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Devlin's Angle

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Tuesday, March 1, 2016

The Math Myth that permeates “The Math Myth”

March 1 saw the publication of the book *The Math Myth: And Other STEM Delusions*, by Andrew Hacker. MAA members are likely to recognize the author’s name from an opinion piece he published in the New York Times in 2012, with the arresting headline "Is Algebra Necessary?"

Yes, I thought you’d remember it! It’s almost up there with John Lennon’s murder in terms of knowing where you were at the time you first heard of it. But just to be sure we are all on the same page, let me recap that, in that essay, Hacker, a retired college professor of political science who over the years had taught some non-majors math courses, laid out a case for dropping algebra as a required course in K-12 and college.

Before I dive into Hacker’s new book, you would be advised to refresh your memory of the case he presented in that article, since his book is essentially an extension of what he said then, expanded to cover the entire Common Core Mathematical Standards. Prior to writing this review, I wrote an article for the Huffington Post in which I summarized, with my commentary, his 2012 article, together with a recent interview he gave to the Chronicle of Higher Education.

In my article, I noted that Hacker has no idea what algebra really is. His focus is entirely on school algebra as it is very often taught, as a collection of rules for manipulating symbolic expressions. What his argument actually establishes, with sound arguments and good examples, are two conclusions I would agree with:

1. Algebra as typically taught in the school system is presented as a meaningless game with arbitrary rules that does more harm than good.
2. There are strong arguments for teaching algebra as it was originally developed and how professional mathematicians today view it.

I’ll leave you to read my HuffPost piece for more of the gory details. For the benefit of lay readers who may come to this site, I should though repeat here the brief summary I gave in that article of the difference between algebra (as mathematicians understand and practice it) and the rule-based-manipulation-of-symbolic-expressions that so often passes for algebra in our schools.

First codified by the Persian mathematician al-Khwarizmi in his book *The Compendious Book on Calculation by Completion and Balancing* (balancing = *al-Jabr*), written in Bhagdad around CE 820, algebra is a powerful method for solving numerical problems more efficiently than by arithmetic. It does so by introducing two new ways of handling numerical problems.

First, algebra provides methods for handling entire classes of numbers, rather than specific ones. (That’s where those x’s, y’s, and z’s come in, but that’s just an implementation detail introduced in France several centuries later.)

Second, it provides a way to find numerical answers not by computing, which is often very difficult, but by reasoning logically to hone-in on the answer, using whatever information is available. Thus, whereas in arithmetic you work forwards, starting with numbers and computing with them to arrive at an answer, in algebra you work backwards, starting by postulating an answer and reasoning logically to figure out what it is. True, this powerful application of human logical reasoning capacity frequently gets boiled down to mastering various symbolic procedures

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to “Solve for x,” but again that’s just a particular implementation. Numerical forensics would be a sexier, and more descriptive, term for the real thing.

The familiar symbolic expressions calculus usually taught in schools as “algebra” was a particular implementation of al-Khwarizmi’s ways of thinking introduced by the French mathematician François Viète in the 16th Century (700 years after algebra first began) to streamline paper-and-pencil problem solving. A more recent implementation of algebra is the computer spreadsheet.

Since his new book follows the same line of attack as his 2012 opinion piece, but with his sights widened from school algebra to the Common Core, instead of crafting another analytic essay, I will do what Hacker himself does, and list a number of examples to make my case. More precisely, I’ll select some of the 20 instances (in a book of just over 200 pages) where I found a claim that is either plain wrong, wildly misleading, or otherwise problematic, and ask where he went wrong. In marking 20 pages, it’s likely I missed some. There were so many wild and inaccurate claims, I frequently found myself skimming through.

First though, I should repeat what I said in my HuffPost article about his algebra piece. Just as his essay actually amounted to a strong argument in favor of teaching algebra to all students (albeit not the rule-based manipulations of formulas so often presented in place of algebra), so too his book includes a strong argument in favor of Common Core Math. In the same way that Hacker mischaracterized algebra in 2012, so too his portrayal of the CCSSM (Common Core State Standards for Mathematics) is totally at odds with the real thing—though not quite so far off if you turn your attention from the Standards themselves to some *implementations* of the CC.

One of the book’s flaws is that Mr Hacker seems to get carried away with the flow of his rhetoric, since for the most part his argument consists of the erection of a series of straw men which he then, in time-honored tradition, proceeds to attack.

“It’s a waste of time forcing kids to master azimuths and asymptotes,” he cries [not an exact quote] as early as page 2.

I had to look up the word azimuth, since in my entire career as a mathematician and mathematics educator, I had never come across it. According to Wikipedia, azimuth is a “concept used in navigation, astronomy, engineering, mapping, mining and artillery.” I ran a search for the word on the entire, 93-page CCSSM document and, as I expected, it did not turn up. Straw man.

Asymptotes are a different matter, of course, since a general sense of asymptotic behavior of functions is useful in many walks of life. The word is mentioned, but just once, in the CCSSM, in the section on Interpreting Functions (F-IF), where it says:

Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

That’s it. One mention, buried towards the end of the document, in the section that says the student should:

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

From the overall thrust of Hacker’s argument, I think it’s clear he believes this kind of knowledge is indeed important for everyone to have. But it’s also clear it is not a central pillar of the CC, to be used on page 2 to set the scene for what his book is about.

Unfortunately, this example is indeed a good characterization of his overall argument: to knock down straw men.

“*We’re told that if our nation is to stay competitive, on a given morning all four million of our fifteen-year-olds will be studying azimuths and asymptotes*,” he writes. (I am still on page 2, with over 200 more pages to go.) He provides no citation regarding who, exactly, is making this proclamation for the nation’s future. It’s not just disingenuously misleading, it’s about as far from reality as you could imagine, and not because of those azimuths. (See momentarily for the real story.)

He continues, “*Then, to graduate from high school, they will face tests on radical notations and*

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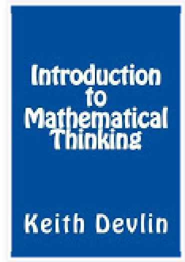

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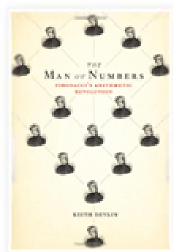

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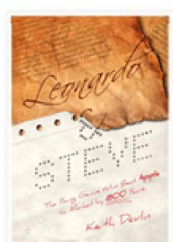


Leonardo and Steve: The Young Genius Who Beat Apple to Market by 800 Years *elliptical equations*."

To be sure, you will find mention of the word radical in the CCSSM, in the context of "Work with radicals and integer exponents" in the Section on Expressions and Equations (8.EE), which provides the helpful illustration,

"*For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger*."

Again, this is exactly the kind of thing Hacker says (towards the end of his book) students should be able to do! And it is entirely reasonable that they be asked to demonstrate that ability on a test.

"Elliptical equations" is another straw man.

The point is, what Hacker keeps attacking are straw men. The CCSS is just what its name implies, a set of standards. It is not a curriculum, nor does it specify anything remotely like a daily, or even weekly timetable. How and when teachers across the land cover the various standards is for them, or perhaps their school district, to decide. As far as the CCSS are concerned, teachers can operate fluidly, depending on how their class progresses. (And no one will even suggest that they mention azimuths, let alone force the class to master them.)

I would hazard a guess that Hacker has never looked at the CCSS document. Nor sat in on many math classes, as I have, and observed what actually goes on in today's schools.

Caveat: I get to see classes to which, for one reason or another, I have been invited to visit. Likely they are some of the best, since their teachers invite me along so their students can talk for a while with someone who has devoted a career to mathematical research. I hear enough stories to be prepared to believe things are often a lot worse. Perhaps even as bad as Hacker says. But his book is purported to be about educational policy, not what you can actually find in good or bad classrooms.

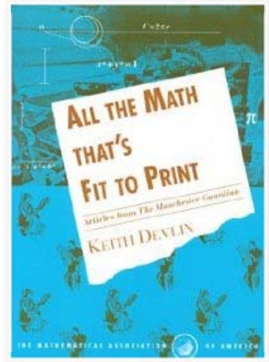
Not only does Hacker give no indication he is familiar with the Common Core—the real one, not the azimuth-strewn, straw-man version he creates—he gives every indication he does not understand mathematics as it is practiced today. (He also does not know that pi is irrational, but I'll come to that later.)

Certainly, the examples he selects to illustrate the irrelevancy (in today's world) of some of the test problems students are asked to solve simply demonstrate that he is lacking the basic, every-day, number sense he is arguing for. Let me give just three examples.

On page 48, Hacker presents a question he took from an MCAT paper. It provides some technical data and asks what happens to the ratio of two inverse-square law forces between charges of given masses when the distance between them is halved. The context Hacker provides for this question is that medical professionals needs to be able to read and understand the mathematics used in technical papers. His claim is that this requirement does not extend to the physics of electrical and gravitational forces. In that, he is surely correct. But anyone with a grain of number sense will recognize at once that the setting is totally irrelevant. It's a simple question about what happens to a ratio when the underlying scale is changed. The answer, of course, is nothing happens. It's a ratio. The changes to the numerator and denominator cancel out. The ratio remains the same.

What this question is asking for is, *Do you understand what a ratio is*? Surely that is something that any medical professional who will have to read and understand journal articles would need to know. Hacker completely misses this simple observation, and presents the question as an example of baroque mathematical testing run amok.

On page 70, he presents a question from an admissions test for selective high schools. A player throws two dice and the same number comes up on both. The question asks the student to choose the probability that the two dice sum to 9 from the list 0, 1/6, 2/9, 1/2, 1/3. Hacker's problem is that the student is supposed to answer this in 90 seconds. Now, I share Hacker's disdain for time-limited questions, but in this case the answer can only be 0. It's not a probability question at all, and no computation is required. It just requires you to recognize that you can never get a sum of 9 when two dice show the same number. As with the MCAT question, the question is simply asking, *Do you understand numbers*? In this case, do you recognize that the sum of two equal numbers can never be odd.



All the Math That's Fit to Print

Finally, on page 101, Hacker presents a list of mathematics requirements high school students must meet in order to study at Harvard and similar universities. The list includes the names of various kinds of analytic functions. As usual, Hacker seems overwhelmed by the technical terms, or worries that the students will be, but all the list is asking for is that students can read graphs and charts and know what they represent in terms of growth and change. An essential skill, surely, for anyone in today's information-rich world, not just students at elite universities.

You get the pattern surely? Hacker's problem is he is unable to see through the surface gloss of a problem and recognize that in many cases it is just asking the student if she or he has a very basic grasp of number, quantity, and relationships. Yet these are precisely the kinds of abilities he argues elsewhere in the book are crucial in today's world. He is, I suspect, a victim of the very kind of math teaching he rightly decries—one that concentrates on learning rules and mastering formal manipulations, with little attention to understanding.

This, surely, explains why he would write (page 96), "*Reasoning mathematically may be a nice skill, but it is not relevant to most of life. We reason about many things: parenting, marriage, careers. Do we learn how to reason about these things by learning algebra?*"

If he had asked instead if we learn such reasoning in a typical school algebra class, I would agree with his implied answer of "No." But algebra arose by codifying the everyday reasoning people carried out—and still carry out today—about the numerical or quantity aspects of any human activity that involves them. (Trade, commerce, and civil engineering were the original applications.)

From that historical perspective, it is absolutely clear that learning algebra can help us master such reasoning. It helps by providing an opportunity to carry out that kind of reasoning free of the complexities a problem generally brings with it when it arises in a real world context.

The tragedy of *The Math Myth* is that Hacker is actually arguing for exactly the kind of life-relevant mathematics education that I and many of my colleagues have been arguing for all our careers. (Our late colleague Lynn Steen comes to mind.) Unfortunately, and I suspect because Hacker himself did not have the benefit of a good math education, his understanding of mathematics is so far off base, he does not recognize that the examples he holds up as illustrations of bad education only seem so to him, because he misunderstands them.

The real myth in *The Math Myth* is the portrayal of mathematics that forms the basis of his analysis. It's the same myth you see propagated in Facebook posts from frustrated parents about Common Core math homework their children bring home from school.

In the interests of their overall cardiovascular health, I have to recommend that math educators do not read *The Math Myth*. But if you do, perhaps you should start with the final chapter, titled "Numeracy 101." Here, at least, you will find things you are likely to agree with, as he lays out what he believes would be a good quantitative literacy course for college students.

But even there, where all seems warm and friendly and positive, you will be jolted by Hacker's fundamental lack of knowledge of mathematics. He writes,

"*Along with phenomena like earthquakes and cyclones, nature also has some numbers that control or explain how the world works. One of them is pi, whose* 3.14159 *goes on indefinitely, at least as far as we know*."

Yes, you read that last part correctly.

"Few people writing today ... can make more sense of numbers" proclaims the *Wall Street Journal* on the cover of Hacker's book. Well, if that's the view of the newspaper that purports to have the expertise to cover the nation's financial markets, it is only a matter of time before we have another financial meltdown.

Posted by Mathematical Association of America at 10:11 AM No comments: Links to this post

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Thursday, February 11, 2016