to think statistically. And that the effort by statistics teachers and students will pay back correspondingly in all that our students do. Math is sometimes said to be the language of science (and much social science), but statistics should teach students the structure for what it communicates.

Is the effort to teach the modern course worth it? We believe that it is. Rather than a collection of techniques or a "cookbook" of situations and formulas, a modern course in statistics must teach students to reason about the world. Although that makes the course more difficult to teach and to assess, it will make a difference in students' lives and serve them for the rest of their academic careers and beyond.