THE BLOG

Andrew Hacker and the Case for and Against Algebra:

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March 1 saw the publication of the book *The Math Myth: And Other STEM Delusions*, by Andrew Hacker. I have published my review of that book in Devlin's Angle, my regular column for the Mathematical Association of America. In this article, I'll set the scene by describing an earlier essay Hacker published, which in essence is a precis of the book, and explain how he manages to make a number of very good observations about current mathematics education but, because of some dramatic misunderstandings about mathematics, ends up drawing a conclusion totally opposite to the one his arguments actually establish.

The saga starts in 2012, when the New York Times published an opinion piece by Mr. Hacker, with the attention grabbing headline Is Algebra Necessary? As the headline writer surely intended, the article generated a large number of online comments, tweets, and discussions in various blogs. Since Hacker clearly has a valuable connection to the nation's premier national newspaper, it is then a pity he pitched his article the way he did. For, although Hacker says that he did not write the headline himself, his article was indeed promoting the removal of algebra as a required course in K-12 education (as does his new book). Not only did that suggestion alienate accomplished scientists and engineers and a great many teachers--groups you'd want on your side if your goal is to change math education--it distracted attention from what was a very powerful argument for introducing the teaching of algebra into our schools, something I and many other mathematicians would enthusiastically support.

Yes, you read that right: *introducing* algebra. For a variety of reasons, the subject now taught in schools under the name of algebra is a travesty of the powerful way of thinking and problem solving developed in the Muslim world in the 8th and 9th Centuries, called "algebra" today after the Arabic term *al-Jabr*. If Hacker had instead used his NYT connection to argue for a major make-over of "*school* algebra" (as I think we should call the object of his criticisms), he would have garnered massive support from the pros, including me. As it is, his support came exclusively from those who, like Hacker himself, have no idea what algebra is or how significant it is in today's world.

Taking two examples from his article, Hacker lamented that there was no good

reason to require K-12 students to "master polynomial functions and parametric equations" or to "force them to grasp vectorial angles and discontinuous functions." His use of these examples as illustrations highlights how narrow and off base is his understanding of mathematics.

He is simply wrong about his first example. There is good reason to study both polynomial functions and equations (parametric or otherwise), provided it is done properly. He would, however, be absolutely correct if you were to take his phrases to refer to mastering certain procedures for manipulating symbolic expressions (as he clearly does), which is what that valuable educational goal has largely morphed into in our classrooms and textbooks.

As for his second example, I had to google the term "vectorial angle." It turns out to be an uncommon (and unfamiliar to me) name for measuring an angle as you go around a circle, so I doubt any child needs to be forced to "grasp" that, even a child brought up in an age when most clocks are digital. And "discontinuous functions" have nothing to do with algebra; they are topics in the subject known as Real Analysis, which typically is studied only by university level math majors.

There is always a danger in setting oneself up as an advocate for change in a discipline one does not know. Hacker is not a mathematician. He is a retired college professor of political science, who has taught some courses in mathematics to non-majors. As a result of his experiences, he has arrived at some conclusions about K-12 mathematics education that in many ways are close to my own, and which I have written about extensively in my Devlin's Angle and my personal blog profkeithdevlin.org.

[In many ways, my position was articulated far more eloquently and passionately than I ever managed by math teacher Paul Lockhart in his essay *A Mathematician's Lament*, which went viral after I introduced it to the world in my MAA blog of March 2008.]

As I say, Hacker and I have very similar views about the abysmal state of much of today's K-12 mathematics education, and the negative effect it has on generations of school students who, as a result, graduate with a poor understanding of, and often great antipathy towards, mathematics. But, given the importance of mathematics in today's world, we absolutely should not abandon the obligatory teaching of algebra, as he advocates; rather, we should teach it right. Unfortunately, since Hacker plainly does not understand what algebra is, or more generally what mathematical thinking is, he instead proposes we throw away the healthy but neglected baby along with the depressing pool of lukewarm, dirty bathwater it currently hides in.

First codified by the Persian mathematician al-Khwarizmi in his book *The Compendious Book on Calculation by Completion and Balancing* (balancing = *al-Jabr*), written in Bhagdad around CE 820, algebra is a powerful method for solving numerical problems more efficiently than by arithmetic. It does so by introducing two new ways of handling numerical problems.

First, algebra provides methods for handling entire classes of numbers, rather than specific numbers (which is what arithmetic does). (Those *x*'s, *y*'s, and *z*'s come in to talk about classes of number, but that's just an implementation detail introduced in France several centuries later.)

Second, algebra provides a way to find numerical answers not by computing, which is often very difficult, but by reasoning logically to hone in on the answer, using

whatever information is available. Thus, whereas in arithmetic you work forwards, starting with numbers and computing with them to arrive at an answer, in algebra you work backwards, starting by postulating an answer and reasoning logically to figure out what it is. True, this powerful application of human logical reasoning capacity frequently gets boiled down to mastering various symbolic procedures to "Solve for *x*," but again that's just a particular implementation. "Numerical forensics" would be a sexier, and more descriptive, term for the real thing.

When al-Khwarizmi wrote his book (the world's first algebra textbook), apart from the familiar ten symbols for numerals, there was not a single symbol anywhere. No formulas or symbolic equations to be seen. Al-Khwarizmi was showing the traders and engineers of the 9th Century how to solve the numerical problems they faced in their daily lives. The focus was on *how to think* about problems, and had nothing to do with manipulating symbols. That is algebra. It is exactly the mental toolkit that Hacker says repeatedly is crucially important and should be taught in schools.

Where Hacker goes wrong is confusing algebra with a specific implementation of algebra introduced by François Viète in 16th Century France. More accurately referred to as "symbolic algebra," it provides a set of formulaic procedures for carrying out algebraic reasoning in a largely mechanical fashion. It's very efficient, which is why it rapidly gained broad acceptance and widespread use. But the symbolic implementation is a procedural mental aid that only makes sense when learned and practiced in the context of real problems. If the symbolic method gets separated from the real world domains it was developed to handle, it ends up seeming like a meaningless and pointless game. That is what has happened with school algebra, as it has become codified in today's textbooks. [Some of us actually like that game, and it has proven time and time again to be valuable to society for some of us to play it. But that is a separate issue.]

As it happens, the separation of symbolic algebra from applications is no longer of much consequence in mass K-12 education. For there is a second, much more recent implementation of al-Khwarizmi's method of algebra--one of immediate use to everyone in today's world: the electronic spreadsheet. A perfectly correct description of Microsoft Excel is that it is a computer implementation of algebra, just as Viète's symbolic algebra was an earlier, paper-and-pencil implementation.

[Check back on my summary of what algebra is. The columns in a spreadsheet allow you to reason with entire classes of numbers, and the spreadsheet's macros are the algebraic formulas. Because computers are highly efficient at performing many arithmetical calculations in essentially the same fraction of a second, there is no need to work backwards as with symbolic algebra; you can solve problems in a different manner, by working forwards on entire columns at once. There are also computer systems that can carry out algebraic symbol manipulations as well; they are called Computer Algebra Systems.]

As I say, it is a pity that, because he is so far removed from mathematics as it is actually practiced in today's world, Hacker misses the large target that I am pretty certain he is trying to hit--a target that deserves to be hit. Namely, the degree to which the mathematics taught in many of the nation's schools has drifted away from the real thing used every day by large numbers of people, to the point where much of what is taught is not only of little use, but can do real harm. Kids who are put off math in school will find their life choices significantly narrowed.

How far is Hacker off base? Two examples jumped out of the page at me in a recent interview he gave for the Chronicle of Higher Education.

In arguing for teaching coding in schools, Hacker says, "*Coding is not based on mathematics ... Most people who do coding, programming, software design, don't do any mathematics at all*." Well, I've been coding since the 1960s, when Fortran and Algol were the leading high-level languages, and have even programmed in various assembly languages, and what Hacker says is totally false. Coding is entirely mathematical. It just has little in common with most *school* mathematics. (The worse for the school math curriculum, I would say, given that computing is more immediate in most people's lives and careers today than physics or engineering.)

Second example: As part of his argument that learning mathematics is irrelevant in much of today's world, Hacker says, "*Would you go to a mathematician to tell us what to do in Syria? It just defies comprehension*." Well, it may defy Hacker's comprehension, but using mathematics to help prevent events in the Middle East result in another Twin Towers attack is *exactly* what the US Defense Department approached me and many other mathematicians (by way of large commercial defense contractors) to start doing following 9/11, work I continued to do over the ensuing decade for various DoD agencies. Much of that work has involved seeing how far algebra-like symbolic representation can take us in tracking events in different parts of the world, bringing algebra back full circle to the Middle East where it began 1200 years earlier. So much for Mr Hacker's Syria comment.

The pity is, Mr Hacker is right on target with his analysis of much that is wrong with what goes on in school math classes (through no fault of the teachers, I should add, since the majority have to teach what is mandated, and tested ad nauseam), and he is fortunate to have access to a large megaphone--the NYT--to make his analysis known. Unfortunately, his narrow, and in many cases out-of-date perception of what mathematics is, together with his many misunderstandings of the nature of subjects such as algebra and the importance of, say, parametric representations, mislead those who have similar misunderstandings, and alienates those of us in the math biz who would otherwise be lining up alongside him. For these reasons, I give his essays an A for observation, C for background knowledge, and an F for drawing the wrong conclusions.

No, make that a D/F for his conclusions. His arguments did yield correct conclusions, he just did not realize they did, and claimed the opposite.

I would give similar grades to Hacker's new book. In reading an advance copy to write my review, I annotated 20 pages (out of a total of 200) where he makes significant errors due to a lack of knowledge of, or a misunderstanding of, mathematics. That's an error rate of 10%; way too high for significant errors. You can read my review in the March 1 issue of Devlin's Angle.

MEMO TO SELF: Don't write essays or books on revolutionizing political science education.

FURTHER READING: For more background on the early development of algebra and its present day nature, see my earlier article What is Algebra?, published in 2011.

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