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## Coincidence in Runs and Clusters

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Slides at [www.StatLit.org/pdf/2012Schield-eCOTS-6up.pdf](http://www.StatLit.org/pdf/2012Schield-eCOTS-6up.pdf)

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## Coincidence?

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### Run of Heads (Red Cells): Chance of 5 Touching: 1 in 32 ( $2^5$ )

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AAA	AAAA	AAAAA		
1	Fair coin: find longest run of heads in a row																															
2	5	1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0																														
3		1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0																														
4	3	1	1	1	1	0	1	1	1	0	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	
5		1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0																														
6	4	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0		
7		1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0																														
8	5	0	1	1	1	1	1	0	0	0	1	1	0	1	1	0	0	0	1	1	1	0	1	1	0	1	1	0	1	1		

[www.StatLit.org/Excel/2012Schield-Runs.xls](http://www.StatLit.org/Excel/2012Schield-Runs.xls)

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### Run of 10 heads (Ones): Chance = 1 in 1,024 ( $2^{10}$ )

[www.StatLit.org/Excel/2012Schield-Runs.xls](http://www.StatLit.org/Excel/2012Schield-Runs.xls)

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### Clusters of Rice Chance of Red: One in 10

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
3	9	3	2	9	9	4	1	9	9	9	2	2	5	3	5	0	5	5
4	8	0	6	4	1	6	7	4	0	2	2	0	3	7	0	9	8	0
5	3	1	7	3	5	2	5	6	8	7	2	0	4	8	9	2	9	6
6	9	0	1	4	3	4	2	8	9	2	6	6	4	7	7	9	2	3
7	9	6	2	1	9	0	4	3	8	6	2	7	5	7	5	1	3	3
8	4	3	6	1	5	8	1	9	4	8	4	9	2	6	1	8	7	2
9	0	0	2	4	3	0	5	5	9	3	1	6	9	5	3	5	8	4
10	9	6	6	7	5	0	6	6	1	2	6	6	0	9	3	6	7	8
11	9	1	0	4	7	4	2	4	4	0	4	3	8	8	4	9	8	5
12	9	8	0	1	4	6	0	8	2	0	4	2	3	5	6	4	5	7

[www.StatLit.org/Excel/2012Schield-Rice.xls](http://www.StatLit.org/Excel/2012Schield-Rice.xls)

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### Chance of a Red Cell: 1 in 10 Chance Two Reds Touch: 1 in 100

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI			
1	Rice: 10 sheet Find the largest group of high cells (red fill) that are touching each other.																																					
2	0 Low 9 High a. touching on sides in a row b. touching on sides (but not just on points)																																					
3	1	4	1	0	1	4	9	9	0	2	3	9	2	9	2	1	9	8	2	7	6	7	8	7	4	6	4	6	5	0	0	9	6	0				
4	3	9	3	0	8	2	5	2	7	8	6	0	8	6	7	1	8	6	6	6	5	6	8	7	8	9	9	7	5	8	0	5	0	7				
5	3	9	9	9	6	5	6	0	3	1	7	9	9	2	0	5	7	1	3	0	6	4	8	1	1	9	6	3	4	4	7	2	3	9				
6	6	7	1	2	8	0	1	2	0	9	5	7	2	0	9	5	7	2	0	4	8	0	8	2	4	8	0	2	4	8	0	6	5	9	2			
7	4	5	2	5	2	3	0	0	3	6	5	8	4	8	0	7	3	6	9	5	0	8	2	4	8	0	2	4	8	0	6	5	9	2	3			
8	0	7	9	6	0	3	2	2	5	8	4	4	7	1	2	0	5	1	9	4	6	7	5	5	8	3	2	2	7	3	6	9	2	9	6			
9	0	3	8	5	2	0	6	2	7	7	6	8	4	0	1	0	3	0	8	6	7	4	0	3	3	4	8	4	9	4	9	0	9	0				
10	2	3	7	1	9	0	4	5	3	7	8	1	5	9	8	7	4	7	5	6	3	8	6	2	3	8	5	3	2	0	2	0						
11	1	8	2	3	7	5	7	6	0	4	1	8	0	1	4	8	1	5	7	1	2	3	6	1	3	0	7	5	1	4	7	2	1					
12	1	1	6	3	6	1	8	5	5	2	1	2	8	2	8	8	5	6	7	4	3	9	2	1	1	0	8	3	2	6	9	1	4	8	5			
13	7	6	6	5	4	6	5	2	0	3	9	9	5	9	4	5	8	2	5	3	8	9	6	6	0	2	7	1	2	9	4	0	6					
14	4	8	7	9	3	0	9	3	6	5	8	1	3	2	6	7	1	0	8	0	9	5	2	7	8	4	5	1	6	0	0	3	6	3	1			
15	2	5	9	3	8	0	2	7	0	1	3	8	0	6	7	9	3	2	5	3	0	8	4	1	9	2	3	0	5	6	9	3	1	3	1			
16	0	5	3	1	8	9	8	2	4	1	2	1	7	4	4	8	2	7	8	5	3	2	7	4	1	4	1	7	1	8	0	6	5	6	0			
17	9	6	8	4	6	4	3	8	5	2	9	5	4	8	8	1	9	1	8	0	8	0	3	8	9	9	1	5	6	2	4	5	6	4	3	4		
18	3	2	1	0	2	3	4	0	3	9	9	6	6	6	8	4	8	0	2	0	6	6	7	1	1	1	4	1	1	1	4	1	9	3	4	6		
19	7	4	7	9	9	7	1	1	3	7	8	3	1	6	9	0	0	3	9	3	0	6	6	6	9	2	4	0	3	5	0	5	1	4	0	9		
20	5	6	9	1	8	3	4	8	9	5	6	5	0	1	5	3	7	5	4	2	8	3	7	7	9	0	6	2	1	3	9	8	9	2	9			
21	6	3	7	5	3	7	8	1	2	5	3	5	0	0	9	3	0	0	2	6	4	4	6	2	8	7	1	6	1	5	0	2	4	0	9			
22	4	0	2	9	3	6	8	0	6	4	7	6	3	7	2	1	9	3	0	9	3	4	2	9	6	6	5	4	6	2	6	5	3	9	3			
23	0	3	0	3	7	4	6	1	4	0	3	7	1	1	9	7	5	5	9	3	2	8	9	8	7	5	0	8	5	7	0	5	4	8	0			

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**Runs and clusters are much more likely than expected!**

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When students press F9, they often get:

RUNS	CLUSTERS
a run of 10 heads:	a cluster of six squares:
one chance in $2^{10}$	one chance in $10^6$
a “thousand-year flood” every year	a “million-year flood” every year


**They get unlikely results every time!**

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**Runs and clusters are much more likely than expected!!!**

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WHY?



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**Explanation #1**

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What is the chance of that?

I. The question is ambiguous?

- Before or after the fact (ex post vs. ex ante)
- Specific place or anywhere
- Painting the target before or after the shooting

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**Explanation #2**

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Law of Very-Large Numbers:

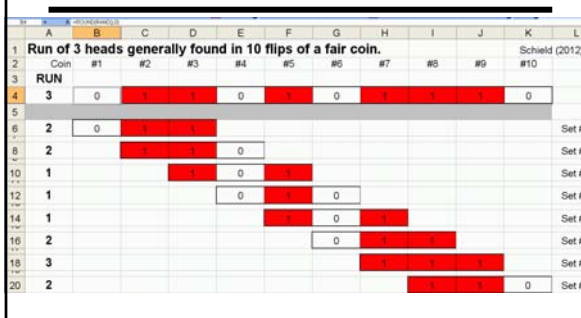
**‘Impossible’: almost certain given enough tries**

The Law of Very-Large Numbers isn’t covered in most intro-stats courses where the focus is on sampling error in small samples.

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**#3 Compression: Flipping 10 coins. Almost the same as 24: 8 sets of 3**

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Run of 3 heads generally found in 10 flips of a fair coin.

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**Students must “see” that coincidences:**

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are much more common than expected

1. involve an ambiguity (ex-ante/ex-post)
2. involve the Law of Very-Large Numbers
3. involve compression
4. may still be signs of causation.

*Example: Cholera outbreak in London around a particular pump.*