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Von Mises' Frequentist Approach to Probability

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Richard von Mises

Richard von Mises (1883-1953) is not well known by statistical educators even though

- he first proposed the classic "birthday problem" in 1939

- he introduced the term 'the label space' which Kolmogorov later called 'the sample space.'

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Frequentist Approach to Probability

Richard Von Mises formulated a strict Frequentist approach to *probability*.

This approach was limited to observations for which there are sufficient reasons to project future stability -- to believe that the relative frequency of the observed attribute would tend to a fixed limit if the observations were continued indefinitely by sampling from a "collective."
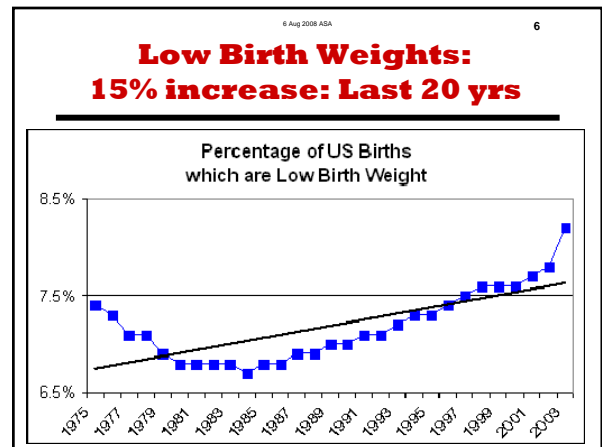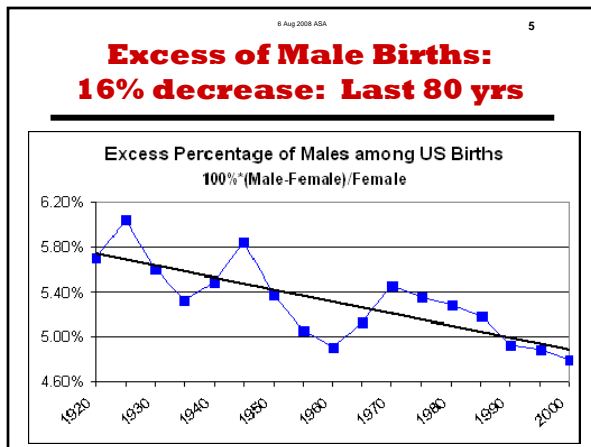

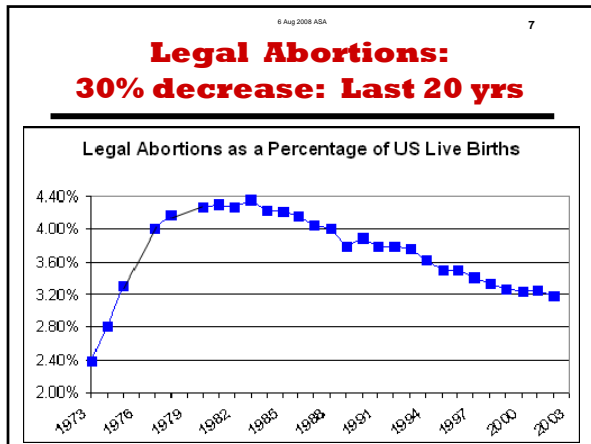6 Aug 2008 ASA   4

Von Mises' concept of "collectives"

Von Mises' concept of "collectives" is an ideal which is fully realized only in physics; gambling equipment comes pretty close; but it's down hill from there.

All bio/socio/econo/politcal phenomena are well known to suffer from numerous influences which change frequencies over a variety of time scales.

Use of strict probability in these areas is always contingent on "ceterus paribus"; continuous rechecking is required to ensure that "ceterus" really are still "paribus".

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Excess of Male Births: 16% decrease: Last 80 yrs

Excess Percentage of Males among US Births
100%*(Male-Female)/Female

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Low Birth Weights: 15% increase: Last 20 yrs

Percentage of US Births which are Low Birth Weight

Legal Abortions:
30% decrease: Last 20 yrs

Legal Abortions as a Percentage of US Live Births

Implications of
Strict Frequentism

One consequence of this approach is that statements of probability are never applicable to a single individual.

Your chance of dying depends on your group:

Chance as a random person in the US.
Chance as a random person of your age
Chance as a random person of your gender.
Chance as a random person of your age & gender.

"Per" grammar
Subjective Probabilities

Some probabilities can be stated in "per" grammar: percent, percentage or rate.

No statements involving subjective probability can be stated using "per" grammar.

- 60% chance that Obama will win the election.
- 40% chance the US will win in Afghanistan.
- 20% chance that God exists.

"Per" Grammar
Relative Frequency Probabilities

All probability statement based on a relative frequency can be stated using "per" grammar.

15% chance of unemployment.
•15% of civilian labor force are unemployed.
52% chance the baby will be a boy.
•52% of babies are boys.

95% chance that a 95% confidence interval includes the population parameter.

- 95% of 95% confidence intervals contain the population parameter.

5% chance that a sample statistic is 2 or more standard errors from the population mean.

- 5% of sample means are 2 standard errors or more.

Implications of
Strict Frequentism

von Mises gave criteria for application to real world events which criteria didn't involve any assumptions about prior probabilities or states of mind.

Frequentism is about repetitive events: a category which doesn't include individual entities, propositions or beliefs.

Statistical Literacy and Probability

Since being literate requires clarity about the meanings of key concepts, statistical literacy requires teaching clarity about the different meanings/interpretations of "probability".

The radically different bases (despite much similarity in math) of the main competing notions/theories needs to be understood at least in general way by all

Conclusion

Statistical educators should:

1. Make users aware of the difference between subjective and relative frequency probabilities.

2. Require students to try stating probabilities using "per" grammar.

3. Help students recognize that only relative frequency probabilities can be stated using "per" grammar.

References (English)

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