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Exploring Lognormal Incomes

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Editor: www.StatLit.org
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www.StatLit.org/
pdf/2014-Schield-Explore-LogNormal-Incomes-Slides.pdf
XLS/Create-LogNormal-Incomes-Excel2013.xlsx

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Log-Normal Distributions

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The lognormal is always positive and right-skewed.

Examples:

- Incomes (bottom 97%), assets, size of cities
- Weight and blood pressure of humans (by gender)

Benefit:

- calculate the share of total income held by the top X%
- calculate share of total income held by the 'above-average'
- explore effects of change in mean-median ratio.

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"We … state our belief that the lognormal is as fundamental a distribution in statistics as is the normal, despite the stigma of the derivative nature of its name."

Aitchison and Brown (1957). P 1.

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Lognormal and Excel

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Use Inverse to find cutoffs: quartiles, to 1%, etc.

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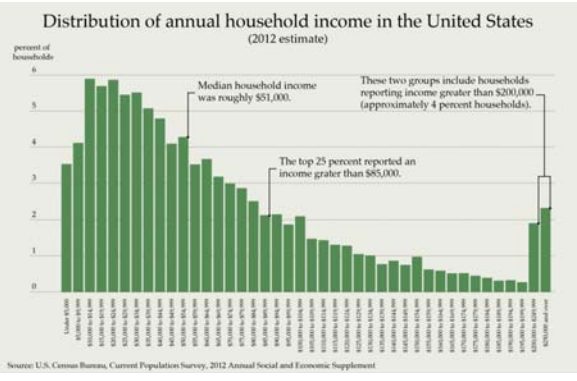

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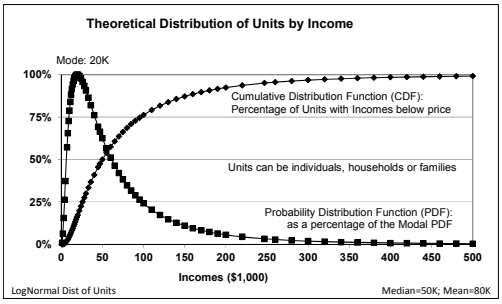
Log-Normal Distribution of Units



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Paired Distributions

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1. Distribution of subjects by X
2. Distribution of total X by X.

Sometime we ignore the 2nd: height or weight.

Sometimes we care about the 2nd: income or assets.

Surprise: If the 1st is lognormal, so is the 2nd.

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Distribution of Households and Total Income by Income

Suppose the distribution of households by income is log-normal with normal parameters mu# and sigma#.

Then the distribution of total income by amount has a log-normal distribution with these parameters: mu$ = mu# + sigma#^2; sigma$ = sigma#.

See Aitchison and Brown (1963) p. 158.

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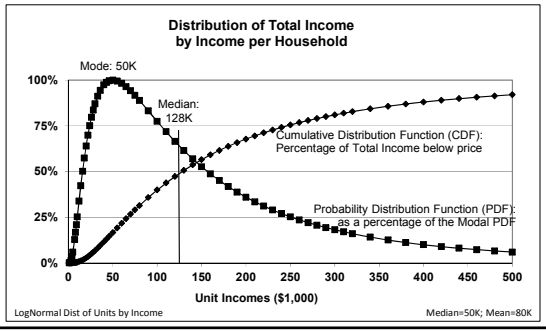
Distribution of Total Income



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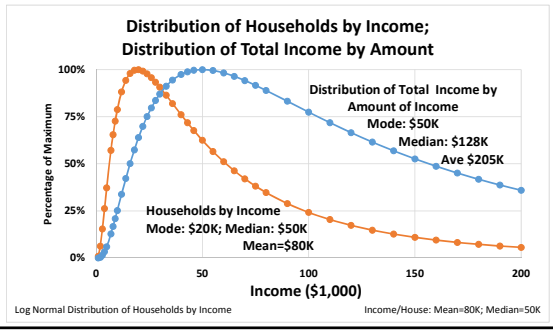
Distribution of Households and Total Income



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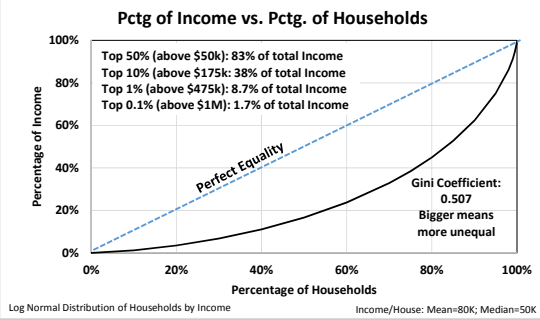
Lorenz Curve and Gini Coefficient



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Champagne-Glass Distribution

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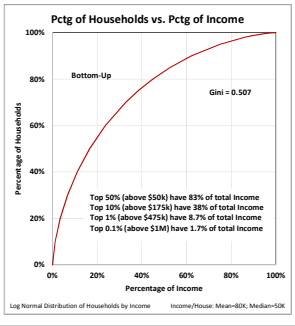
The bigger this ratio the bigger the Gini coefficient and the greater the economic inequality.



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Log-Normal Balance Conjecture

Conjecture: If household (HH) income is distributed log-normally and X% of households have below-average incomes, then X% of all income is earned by HH with above-average incomes.

Example: If 60% of HH have below-average incomes, then 60% of total income is earned by HH having above-average incomes.

Evidence using Excel spreadsheet:
Suppose Mean# = 50K and Median# = 80K.

- 68.61%: Percentage of HH having below-average income
- 68.61%: Percentage of total income that is associated with HH having above-average incomes. QED

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As Mean-Median Ratio ↑ Rich get Richer (relatively)

Log-normal distribution. Median HH income: $50K.

	Top 5%		Top 1%		
Mean#	Min$	%Income	Min$	%Income	Gini
55	103	11%	138	2.9%	0.24
60	135	15%	204	4.2%	0.33
65	165	18%	270	5.5%	0.39
70	193	20%	337	6.6%	0.44
75	220	23%	406	7.7%	0.48
80	246	25%	477	8.7%	0.51
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90	298	29%	623	10.7%	0.56

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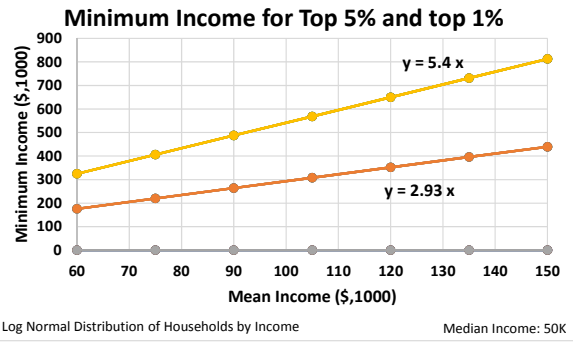

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Which parameters best model US household incomes?

US Median Income (Table 691*)

- $46,089 in 1970; $50,303 in 2008

Share of Total Income by Top 5% (Table 693*)

- 16.6% in 1970; 21.5% in 2008

Best log-normal fits:

- 1970 Median 46K, Mean 53K: Ratio = 1.15
- 2008 *Median 50K, Mean 73K*; Ratio = 1.46

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Conclusion

Using the LogNormal distributions provides a principled way students can explore a plausible distribution of incomes.

Allows students to explore the difference between part and whole when using percentage grammar.

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Bibliography

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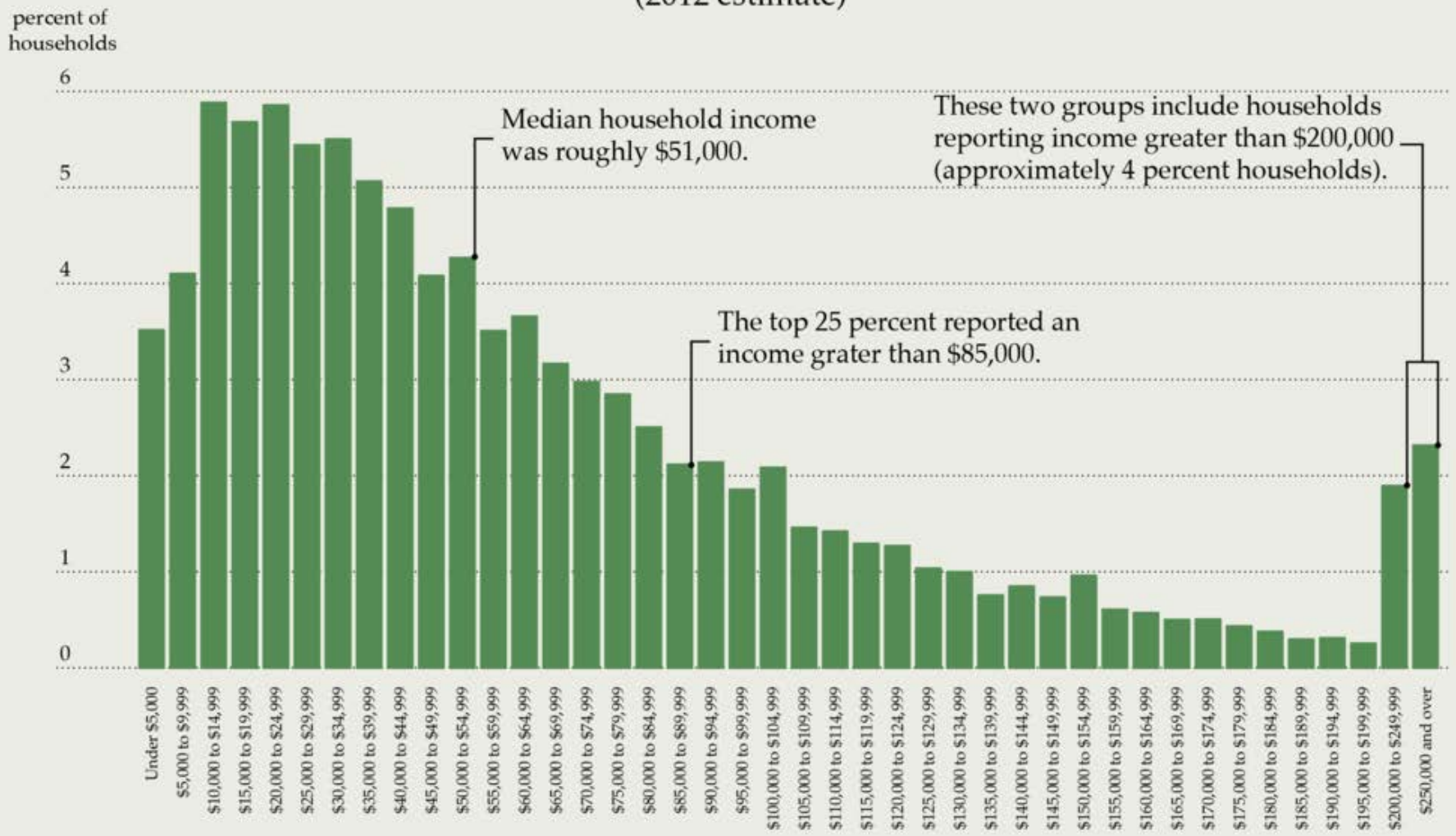
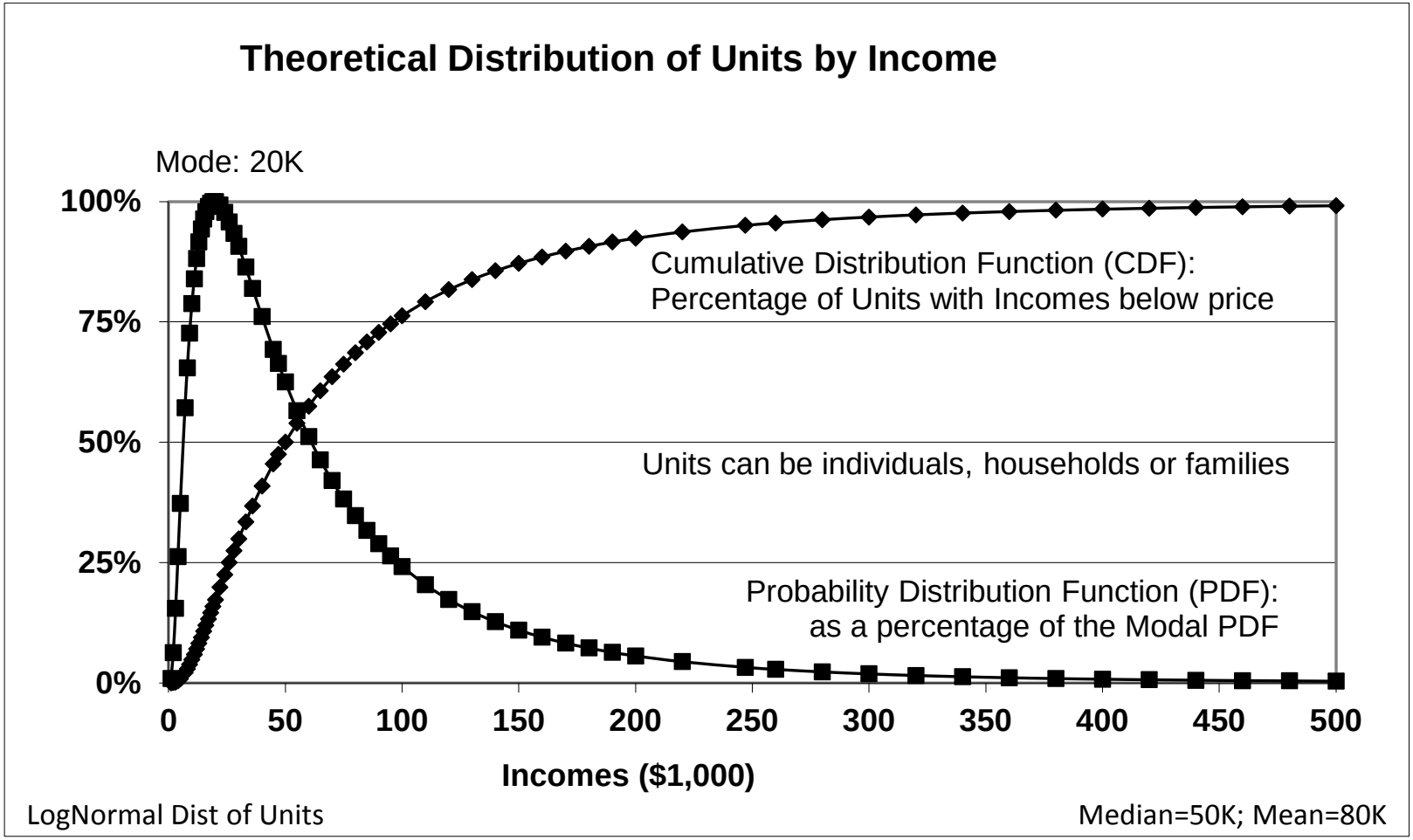
Distribution of annual household income in the United States

(2012 estimate)



Source: U.S. Census Bureau, Current Population Survey, 2012 Annual Social and Economic Supplement

Log-Normal Distribution of Units



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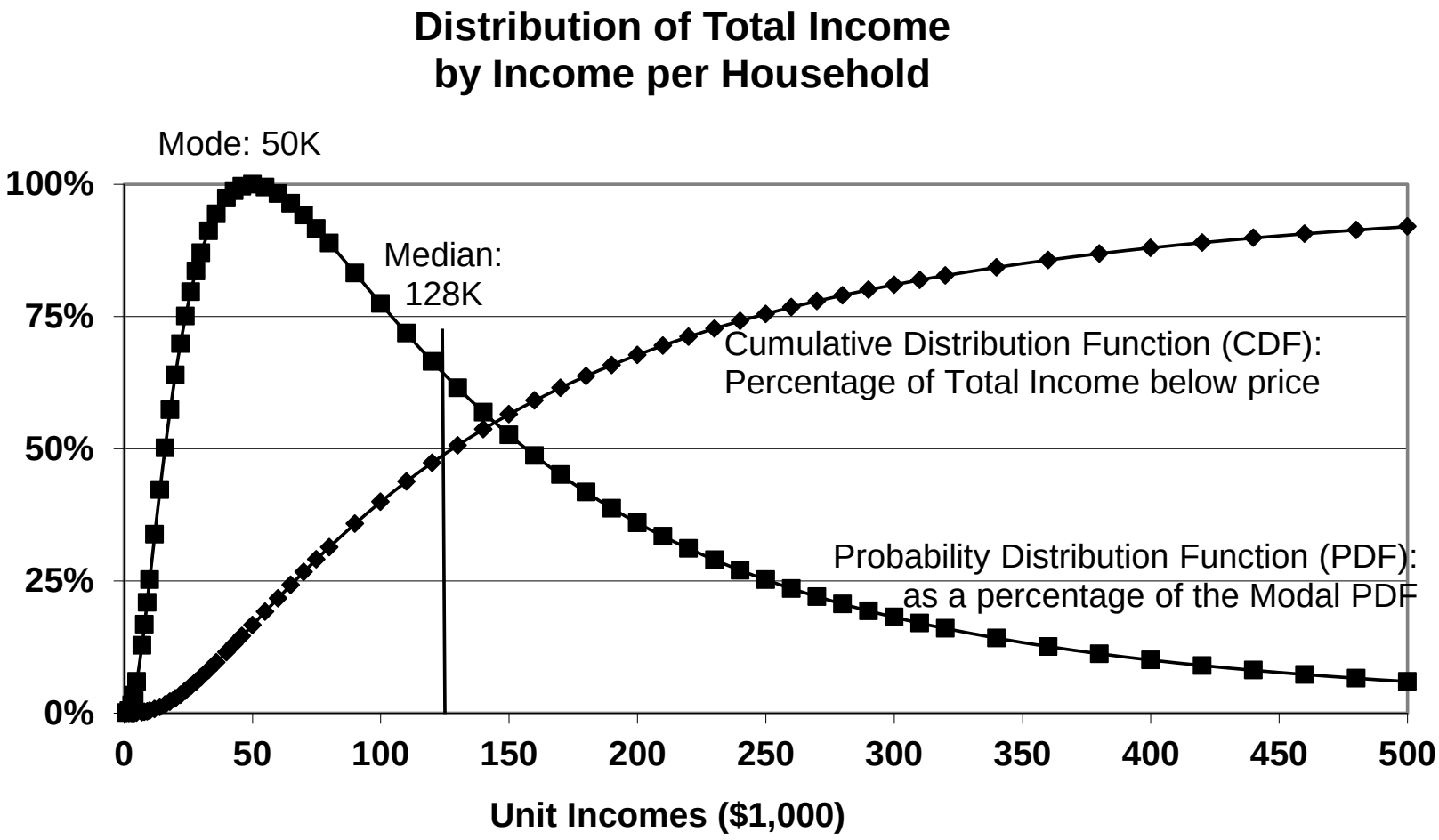
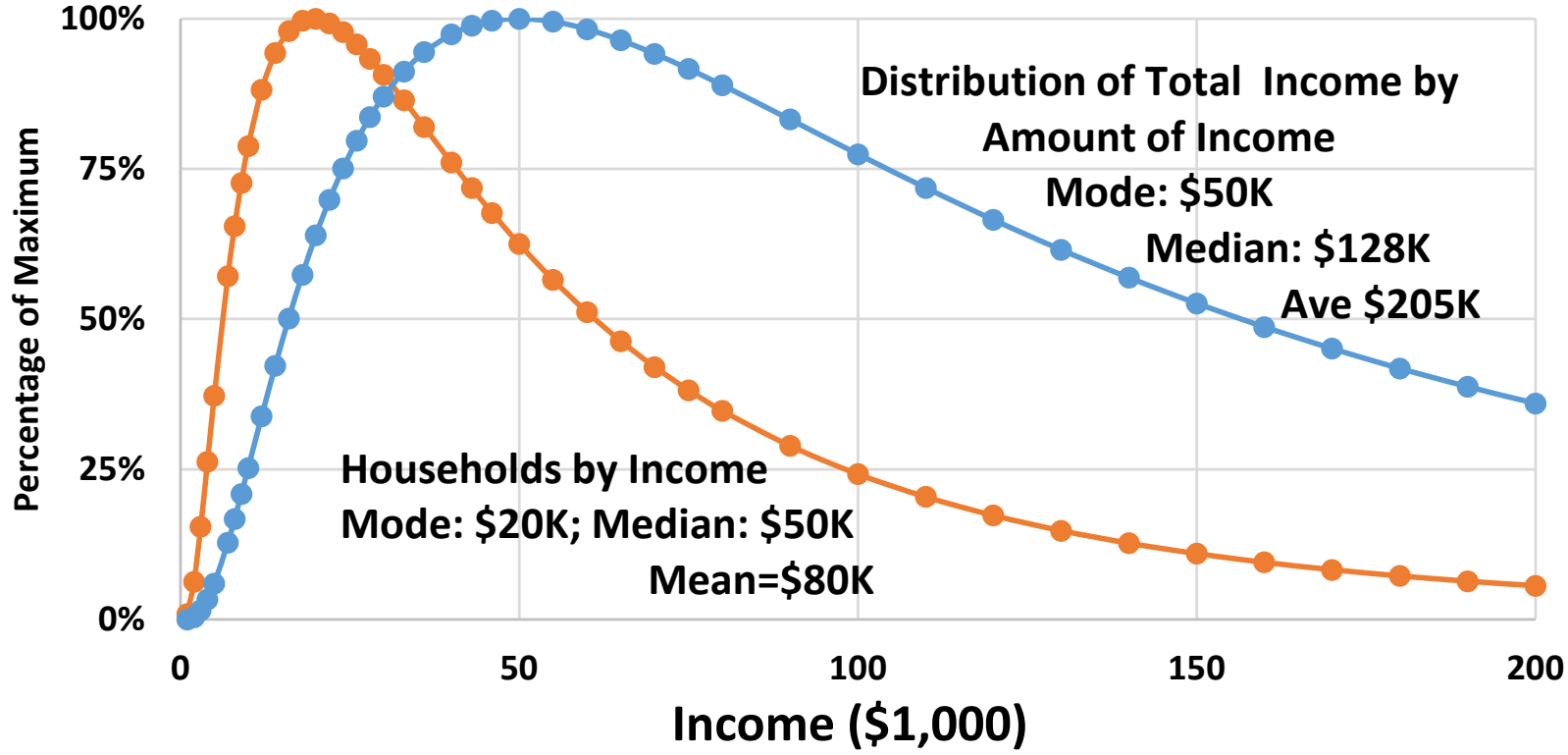


LogNormal Dist of Units by Income

Median=50K; Mean=80K

Distribution of Households and Total Income



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Income/House: Mean=80K; Median=50K

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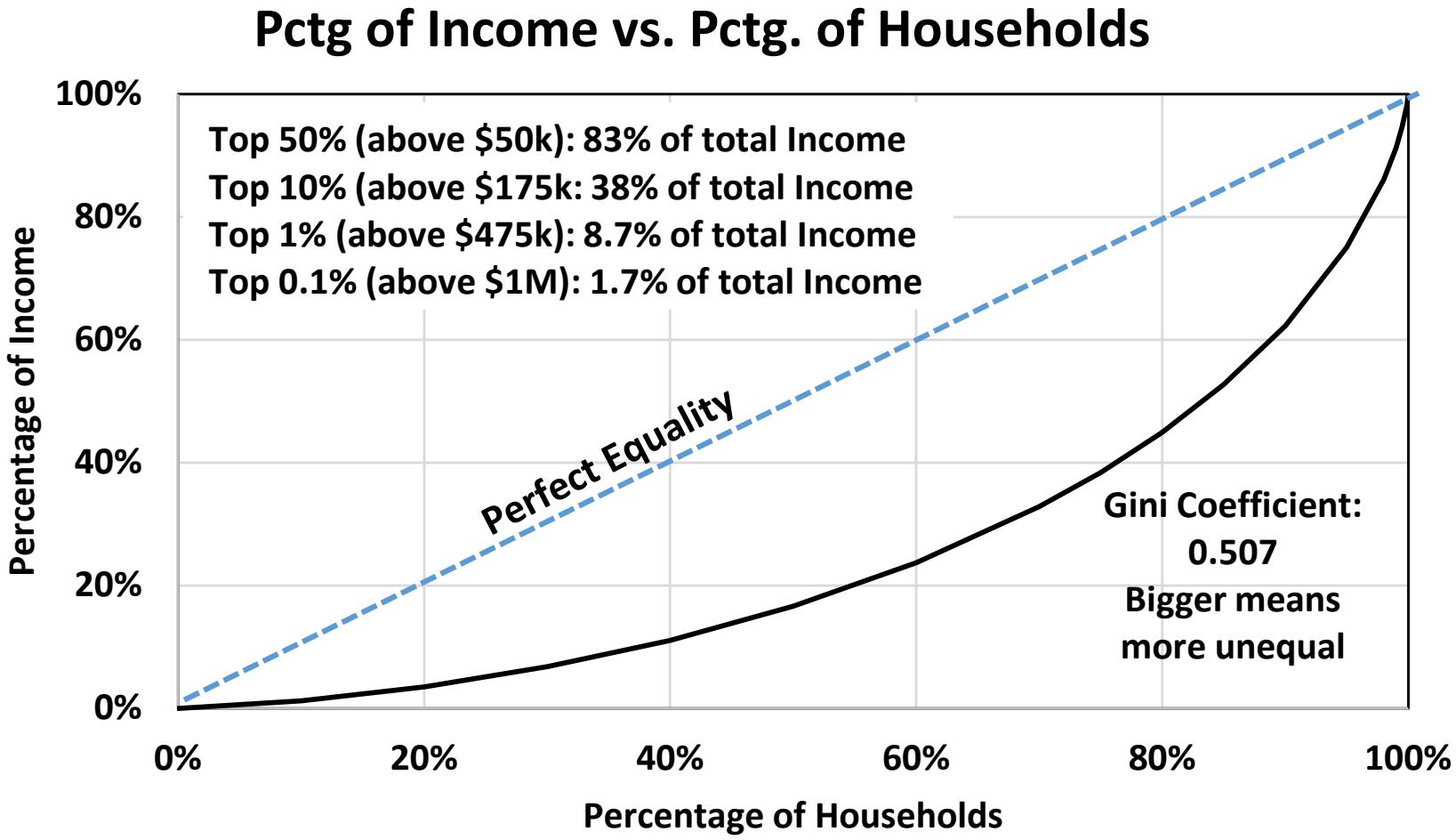

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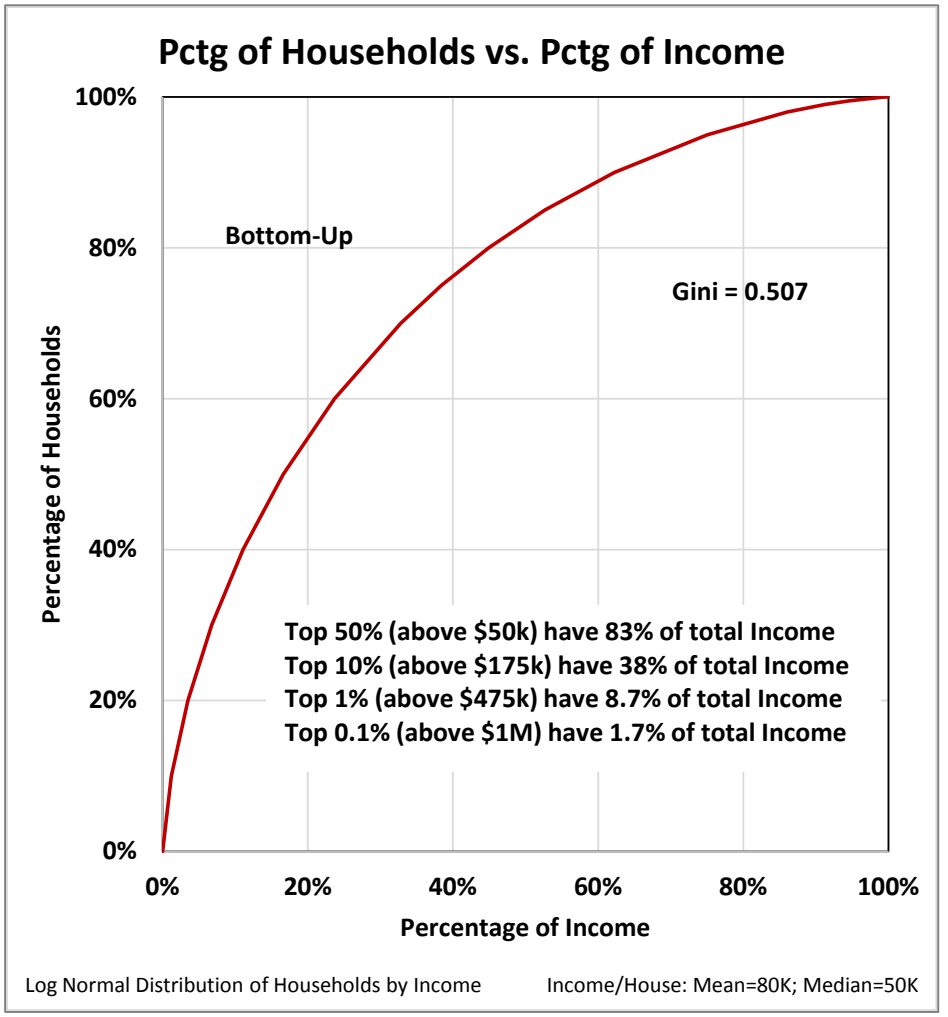

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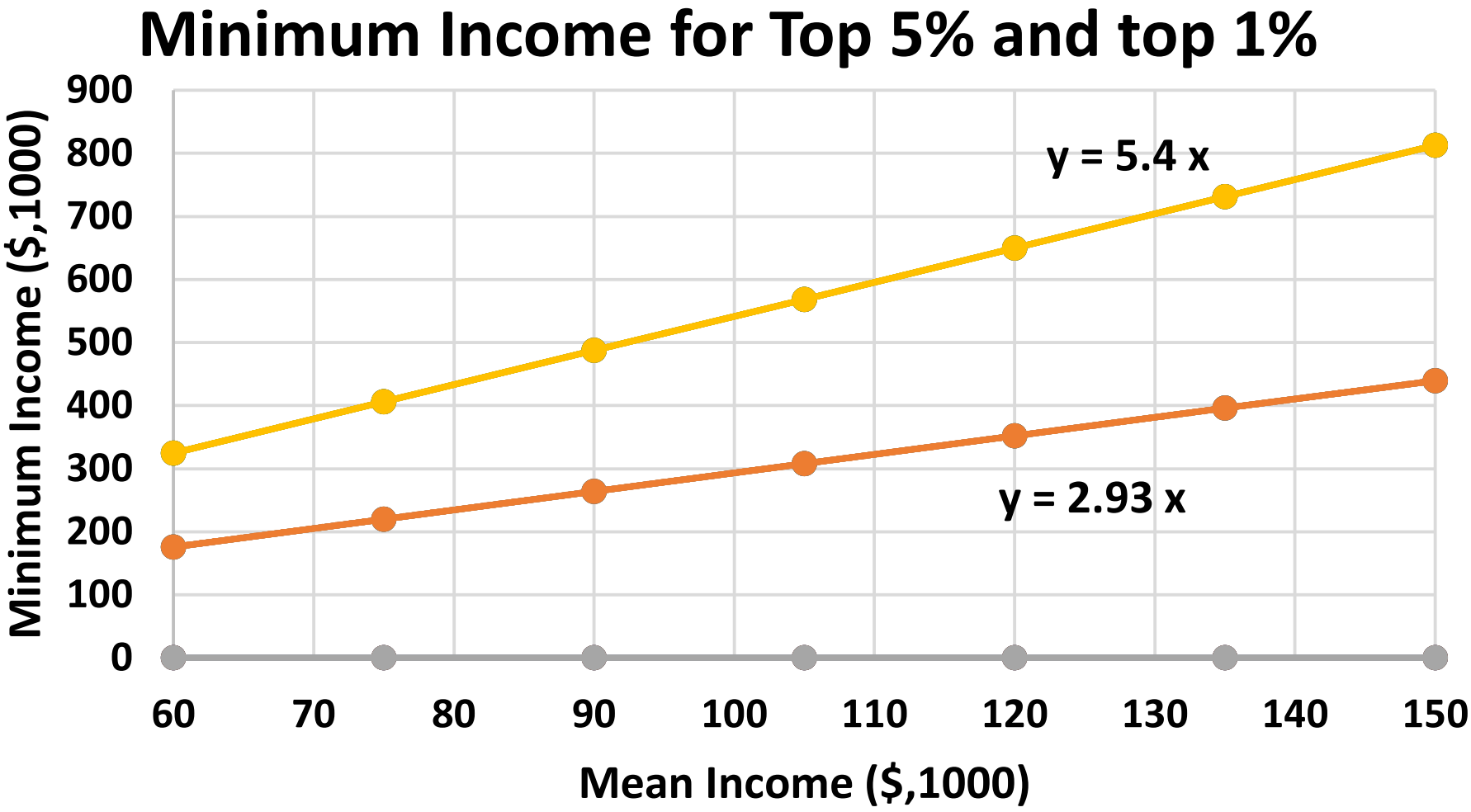

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