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Logistic Regression: MLE vs. OLS3 in Excel 2013

by
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US Rep: International Statistical Literacy Project
Director, W. M. Keck Statistical Literacy Project

Slides and data at: www.StatLit.org/pdf/2014-Schield-Logistic-MLE-OLS3-Excel2013-slides.pdf
Excel/2014-Schield-Logistic-MLE-OLS3-Excel2013.xlsx

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Background & Goals

Doing logistic regression properly requires MLE.
Doing MLE in Excel is not easy. See Schield 2014a

Schield has identified three OLS shortcuts:
OLS1: Model Ln(Odds(p)) where p is near 0 or 1
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These slides compare OLS3 with MLE.
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Predict Gender using Height

Source: Minitab Pulse dataset

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8	60	76	71	170	3	1	0	1
9	60	62	71	155	2	0	0	1
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Model Gender by Height (OLS) Must use logistic regression

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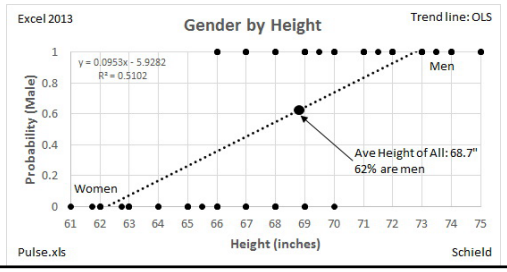

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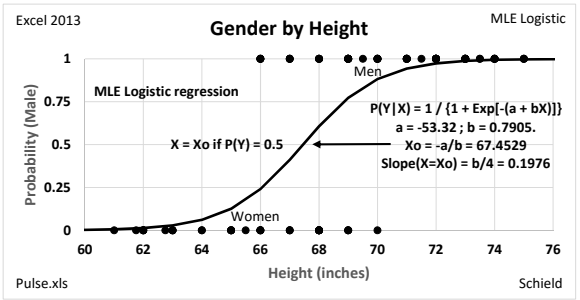
1a: MLE Logistic Regression of Gender by Height



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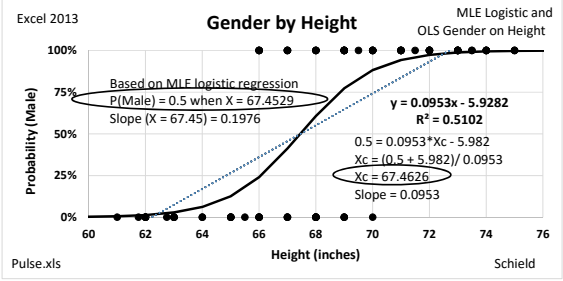
1b: MLE Logistic Regression vs OLS(Y|X) of Gender on Height

Different slopes but they intersect near P(Y) = 0.5



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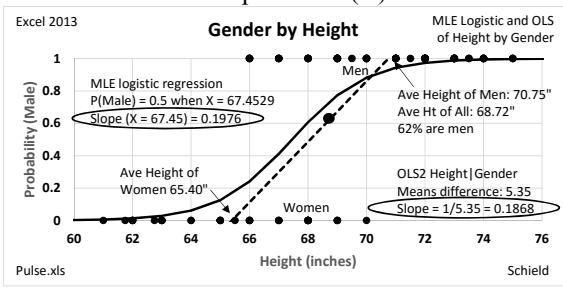
1c: MLE Logistic Regression vs OLS(X|Y) of Height on Gender

Similar slopes near P(Y) = 0.5



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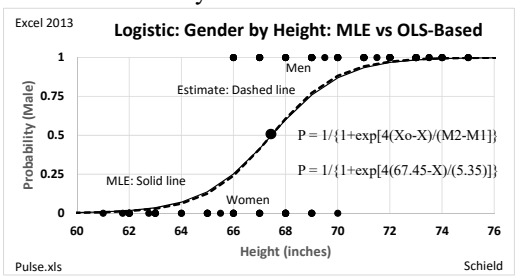
1d: Intersection from OLS(Y|X) Get slope from OLS(X|Y)

Very nice estimate!



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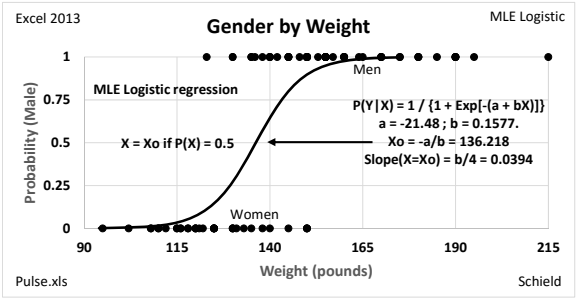
2a: MLE Logistic Regression of Gender by Weight.



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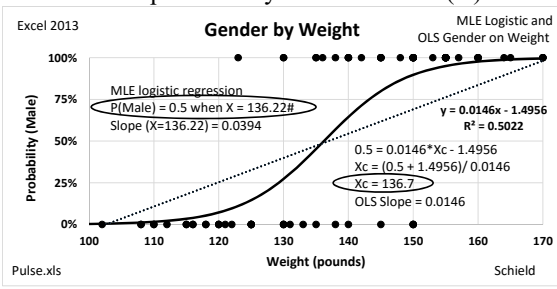
Different slopes but they intersect near P(Y) = 0.5



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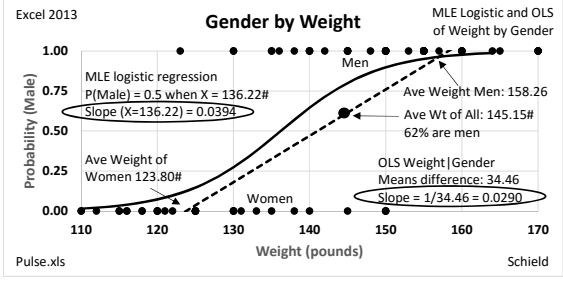
2c: MLE Logistic Regression vs OLS(X|Y) of Weight on Gender

Similar slopes near P(Y) = 0.5



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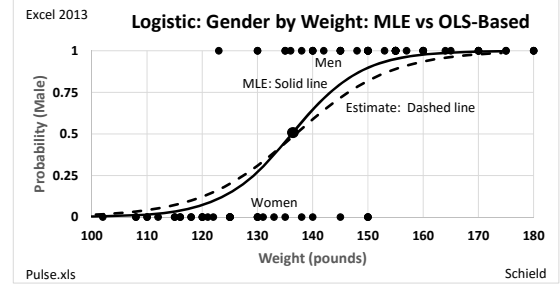
2d: Intersection from OLS(Y|X) Get slope from OLS(X|Y)

Fairly good estimate



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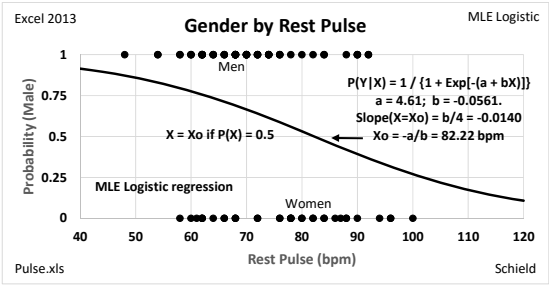
3a: MLE Logistic Regression of Gender by Rest Pulse

Almost flat! Discriminatory power is weak.



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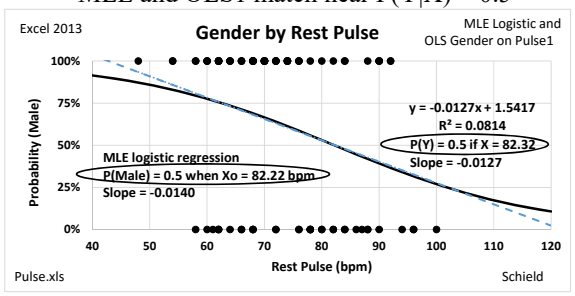
3b: MLE Logistic Regression vs OLS(Y|X) of Gender on Pulse1

MLE and OLS1 match near P(Y|X) = 0.5



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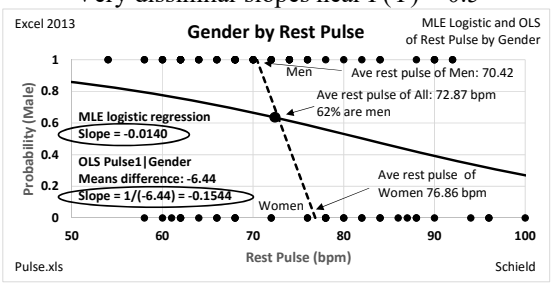
3c: MLE Logistic Regression vs OLS(X|Y) of Gender on Pulse1

Very dissimilar slopes near P(Y) = 0.5



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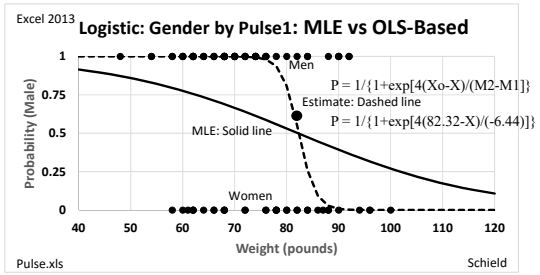
3d: Intersection from OLS(Y|X); Get slope from OLS(X|Y)

Very bad estimate



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Analysis and Conclusion Model Gender:

#1: $R^2 = 51\%$ by Height. "Good estimate"
#2: $R^2 = 50\%$ by Weight. "Fair estimate"
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Conclusion #1: Using OLS(Y|X) for Xo and using OLS(X|Y) for the associated slope works fairly well when the overlap is small or moderate: OLS(Y|X). R-squared is high: > 0.5

Conclusion #2: Must use MLE when the overlap is large: OLS(Y|X) R-squared is low (< 0.5). But why bother if the model explains so little?

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References & Derivation

Schield, Milo (2014a). www.statlit.org/pdf/
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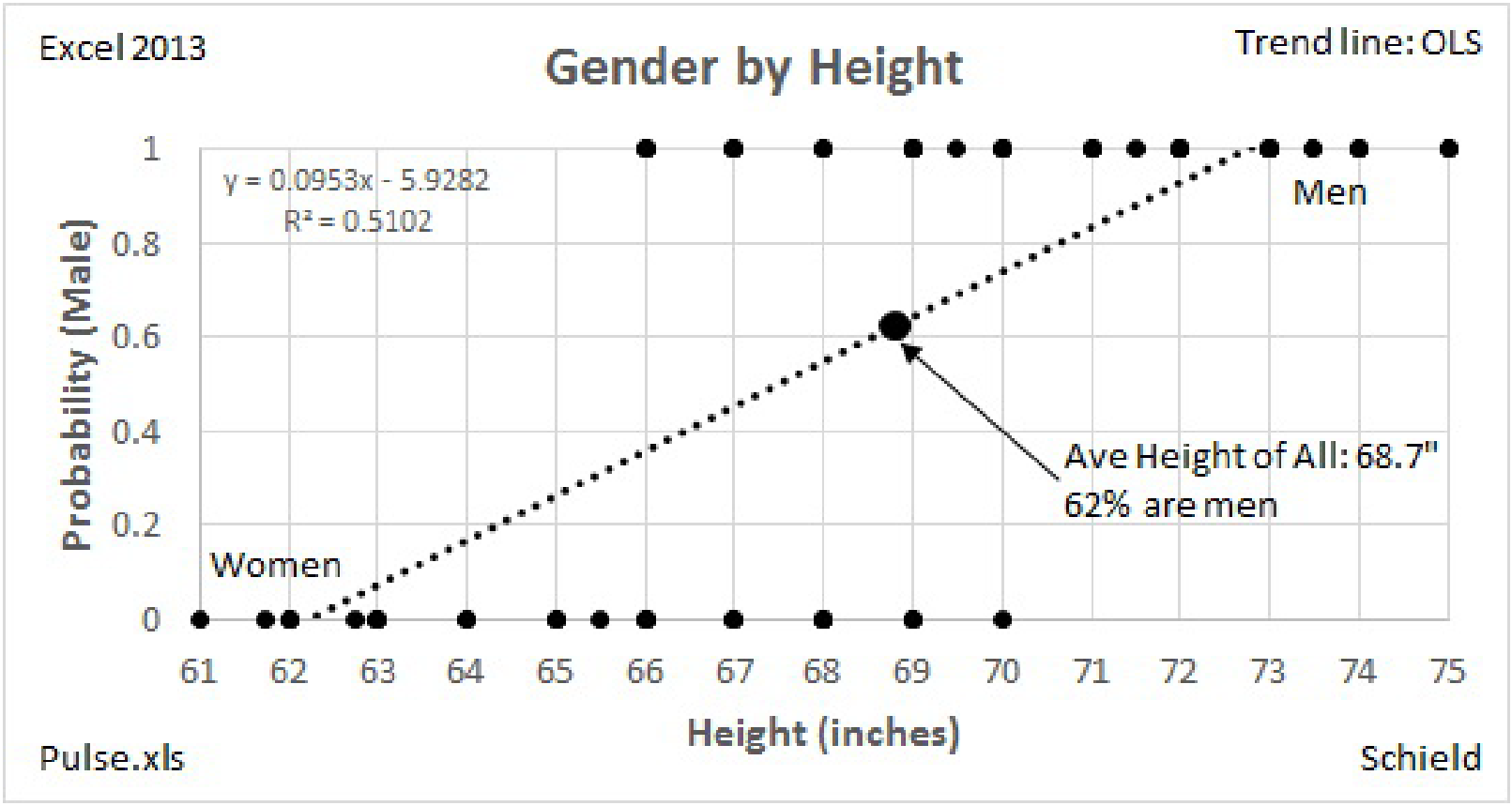

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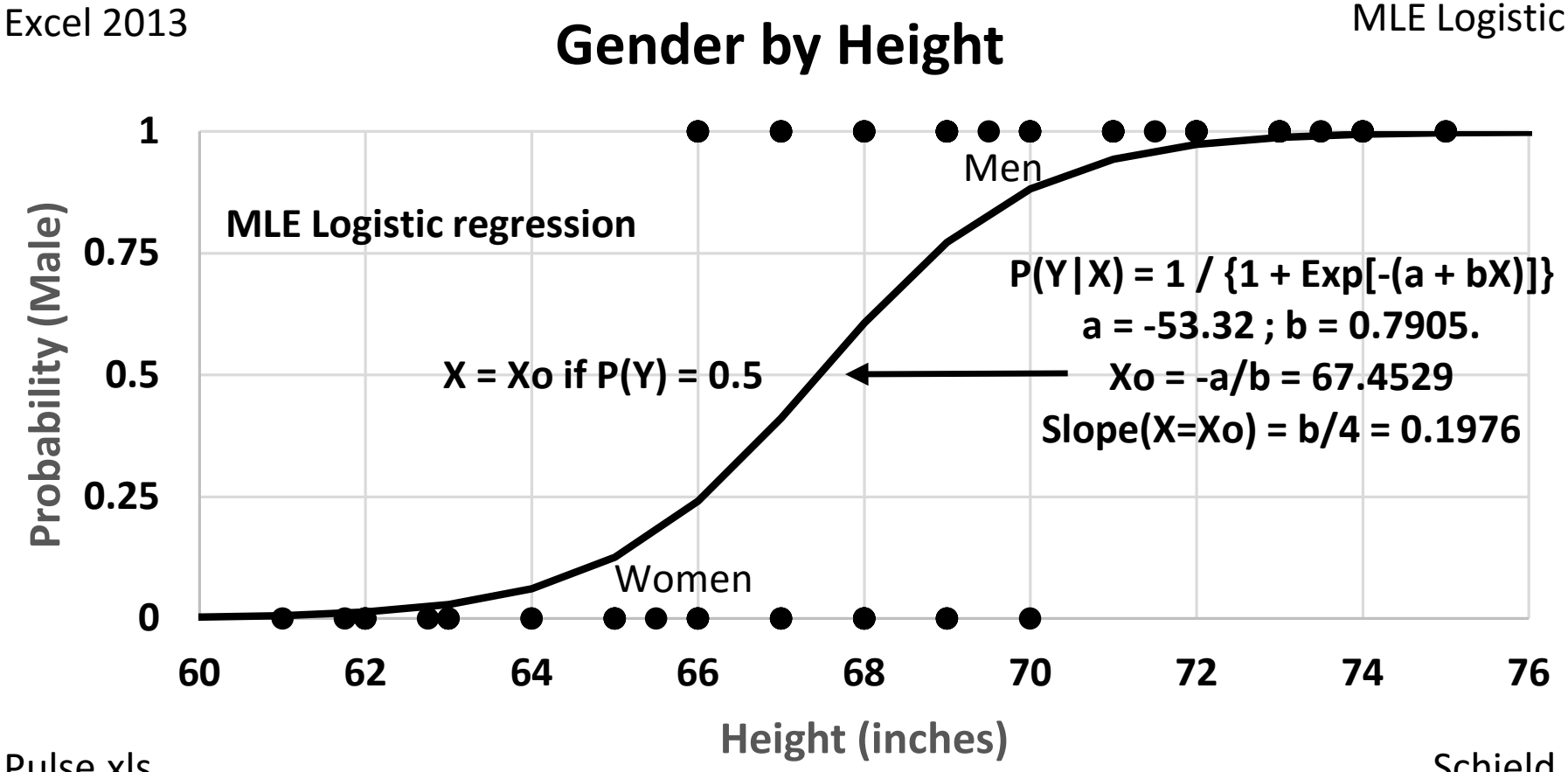

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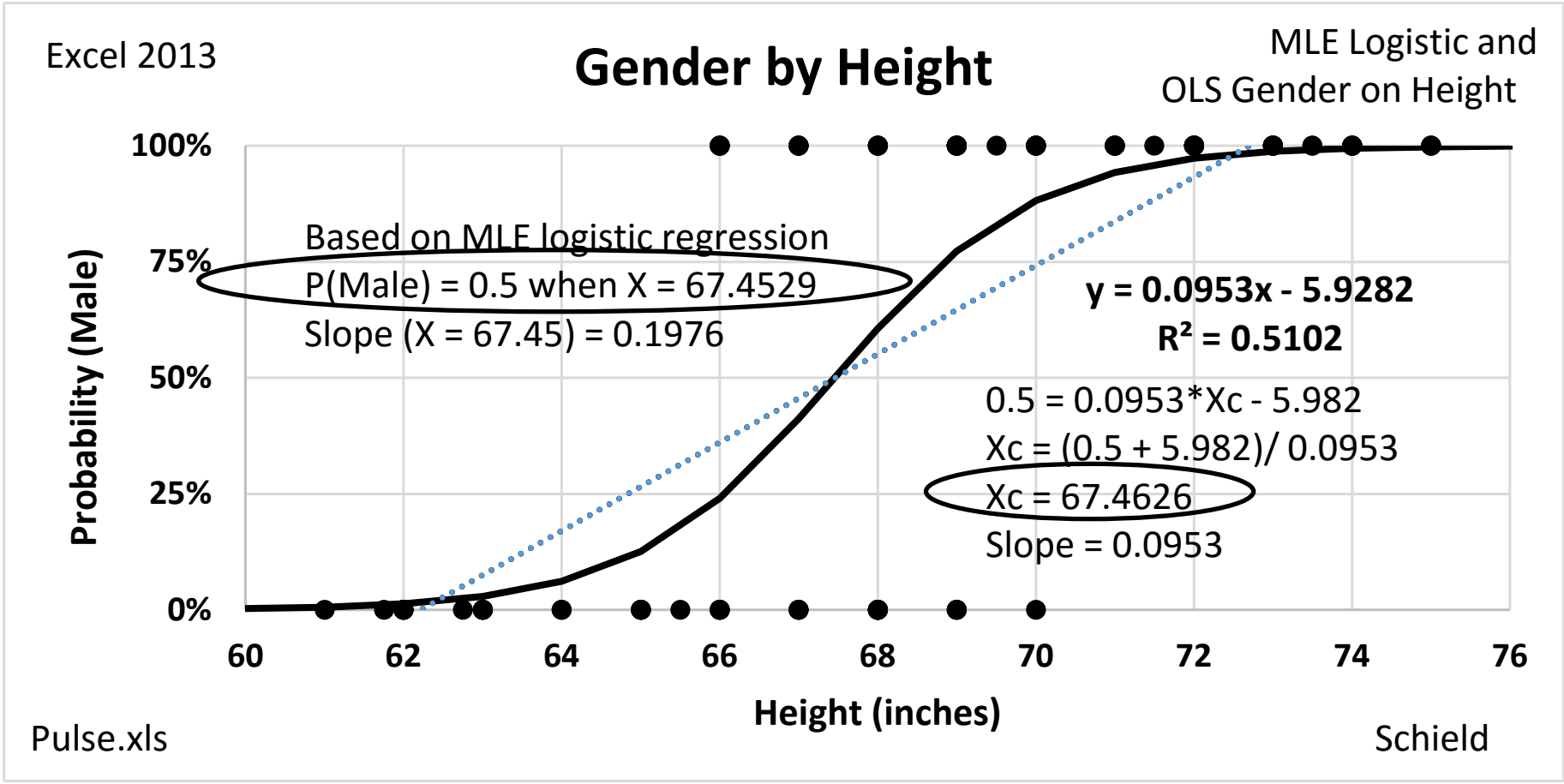
Different slopes but they intersect near P(Y) = 0.5



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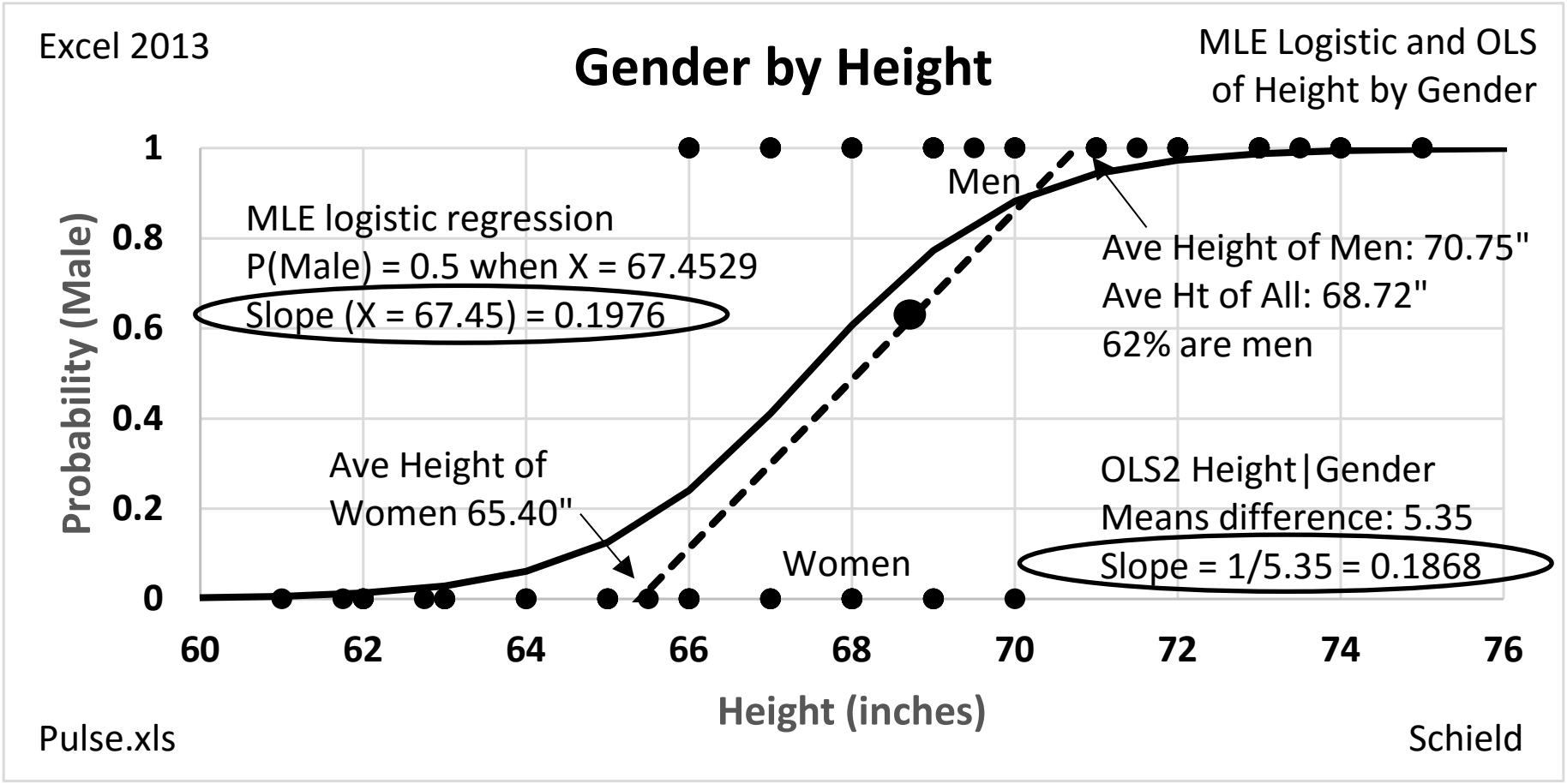
Similar slopes near P(Y) = 0.5



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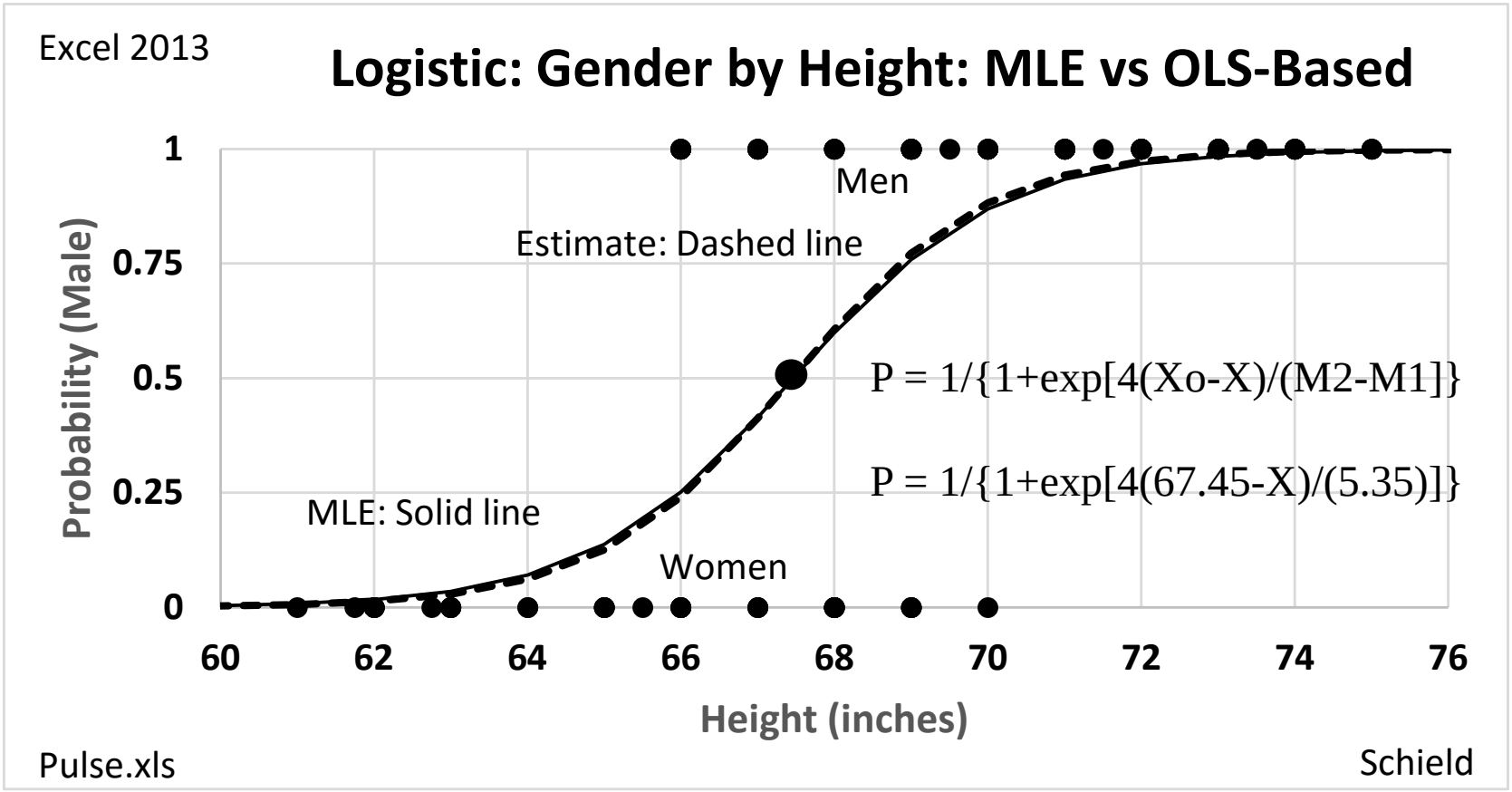

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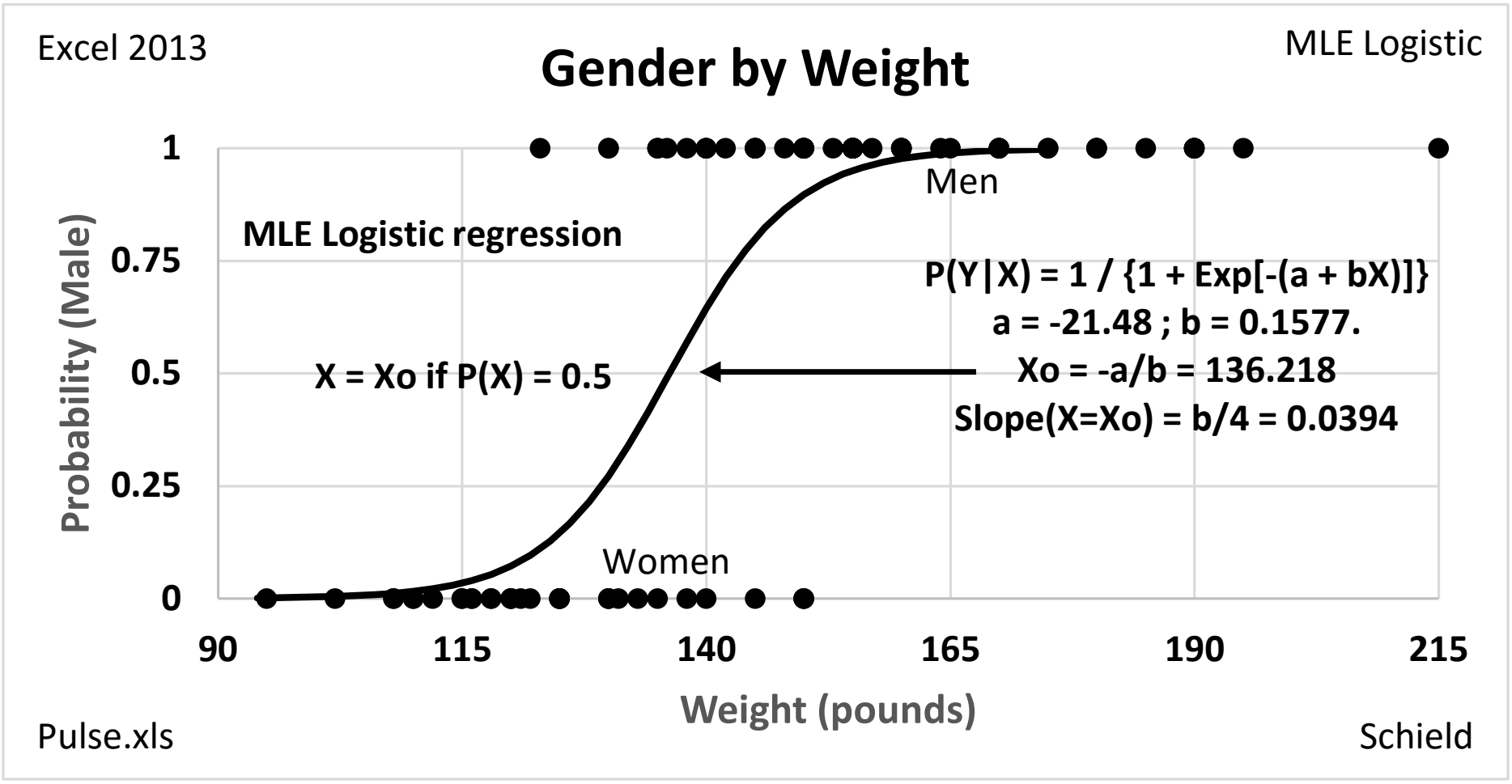

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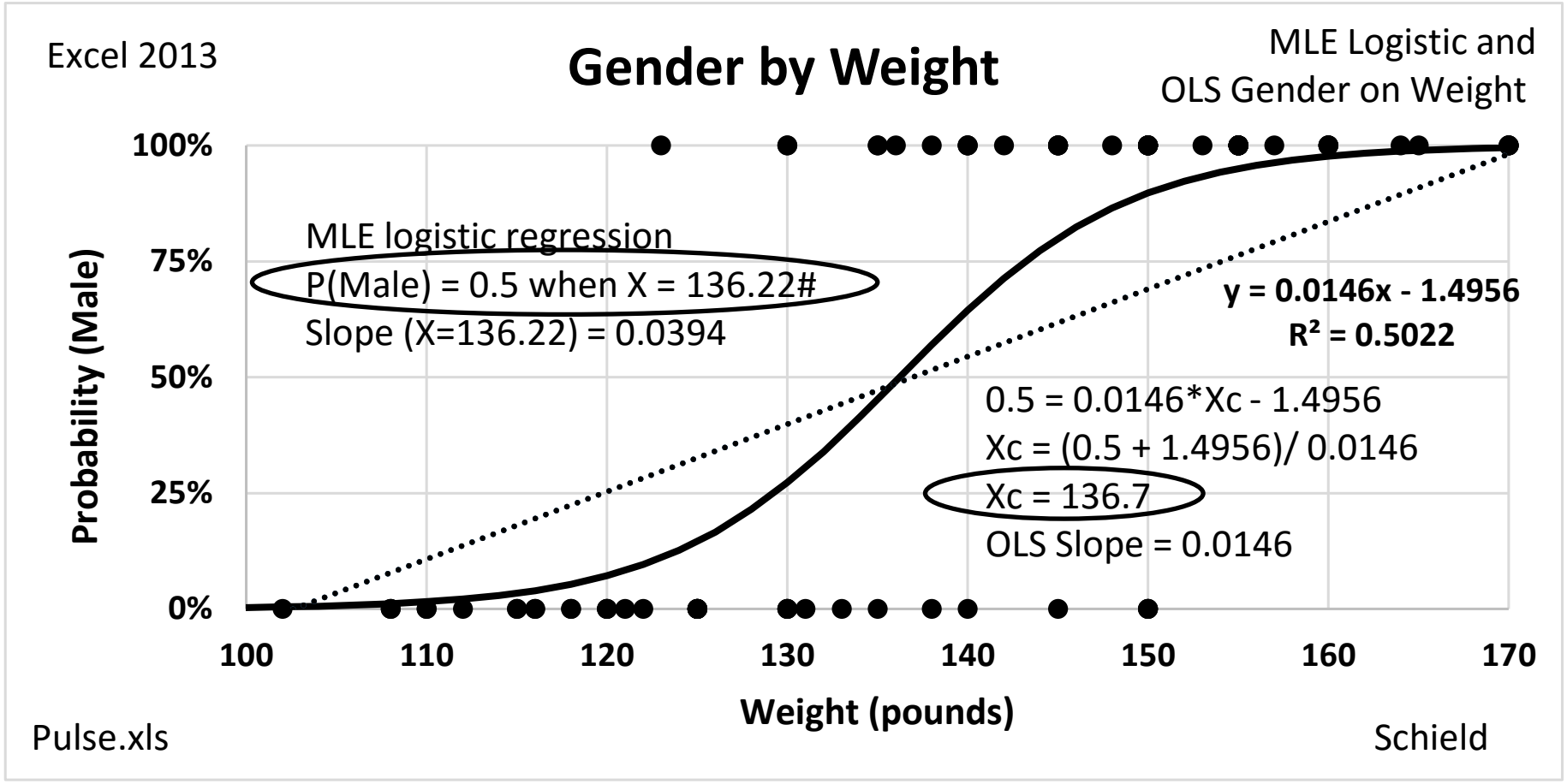

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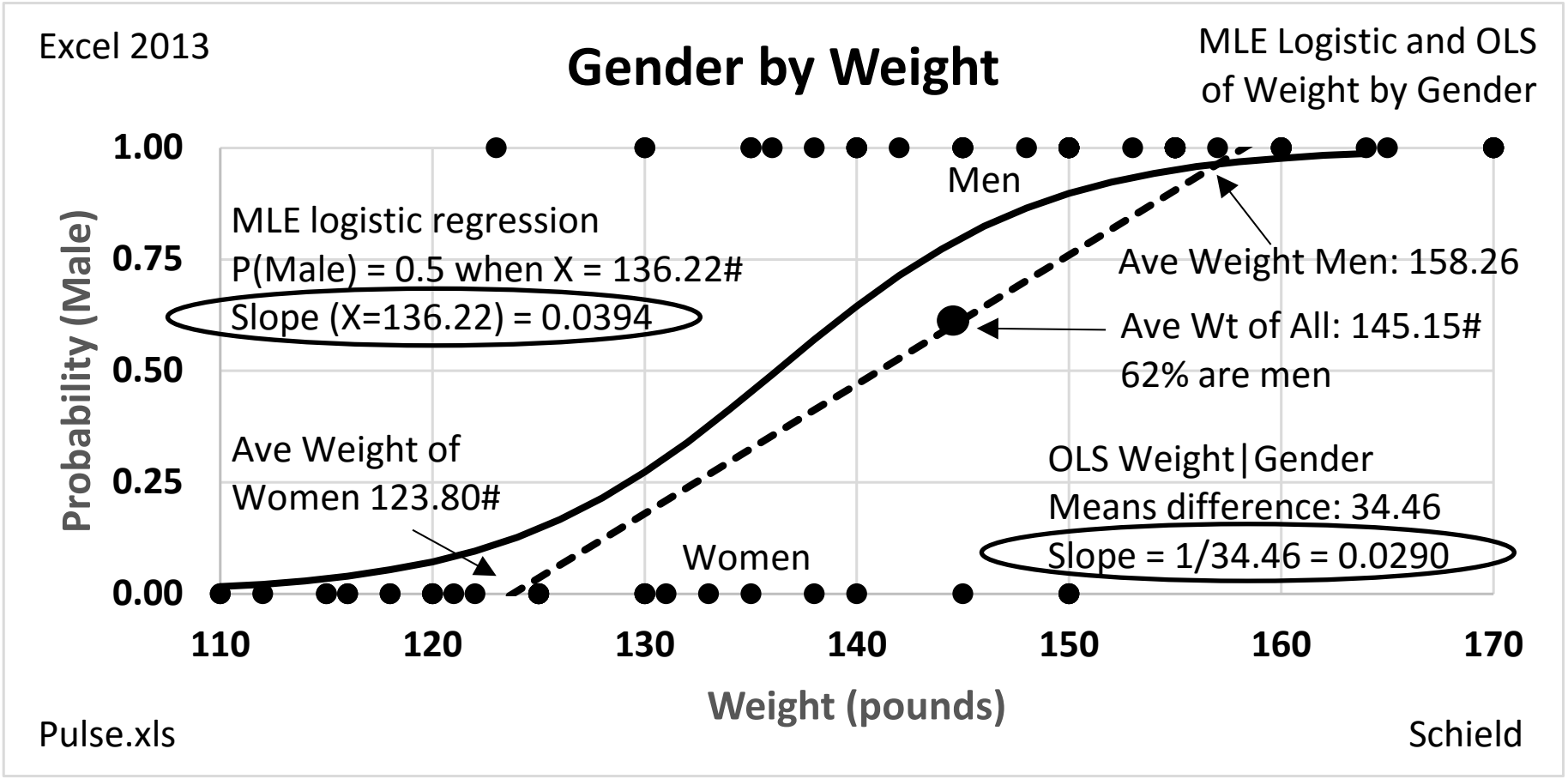

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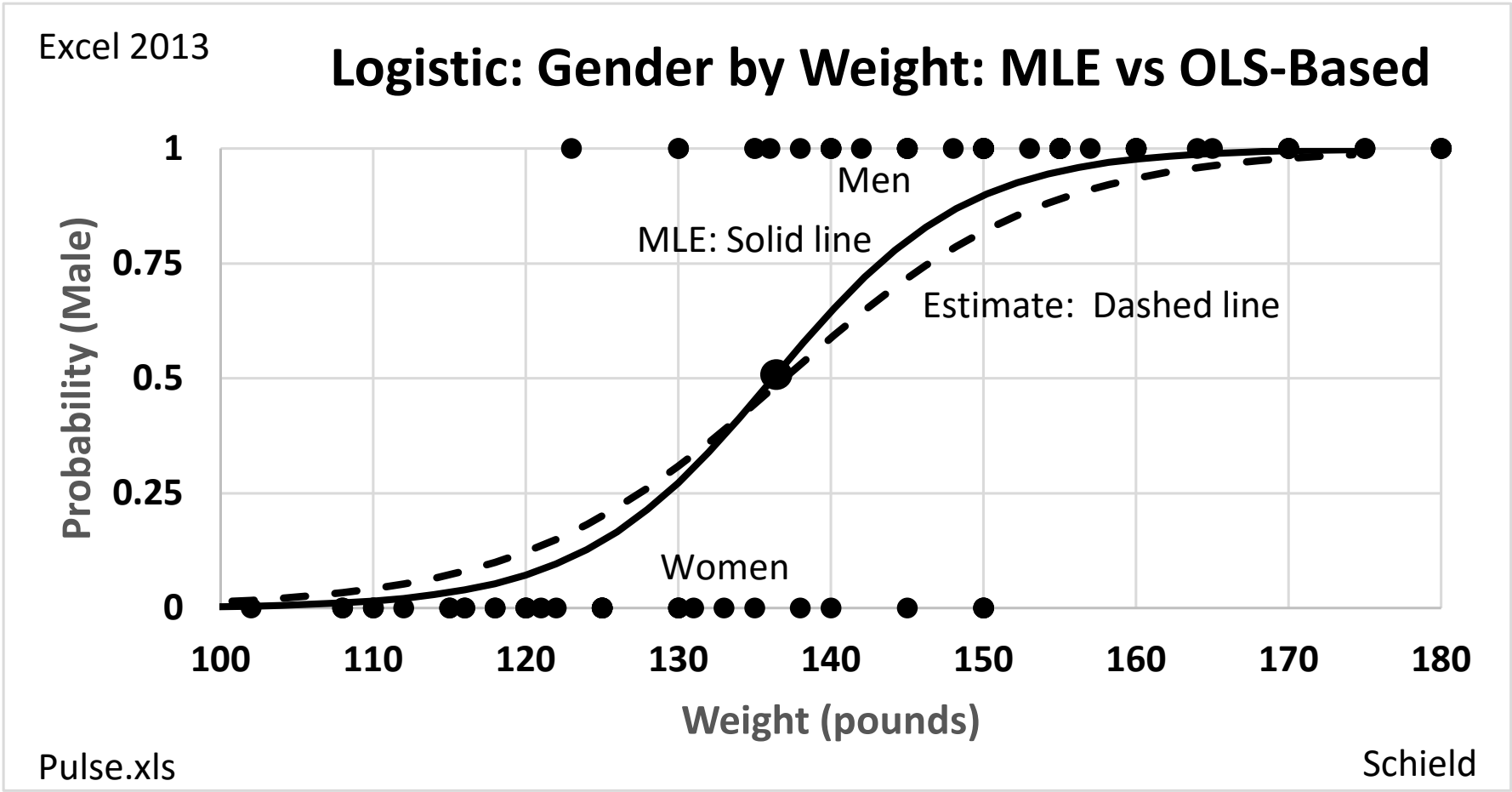

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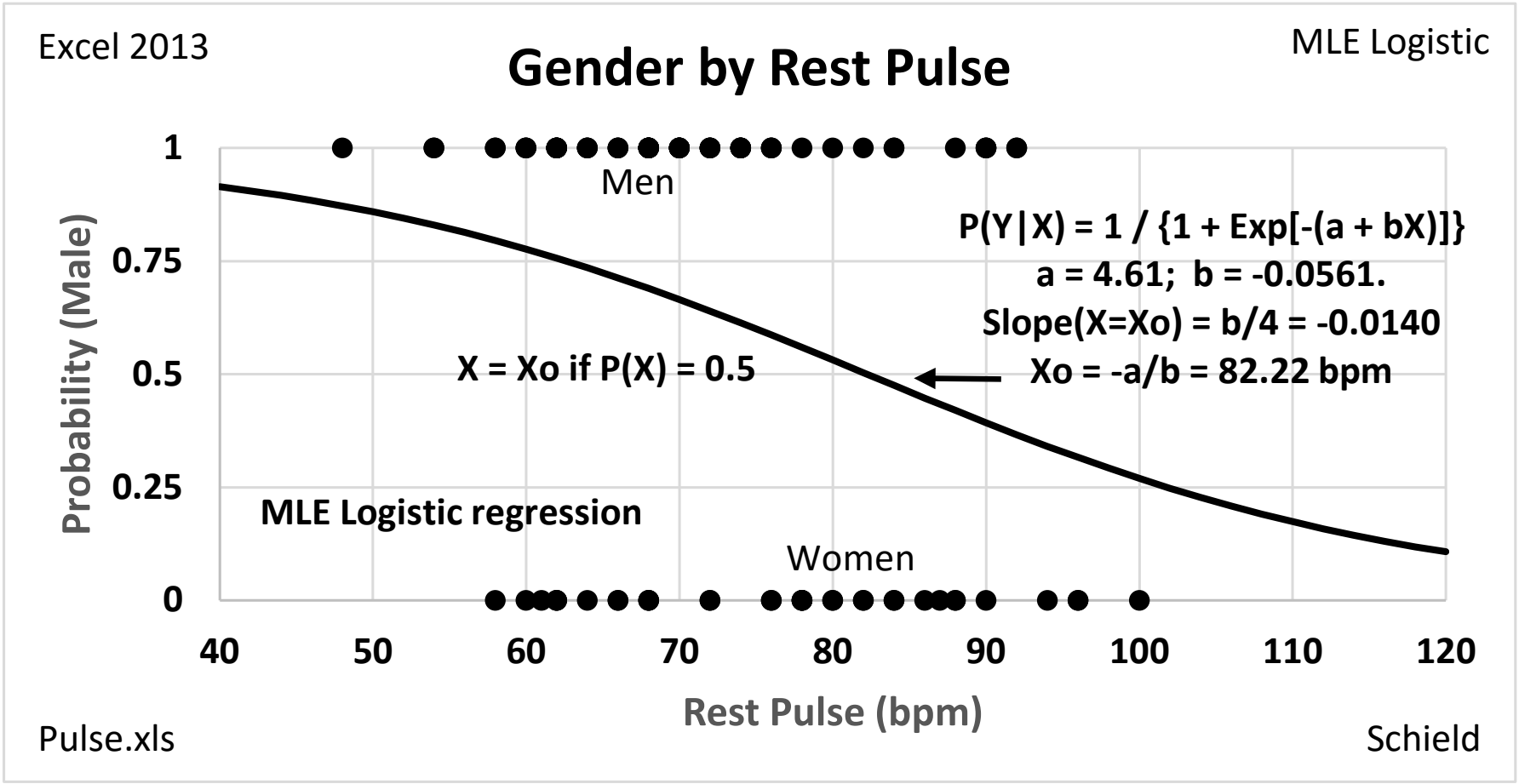

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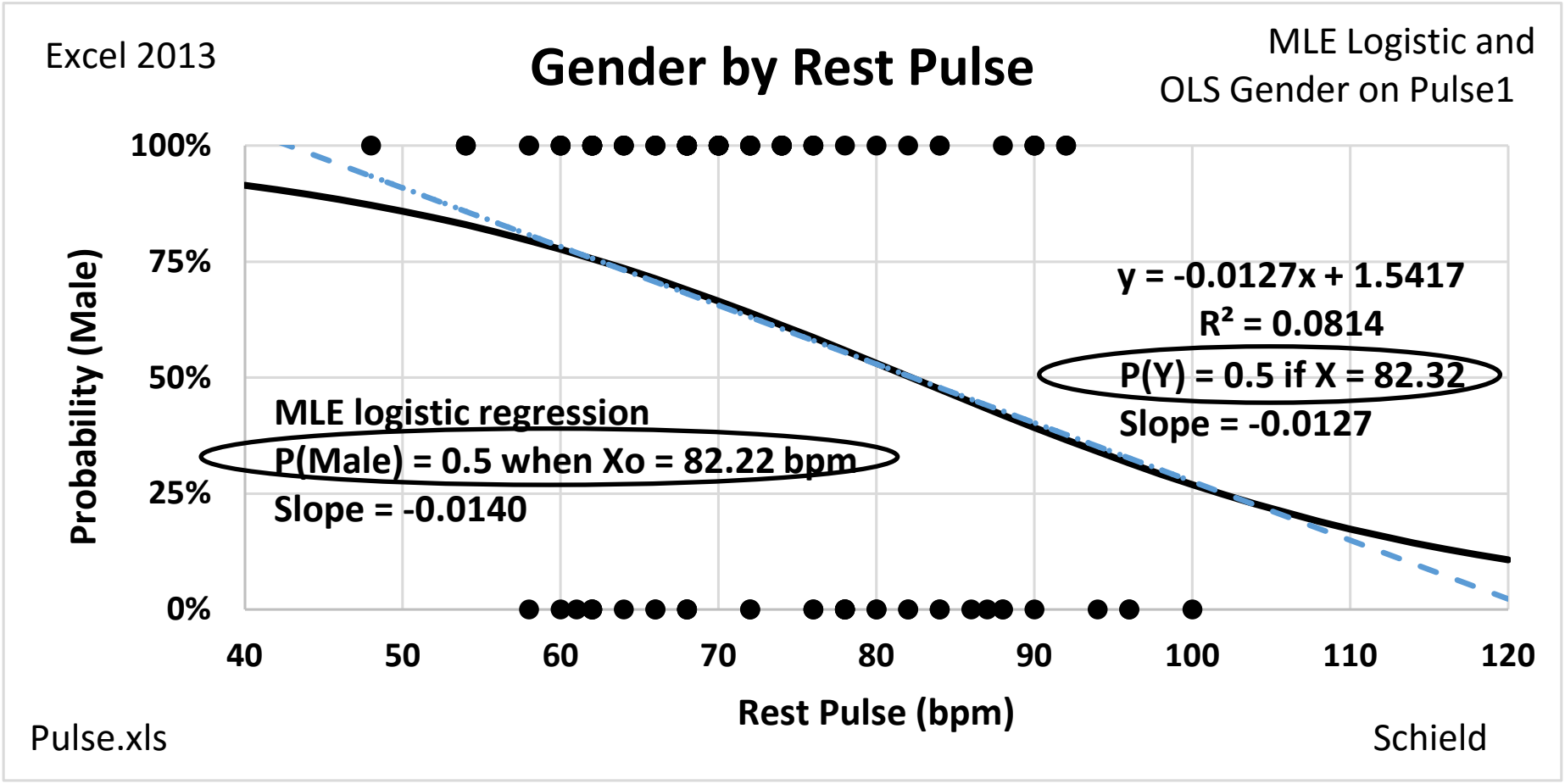

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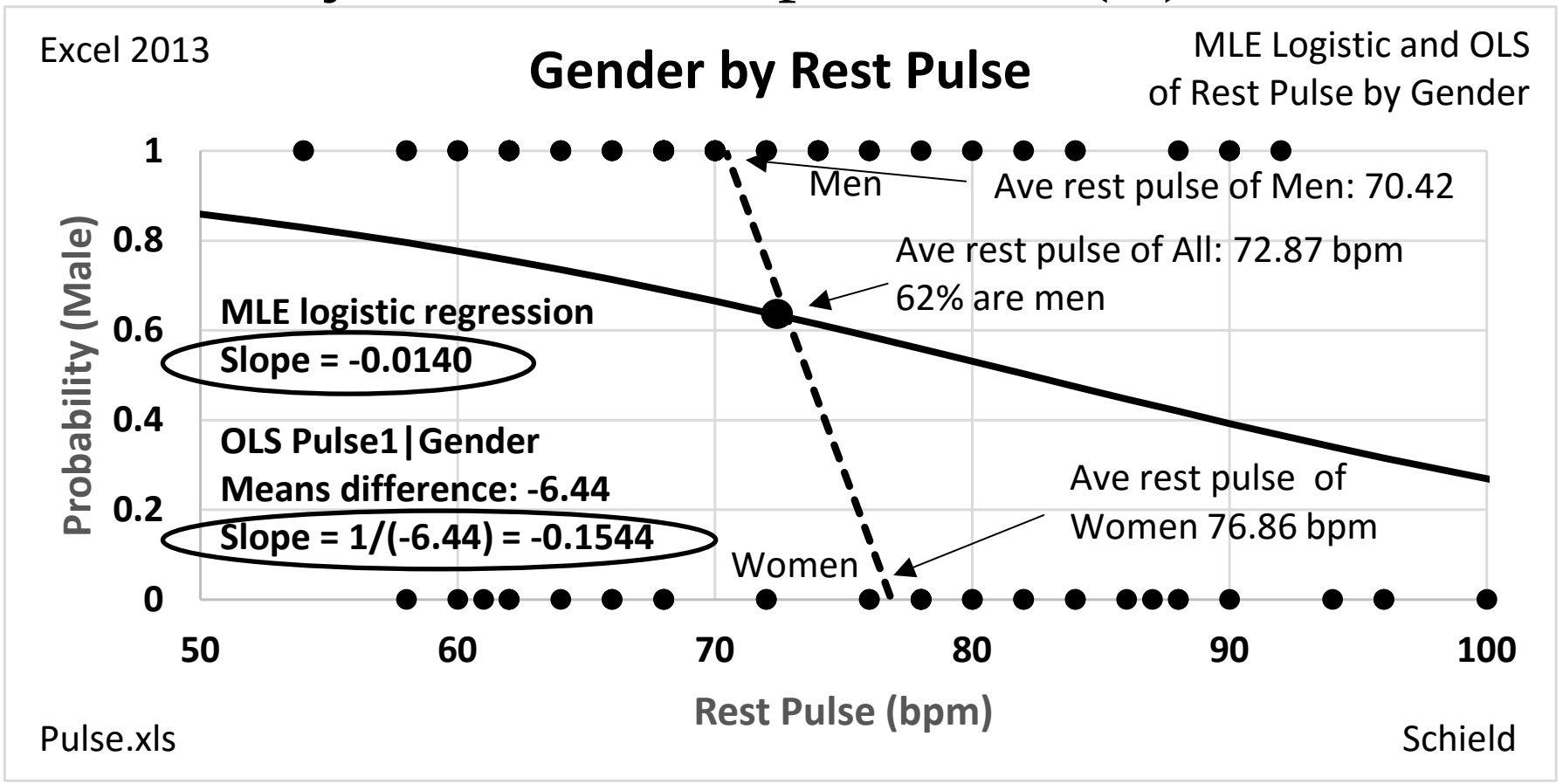

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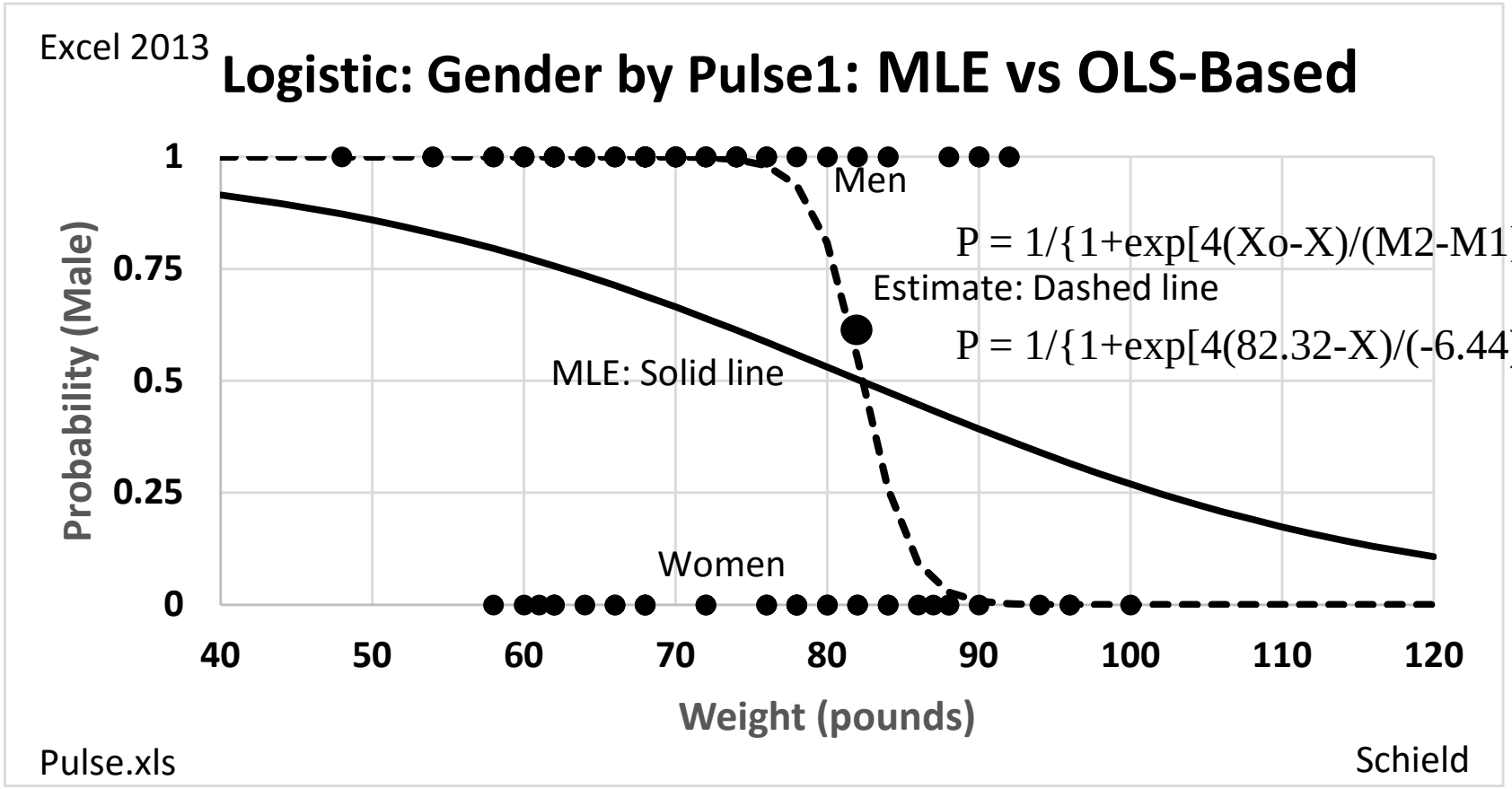

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