Distribution of Households (HH) and Total Income by HH Income is Log-Normal given Mean and Median

INPUTS	**50**	Median	Assume that Households are distributed log-normally by income.
	80	Mean	It follows that total income is also distributed log-normally.
OUTPUT	0.507	Gini Coefficient	
	0.686	Pctg of Tot Income that goes to those Below Ave	

Table 1 : Sort by Percentile: Bottom-up percentiles on left (A-C); Top-down on the right (E-G)

Cell	ID	Definition
A25	%#Up	X: The percentile of subjects by income
B25	$Cutoff#	The income needed for a subject to be at the Xth percentile by income
C25	%$Up	The percentage of total income earned by subjects having an income **below** the cut
E25	%#down	Y: The percentage of subjects having incomes **above** the cutoff.
F25	%$down	Percentage of total income earned by the **top** Y percentage of subjects by income
G25	own:%$ / %	TimesEqualShare: **Top-down** ratio of cumulative pctg of income to cumul. pctg. of s
H25	Ave$	The average income of subjects in the **top Y** percentage of subjects by income
I26	/$Cutoff#	Ratio: Average income above the Xth percentile to Cutoff income at the Xth percenti

A	B	C	D	E	F	G	H	I
----------BOTTOM-UP----------			**Table 1**	---TOP_DOWN---		Times=Share:	Above	AboveAve$
%#Up	$Cutoff#	%$Up		%#down	%$down	Down:%$ / %#	Ave$	/$Cutoff#
0%	0.0	0.00%		100%	100.0%	1.0	80	
10%	14.4	1.22%		90%	98.8%	1.1	88	6.1
20%	22.1	3.51%		80%	96.5%	1.2	96	4.4
30%	30.1	6.76%		70%	93.2%	1.3	107	3.5
40%	39.1	11.07%		60%	88.9%	1.5	119	3.0
50%	50.0	16.61%		50%	83.4%	1.7	133	2.7
60%	63.9	23.69%		40%	76.3%	1.9	153	2.4
70%	83.1	32.81%		30%	67.2%	2.2	179	2.2
75%	96.2	38.40%		25%	61.6%	2.5	197	2.1
80%	113.1	44.91%		20%	55.1%	2.8	220	1.9
85%	136.6	52.67%		15%	47.3%	3.2	252	1.8
90%	173.2	62.25%		10%	37.8%	3.8	302	1.7
95%	246.4	75.03%		5%	25.0%	5.0	400	1.6
98%	366.2	86.09%		2%	13.9%	7.0	557	1.5
99%	477.0	91.26%		1%	8.7%	8.7	699	1.5
99.5%	607.5	94.59%		0.5%	5.4%	10.8	866	1.4
99.9%	1,000.4	98.30%		0.1%	1.7%	17.0	1,358	1.4
99.95%	1,214.8	98.99%		0.05%	1.0%	20.3	1,623	1.3
99.99%	1,840.4	99.70%		0.01%	0.3%	29.8	2,388	1.3

Copy of output at www.statlit.org/pdf/2014-Schield-LogNormal-Income2B-Excel2013-Demo.pdf

Table 2 : Sorted by Income Ignore columns B & C, and F, G & H.

A55	Income	X: The income (in $1,000)
D55	CDF#	The percentage of **subjects** that have incomes **BELOW** X
I55	CDF$	The percentage of **total income** that is earned by subjects having incomes **BELOW**

A	B	C	D	E	F	G	H	I

Table 2 Distribution of Subjects by Income

Income	PDF#	% of mode	CDF#
1	1.20E-04	0.91%	0.00%
5	4.90E-03	37.25%	0.88%
10	1.04E-02	78.79%	4.85%
15	1.27E-02	96.36%	10.72%
20	1.32E-02	99.97%	17.23%
25	1.27E-02	96.81%	23.73%
30	1.19E-02	90.67%	29.91%
35	1.10E-02	83.44%	35.65%
40	1.00E-02	76.08%	40.90%
45	9.09E-03	69.04%	45.67%
50	8.23E-03	62.50%	50.00%
55	7.45E-03	56.54%	53.92%
60	6.74E-03	51.17%	57.46%
65	6.10E-03	46.35%	60.67%
70	5.53E-03	42.03%	63.57%
75	5.03E-03	38.18%	66.21%
80	4.57E-03	34.73%	68.61%
85	4.17E-03	31.65%	70.79%
90	3.80E-03	28.89%	72.78%
95	3.48E-03	26.42%	74.60%
100	3.19E-03	24.20%	76.27%
110	2.69E-03	20.41%	79.20%
120	2.28E-03	17.32%	81.67%
130	1.95E-03	14.79%	83.78%
140	1.67E-03	12.70%	85.59%
150	1.44E-03	10.96%	87.14%
160	1.25E-03	9.51%	88.49%
170	1.09E-03	8.29%	89.66%
180	9.55E-04	7.25%	90.68%
190	8.39E-04	6.37%	91.57%
200	7.40E-04	5.62%	92.36%
220	5.82E-04	4.42%	93.68%
240	4.63E-04	3.52%	94.72%
260	3.73E-04	2.83%	95.55%
280	3.03E-04	2.30%	96.22%
300	2.49E-04	1.89%	96.77%
320	2.06E-04	1.56%	97.22%
340	1.71E-04	1.30%	97.60%
350	1.57E-04	1.19%	97.76%
400	1.03E-04	0.78%	98.40%
500	4.90E-05	0.37%	99.12%
800	8.62E-06	0.07%	99.79%
1,000	3.48E-06	0.03%	99.90%
2,000	1.48E-07	0.00%	99.99%
5,000	1.04E-09	0.00%	100.00%
10,000	1.35E-11	0.00%	100.00%
50,000	7.81E-17	0.00%	100.00%
100,000	1.85E-19	0.00%	100.00%

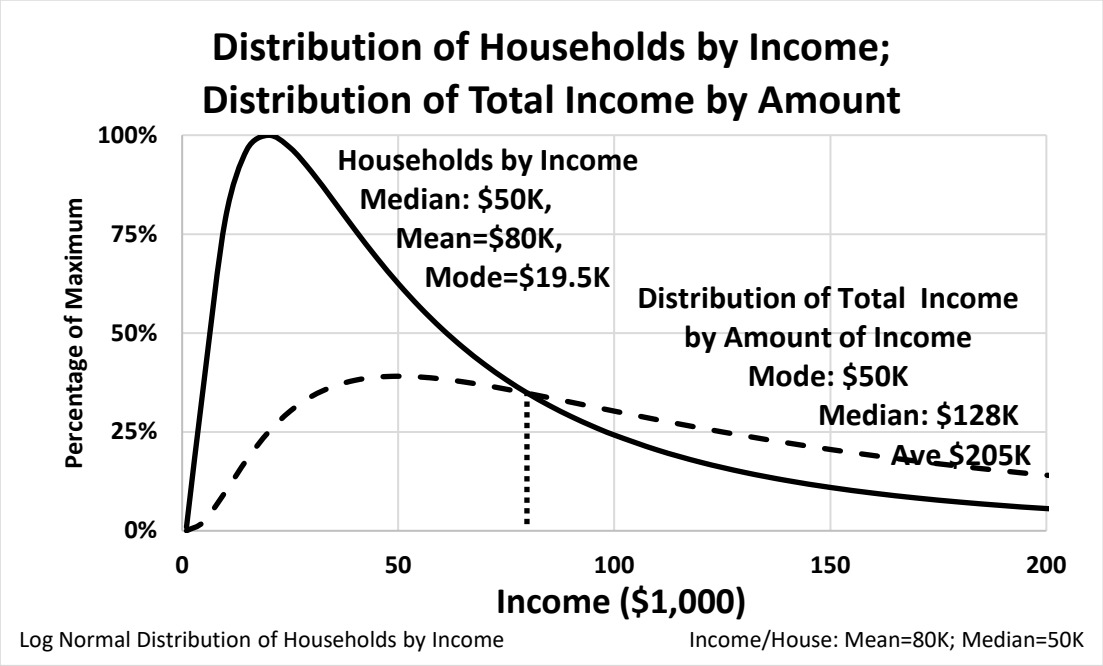
Distribution of Total Income by Amount

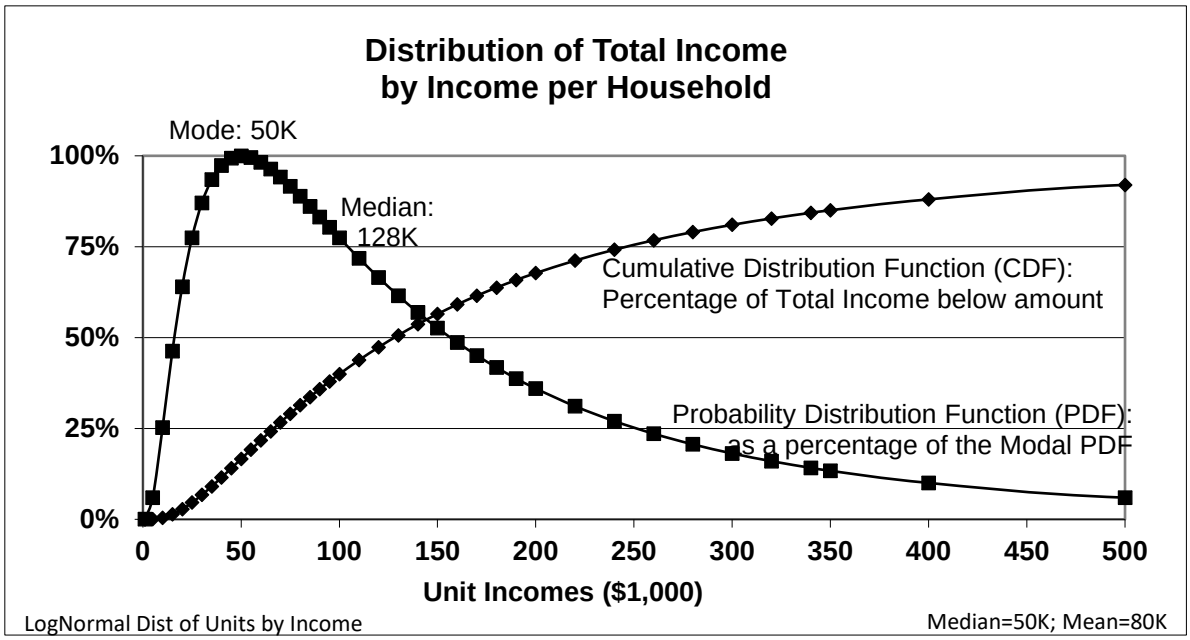
PDF$	% of $mode	%of#mode	CDF$
1.50E-06	0.03%	0.01%	0.00%
3.07E-04	5.96%	2.33%	0.04%
1.30E-03	25.21%	9.85%	0.43%
2.38E-03	46.25%	18.07%	1.35%
3.29E-03	63.98%	24.99%	2.78%
3.98E-03	77.45%	30.25%	4.60%
4.48E-03	87.04%	34.00%	6.73%
4.81E-03	93.46%	36.51%	9.05%
5.01E-03	97.39%	38.04%	11.51%
5.11E-03	99.41%	38.83%	14.05%
5.14E-03	100.00%	39.06%	16.61%
5.12E-03	99.52%	38.87%	19.18%
5.05E-03	98.25%	38.38%	21.73%
4.96E-03	96.40%	37.66%	24.23%
4.84E-03	94.16%	36.78%	26.68%
4.71E-03	91.63%	35.79%	29.07%
4.57E-03	88.91%	34.73%	31.39%
4.43E-03	86.09%	33.63%	33.64%
4.28E-03	83.21%	32.50%	35.82%
4.13E-03	80.32%	31.38%	37.92%
3.98E-03	77.45%	30.25%	39.95%
3.70E-03	71.84%	28.06%	43.79%
3.42E-03	66.52%	25.98%	47.35%
3.16E-03	61.53%	24.04%	50.64%
2.93E-03	56.90%	22.23%	53.68%
2.71E-03	52.62%	20.56%	56.50%
2.50E-03	48.69%	19.02%	59.10%
2.32E-03	45.09%	17.61%	61.51%
2.15E-03	41.78%	16.32%	63.74%
1.99E-03	38.75%	15.14%	65.81%
1.85E-03	35.98%	14.05%	67.74%
1.60E-03	31.11%	12.15%	71.18%
1.39E-03	27.01%	10.55%	74.16%
1.21E-03	23.56%	9.20%	76.76%
1.06E-03	20.62%	8.06%	79.03%
9.32E-04	18.13%	7.08%	81.02%
8.23E-04	16.00%	6.25%	82.77%
7.28E-04	14.16%	5.53%	84.32%
6.86E-04	13.34%	5.21%	85.02%
5.16E-04	10.03%	3.92%	88.00%
3.07E-04	5.96%	2.33%	92.00%
8.619E-05	1.68%	0.65%	97.06%
4.346E-05	0.85%	0.33%	98.30%
3.696E-06	0.07%	0.03%	99.77%
6.489E-08	0.00%	0.00%	99.99%
1.68E-09	0.00%	0.00%	100.00%
4.88E-14	0.00%	0.00%	100.00%
2.32E-16	0.00%	0.00%	100.00%

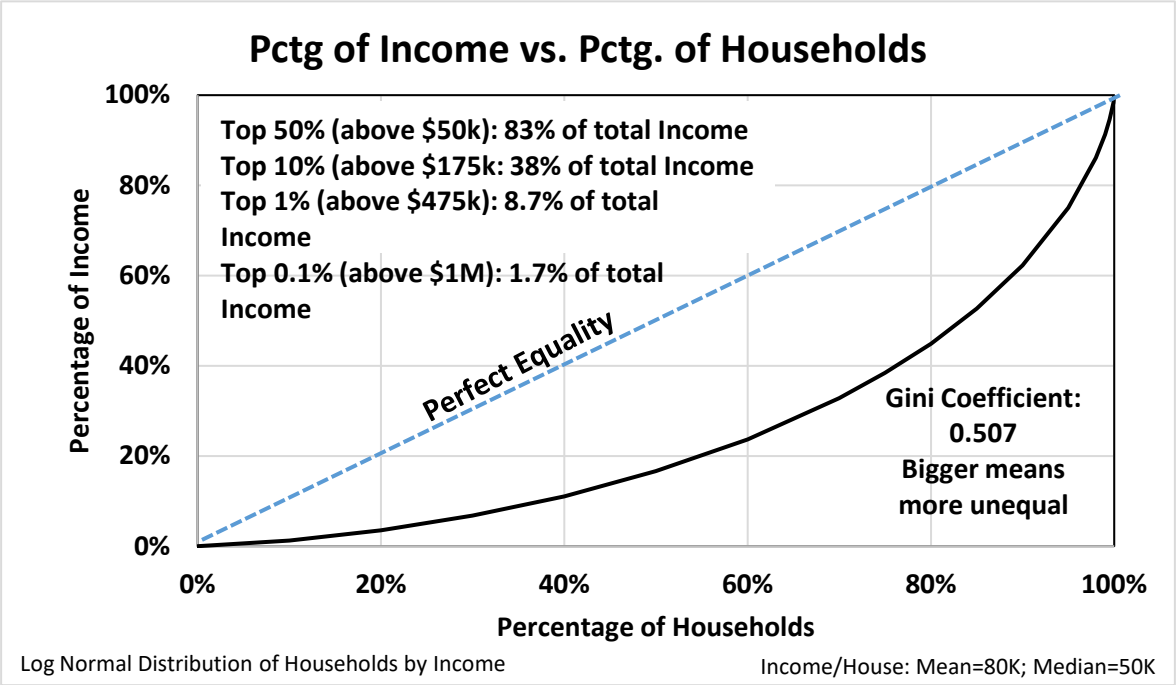
Sample questions (with answers):

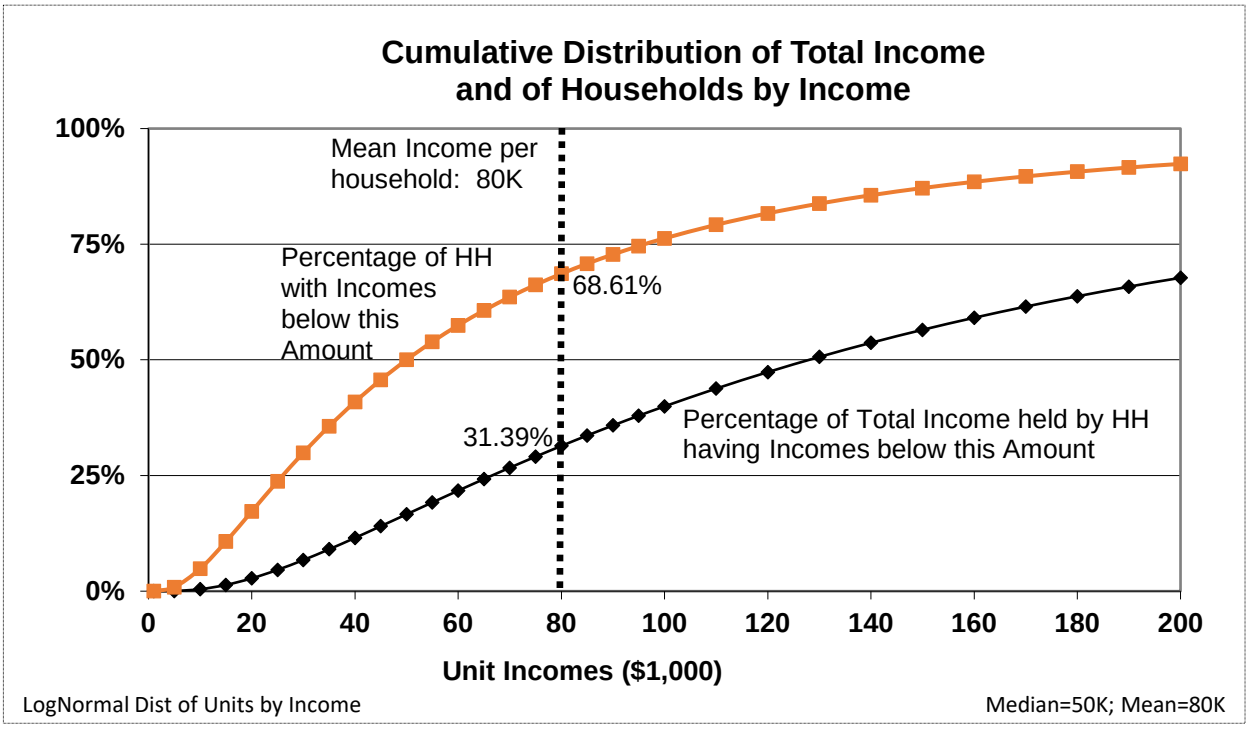
Table 1 is sorted by percentages (bottom up & top down); Table 2 is sorted by Incomes ($1,000)

Answer	Table	**Question**: If Median = 50k and Mean = 80k,
96.2K	1	what is the minimum income ($) needed to be in the top 25% of income earners (#)?
16.6%	1	what percentage of income ($) is earned by the bottom 50% of income earners (#)?
25.0%	1	what percentage of income ($) is earned by the top 5% of income earners?
5.0	2	then the top 5% have ____ times their equal share of total income
68.6%	2	what percentage of subjects (#) have income less than 80K
31.4%	2	what percentage of income ($) is earned by subjects (#) making **less** than 80k
68.6%	arithmetic	what percentage of income ($) is earned by subjects (#) making **more** than 80k









How to protect (and unprotect) cells in a worksheet:
https://support.microsoft.com/en-us/kb/214081

Protect so that users don't accidentally change formulas.

MATH FORMULAS

Households Normal Distribution

mu	3.912	=LN(B2)
mu+S^2/2	4.382	=LN(B3)
Sigma^2	0.940	=2*(R56-R55)
Sigma	0.970	=SQRT(R57)

Mode	19.5	=EXP(R55-R57)
PDF# (Mode)	1.32E-02	=LOGNORM.DIST(M58,R55,R58,0)
Std.Dev	99.9	=SQRT((EXP(R57)-1)*EXP(2*R55+R57))
HH by HH$ < Ave$	0.686	=NORM.S.DIST(SQRT(LN(B3/B2)/2), 1)
Gini Coefficient	0.507	=2*NORM.S.DIST(R58/SQRT(2),1)-1

A	B	C	D	E	F	G	H	I

Since households have a Log-Normal distribution by Income with mu# and sigma#, it follows that total Income has a Log-Normal Distribution by HH Income [Aitchinson & Brown (1963)] with parameters mu$ = (mu# + sigma#^2) and sigma$ = sigma#

Total Income Log-Normal Distribution

Median$	128	=EXP(R68)
Mean$	205	=EXP(R71)
Mode	50.0	=EXP(R68-R70)
PDF (Mode)	5.14E-03	=LOGNORM.DIST(M71,R68,R69,0)
StdDev=Mean*CV	255.8	=M69*SQRT(((M69/M68)^2)-1)
Tot$ by HH$ > Ave$	0.686	=1-NORM.DIST(LN(B3),R68,R69,1)

Total Income Normal Distribution

mu$	*4.852*	=R55+R57
Sigma$	0.970	=R58
Sigma$^2	0.940	=R69^2
mu$+S$^2	5.322	=R68+R70/2

A	B	C	D	E	F	G	H	I

CELL	FormulaText()
Add, B33. C33: Manual entry (Already entered)	
B26	=LOGNORM.INV(A26,R$55,R$58)
C26	=LOGNORM.DIST(B26,R$68,R$69,1)
E25	=1-A25

CELL	FormulaText()
F25	=1-C25
G25	=F25/E25
H25	=B$3*F25/E25
I26	=H26/B26

CELL	FormulaText()
A55	Manual entries (Already enetered)
B55	=LOGNORM.DIST(A55,R55,R58,0)
C55	=B55/M$59
D55	=LOGNORM.DIST(A55,R$55,R$58,1)

CELL	FormulaText()
F55	=LOGNORM.DIST(A55,R$68,R$69,0)
G55	=F55/M$72
H55	=F55/M$59
I55	=LOGNORM.DIST(A55,R$68,R$69,1)

To make reading Table 2 easier, hide columns B-C and F-H. Unhide all to read Table 1