STATISTICAL LITERACY: CRITICAL THINKING ABOUT CONFOUNDING

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Many – if not most – social arguments today involve statistics. These social statistics are quite different from mathematical numbers. They are socially constructed by people with goals and motives. And since they are based on reality they can be influenced. This presentation introduces statistical literacy as a new discipline and as a course (Math 1300) taught at the University of New Mexico. The focus is on introducing those ideas that students need to decode and evaluate the statistics in the everyday media, in tables and in graphs. Confounding is introduced as a strong source of influence. Data shows that vaccinated cases are more likely to die from Covid than unvaccinated. This association is reversed after taking into account (controlling for) a confounder: age. Statistical literacy, quantitative rhetoric, is argued to be a necessary skill in order to deal with data based arguments in a modern society.

STATISTICS VS. NUMBERS

Statistics are different from numbers. Yes, statistics are numbers. But they are numbers in context – numbers in reality. Statistics can be influenced by the context – by the reality.

Statistics are more like words than numbers. Statistics are created by people: people with motives, values and goals. It is so easy to think of statistics as just being numbers.

In arithmetic, one plus one is always two. But in bunny math, one bunny plus one bunny can give more than two bunnies – under certain conditions. In ice-cube math, one ice-cube plus one ice-cube can give less than two ice cubes – under certain conditions. The reality matters.

Dr. Joel Best cut to the heart of the matter when he said, "Statistics are socially constructed." Best is the author of a great book: "*Lies, Damned lies and Statistics*." He identified the social construction of statistics as the most important – the most fundamental – aspect of all reality-based statistics.

What is the best advice to anyone dealing with statistics? *Take care*! Statistics can be influenced.

In Statistical Literacy, all the influences on a statistic are grouped into four categories. The first letter in the name of each category matches the four letters in CARE.

- *C: Confounding*. Statistics are confounded (confused) by related factors. Critical thinking is required to think of what related factors are most likely to influence a given statistic.
- *A: Assembly or assumptions*: statistics are influenced by how they are defined, counted, measured, summarized, compared and presented. Consumers must use critical thinking to think of how a statistic could have been defined, counted, measured, etc.
- *R: Randomness*: Not just the randomness in sample statistics in small samples, but the randomness in extreme outcomes, or patterns.
- *E: Error or bias*. Error includes the confusion of the inverse: confusing the "percentage of women who smoke" with the "percentage of women among smokers". Bias includes subject bias, measurement bias or selection bias. Here are some examples: Subject bias: people understate their age or weight; they overstate their income. Measurement bias: Asking loaded questions. Selection bias: Popes and political leaders live longer than the average person. True, but people do not become pope or a political leader until they are already older. On average, older people live to an older age than do infants.

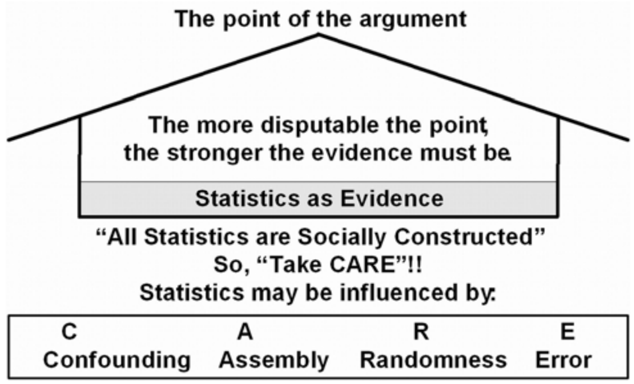
The last two topics are the basis for the traditional introductory statistics course. The first two are the primary focus of a statistical literacy course. The difference is their emphasis. In the traditional course, randomness and error make up at least 90% of the course. In this confounder-based statistical literacy course, they are less than 30%.

So, take CARE when dealing with social statistics. Critical thinking is required to analyze these influences.

To summarize, let's look at arguments – for that is what critical thinking is about. An argument is something mental: an arrangement of less-disputable claims that support a more disputable claim.

In order to understand arguments, here is a physical analog: a house.

Figure 1. An Argument is Like a House



In a house, the basement supports the walls; the walls support the roof and they all support the point of the roof. In an argument, the reasons are like the basement and the walls: they support the point of the argument. The point is something less observable and thus more disputable than the evidence.

The statistics are typically in the basement. As noted before, statistics are socially constructed so they can be influenced and manipulated. The best advice when dealing with statistics is "Take CARE". The influences are grouped into four categories where the first letters match the four letters in CARE as shown in Figure 2.

Students like "CARE". It gives them a structure. When given a list of 25 ideas or topics and asked to rank which ideas they considered the most valuable, most students chose "Take CARE".

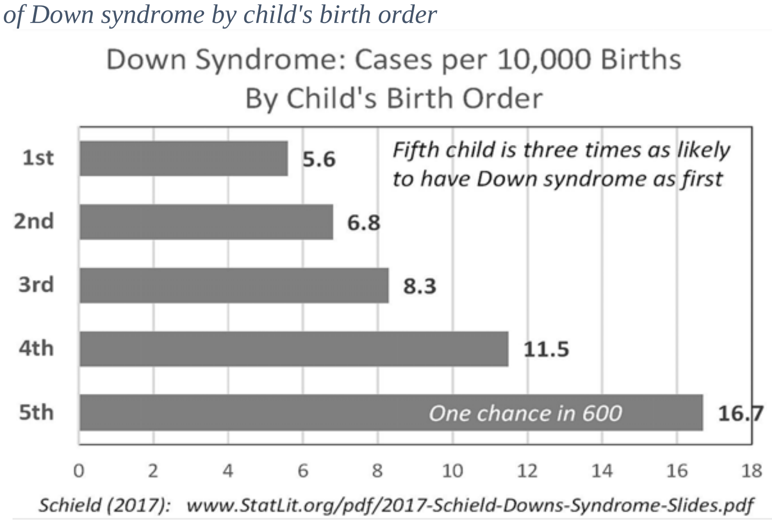
C: CONFOUNDING

Here are some examples of hos statistics can be confounded.

#1: *People who read home and fashion magazines are more likely to get pregnant than people who read car and sport magazines*. We know that pregnancy is not caused by magazines. And we know that only women can get pregnant today. We quickly realize that women are more likely to read home and fashion magazines than men. This crude association could be confounded: confounded by gender.

#2: To better understand confounding, consider Down syndrome: a birth defect.

Figure 2. Frequency of Down syndrome by child's birth order

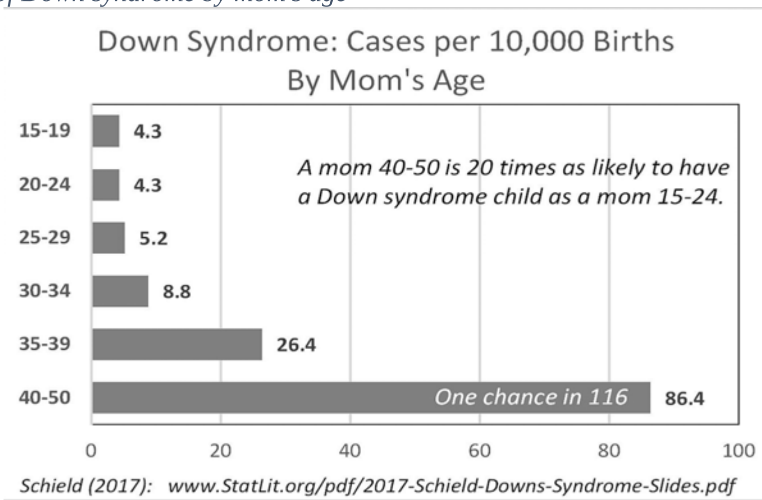


The later the child in the birth order, the more likely they are to have a Down syndrome. The fifth child is three times as likely to have Down syndrome as the first child.

This association is true – given this data. But it is a crude association. If there are other differences between mom's having their first child and mom's having their fifth child, this comparison may be a mixed fruit comparison: an apples and oranges comparison.

What else could influence having a Down syndrome child? Here is the same data organized by the mom's age.

Figure 3. Frequency of Down syndrome by mom's age



As the mom's age increases, the chance the child will have a Down syndrome increases. Moms age 40-50 is about 20 times as likely to have a Down syndrome child as are moms 15 to 24.

Can you understand how these two explanations are connected? The later the child in the birth order, the older the mom. Perhaps, what we saw as influenced by birth order was really spurious. Perhaps the only influence is the mom's age.

TODAY'S STUDENTS

Today's students are interested in arguments. Most stories in the everyday media involve controversial claims. These asserting these claims give reasons – they provide arguments.

Many of these arguments involve statistics. Most of these statistics are social statistics: statistics involving people.

Many of these claims involve a certain kind of statistic: disparities in outcomes. Disparities in education, suspension and graduation. Disparities in policing, crime, arrests, sentencing and prison. Disparities in money: in income, wages, assets, loans and wealth. Disparities in health: in healthcare, homicides and death.

These disparities in outcomes typically involve groups: groups involving gender, race, ethnicity, socio-economic class, religion, politics, age, etc.

All of these outcome disparities by group are associations. These associations rely on statistics: social statistics. Almost all of these statistics are based on observational studies. They are crude associations: they are true, but they tell just a part of the story. In many – if not most – cases, there is a story behind each of these statistics.

Now all this is interesting, but how can we untangle the influence of a confounder? How can we change a mixed-fruit comparison (an apples and oranges comparison) into a same-fruit comparison (an apples and apples comparison)?

CRUDE COMPARISONS

The UK National Health service found that "vaccinated cases were more likely to die of Covid than were unvaccinated cases". Arithmetically, "vaccinated cases were twice as likely to die from Covid as were unvaccinated cases."

Table 1: Covid Death rates per Case: Vaccinated vs. Unvaccinated

Covid Death Rates Per Case		
	Crude Rate	# Cases
Unvaccinated	**0.17%**	151,052
Vaccinated	**0.41%**	117,114
41/17 = 2.4		UK NHS 2021

Both claims are controversial. For many, they may seem to be false. With over 260,000 cases, it doesn't seem likely that this comparison is due to randomness or coincidence.

What else could be related to death among Covid cases? Age! Older people are more likely to die of Covid than younger people. To untangle this confusion, we need more data.

Table 2: Covid Death rates per Case by Age: Vaccinated vs. Unvaccinated

Covid Death Rates Per Case				Death Rates by Age	
	Crude Rate	# Cases		<50	50+
Unvaccinated	**0.17%**	151,052		0.03%	5.96%
Vaccinated	**0.41%**	117,114		0.02%	1.68%
41/17 = 2.4		UK NHS 2021			

The data on the left side is the same as before. The data by age on the right is new. Notice that vaccinated cases are more likely to die of Covid than unvaccinated if the person is under 50. The same is true if the person is 50 or older.

Does this make sense? Vaccinated cases are less likely to die than unvaccinated if they are under 50 or if they are at least 50. But they are more likely to die than unvaccinated if we disregard their age.

This doesn't make sense. It like a team winning the first and second halves of a game, but losing the game. This is a paradox. It is Simpson's paradox.

Simpson's paradox occurs when a two-group comparison has the opposite direction or sign from that in each of the subgroups. It is impossible with counts, but it is possible with ratios.

Simpson's paradox is the elephant in the room for the social sciences. Simpson's paradox shows that an association is definitely a crude association: a mixed fruit comparison.

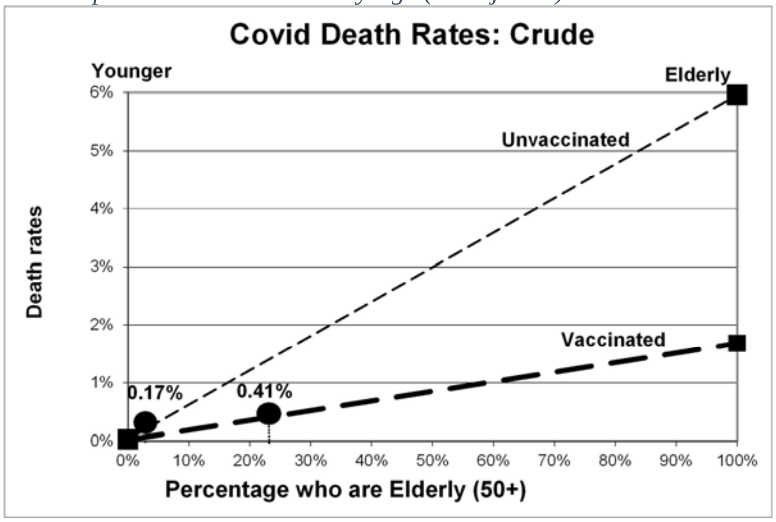
What explains this paradox? It's the mix. Elderly are 23% of the vaccinated, but only 2.3% of the unvaccinated. This is what makes this comparison a mixed-fruit (an apples and oranges) comparison.

How can we untangle this without using computer software or advanced mathematics?

TAKING INTO ACCOUNT (MIX-MATCHING)

Giving both groups the same mixture of the confounder is the simplest way to untangle this confusion. Since the crude rates are weighted averages, this can be done arithmetically or graphically. Figure 4 shows the graphical presentation.

Figure 4. Standardization Graph: Covid Death Rates by Age (Unadjusted)



There's a lot going on in Figure 4, so let's take it slowly. The vertical axis is the death rate in percent. The horizontal axis is the percentage who are elderly: at least 50 years old.

- The far left side (0%) has no elderly. Everyone is younger than 50.
- The far right side (100%) has all elderly. Everyone is at least 50.

Individuals are either on the far left (0%) or on the far right (100%) based on their age.

In Table 2, note the death rates for the elderly: roughly 6% and 1.5%. Those match the dots on the far right side of this graph. Now note the death rates for the younger non-elderly: roughly 0%. Those match the dots on the far left,

The dashed lines connect the dots for the unvaccinated (the top line) and the vaccinated (the bottom line). These lines are the weighted average lines. If half the people in the vaccinated group are elderly, then the average death rate for the vaccinated would be nearly 1%. In this case, 23% of the vaccinated are elderly. So we find 23% on the horizontal axis and draw a vertical line upward until it intersects the vaccinated weighted average. The death rate at that point is the weighted average for the vaccinated: about 0.4%.

After doing the same for the unvaccinated (2.3% are elderly), their average death rate is about 0.2%. These match the crude rates on the left side of Table 2. To see close up, checkout Figure 5.

Figure 5. Standardization Graph Close up: Covid Death Rates by Age (Unadjusted)

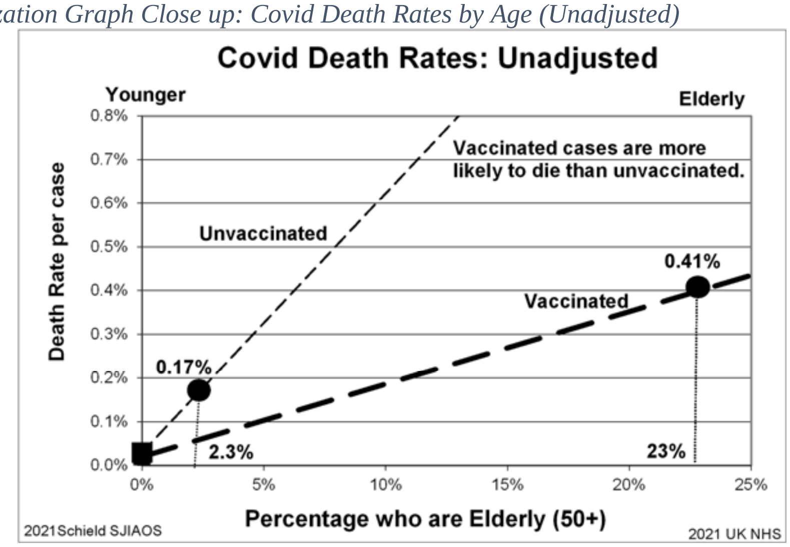


Figure 5 is just the lower-left corner of Figure 4. Now we can see more clearly the weighted averages for the vaccinated (0.41%) and the unvaccinated (0.17%).

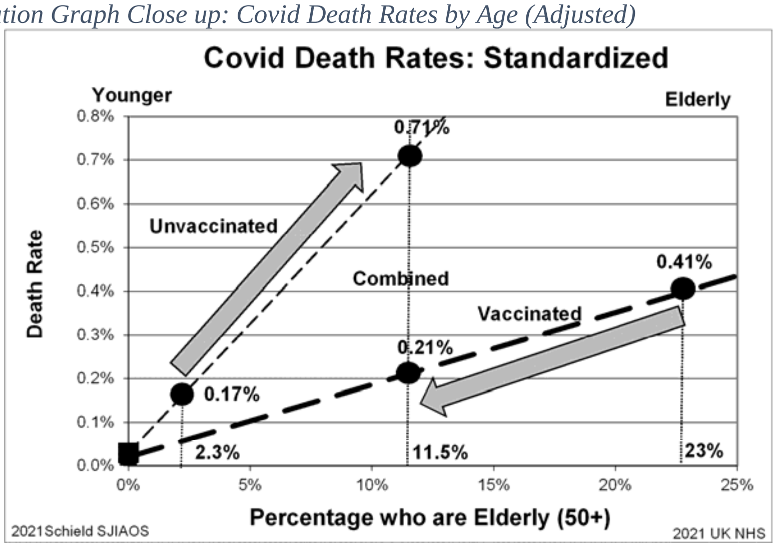
Clearly this is a crude comparison: a mixed-fruit comparison. The two groups have a very different mix of the elderly: 23% for the vaccinated and 2.3% for the unvaccinated.

To standardize, we want to give both groups the same mix of the elderly. One way to do this is to find the percentage who are elderly when both groups (vaccinated and unvaccinated) are combined.

Suppose the percentage in the combined group who are elderly is 11.5%. Figure 6 shows the results. This is a very busy graph. Focus first on the vaccinated diagonal line: the lower line. Start at the right side where the percentage who are elderly is 23%. That clearly corresponds to a death rate of 0.41%.

But if the percentage who are elderly were reduced to 11.5%, the weighted average would move downward to the left and end at 0.21%. Note that we are not changing the Covid death rates for any of the four groups. We are only changing the mix: the percentage who are elderly.

Figure 6. Standardization Graph Close up: Covid Death Rates by Age (Adjusted)



Now focus on the unvaccinated diagonal line: the upper line. Start at the far left where the percentage who are elderly is 2.3%. If that percentage were increased to 11.5%, the weighted average would move up the line to 0.71%. Once again, we are not changing the death rates for any group. We are just giving both groups the same mixture.

Compare the adjusted results. After taking into account age, the unvaccinated are more likely to die than the vaccinated. After controlling for age, the unvaccinated are more than three times as likely to die as are the vaccinated.

Taking into account age by mix-matching reversed the crude association. Taking into account" the influence of age converting a mixed-fruit comparison (apples and oranges) into a same fruit comparison (apples and apples).

Confounding is the elephant in the room for those social sciences dealing with observational data. Statistical educators know about this. It is generally taught in a second course. It should be taught in every introductory course including every research methods course. Typically there isn't time. This is one reason the University of New Mexico introduced Statistical Literacy as a companion course to the traditional introductory statistics course.

STATISTICAL LITERACY AT THE UNIVERSITY OF NEW MEXICO

In fall 2021, the University of New Mexico offered students in non-quantitative majors a new catalog course: Math 1300 Statistical Literacy. In 2021-22, seven sections were offered with 15 to 30 students per section.

This catalog course (Math 1300) satisfies a mathematics requirement in the New Mexico general education curriculum. Here is the catalog description for Statistical Literacy:

> *Participants will study the social statistics encountered by consumers. Investigate the story behind the statistics. Study the influences on social statistics. Study the techniques used to control these influences. Strong influence on confounding.*

Consider some of these phrases:

- *Investigate the story behind the statistics.* In their 12 years of mathematics, students have never seen or heard of a story behind the numbers. They have investigated the story behind the story in their majors: history, journalism, political science, English, etc. They have gone beneath the surface and looked for other connections: other explanations. But in twelve years of mathematics, they have never done anything like that in dealing with a number.

- *Study the influences on social statistics.* In their 12 years of school mathematics, they have never investigated the influence on a number.

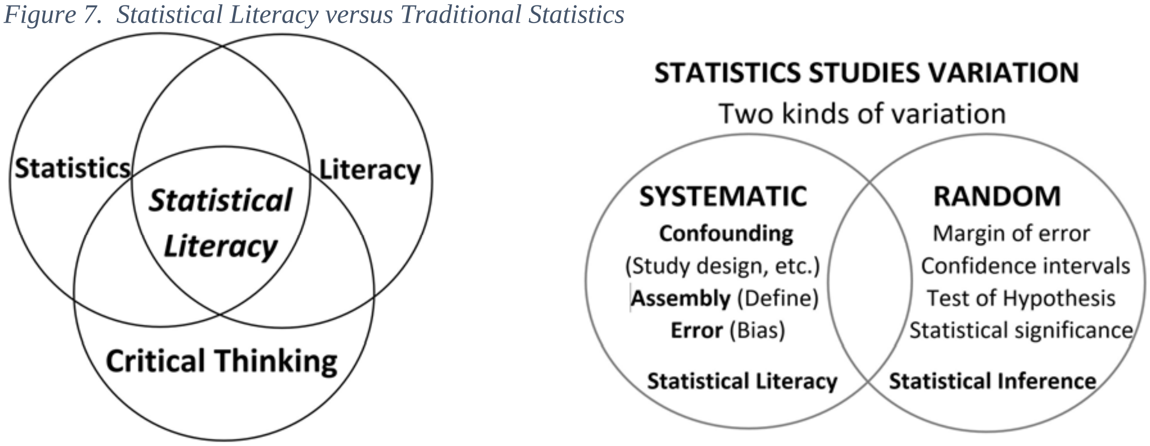
- *Study the techniques used to control these influences.* You don't do that in mathematics – at least not the mathematics they took in high school.

- *Strong influence on confounding.* Most introductory courses never mention the word 'confounding'. Most introductory course don't have 'confounding' or 'lurking variable' in their table of contents. Even if they do, it typically involves a way in which association is not causation. It then disappears from the course entirely.

Math 1300 is a very different from the traditional introductory statistics course. This confounder-based statistical literacy course has less than a 30% overlap with a traditional introductory statistics course.

Math 1300 is a combination of three topics: statistics, critical thinking and literacy. This course isn't a regular math course that uses symbols. This is a literacy course. It uses ordinary English to describe counts, amounts and ratios: rates and percentages. These words are often used in very technical ways: ways that students are not familiar with. But the words are just common ordinary English.

Figure 7. Statistical Literacy versus Traditional Statistics



As shown on the right side of Figure 7, Statistics studies variation. There are two kinds of variation: random and systematic.

- Random variation is studied in a typical introductory statistics course. Random variation is the variation in sample statistics that are randomly selected or assigned. This course deals with statistical inference: topics such as margin of error, confidence intervals, a test of hypothesis, and statistical significance.

- Systematic variation is studied in Statistical Literacy. Systematic variation can be found in small samples or in big data. Topics include the various influences on a statistic indicated by CARE: confounding, Assembly or assumptions, Randomness and Error or bias.

These two courses have two different audiences.

- Traditional statistics is designed for the *producers* of statistics: students in STEM majors (Science, Technology, Engineering and Mathematics).
- Statistical Literacy is designed for the *consumers* of statistics: students in the non-quantitative majors like journalism, political science, and history. Majors like archaeology, film, art and music. Majors like geology and epidemiology. These are majors that don't require their students to take a particular mathematics course.

HYPOTHETICAL THINKING

Confounding and assembly both require hypothetical thinking.

- What else might influence pregnancy than choice of magazine?
- What else might influence a mom having a Down syndrome child besides birth order?
- Why might vaccinated cases have a higher Covid death rate than those who are vaccinated.

MORE CONFOUNDING

- Association is not … causation.   *People who shave their face are taller than those who shave their legs.* We know that shaving does not influence height. What else does influence height? Gender. Men are more likely to shave their face; women are more likely to shave their legs.
- Disparity is not … discrimination. *Tall people are 10 times as likely to be in prison as short people.* Again, we are pretty certain that height is not connected to being in prison. What else influences being in prison? Gender! Men are much more likely to be in prison than women are.   Men are generally taller than women. This disparity, this crude association, is influenced by a confounder: gender.

Statistical Literacy (Math 1300) is a new course. It has a new textbook. It has been field tested by over a thousand students and by more a dozen teachers.

Figure 5. Statistical Literacy textbook

Students find Statistical Literacy (Math1300) valuable. Here are the results from 76 students in an anonymous survey from the fall 2021 class.[1]

Q1. *How valuable is this course in helping you read and interpret everyday statistics?*
Negative (1%), Neutral (4%), Some value (12%), Fair value (38%), Highly valuable (45%).

Q2. How helpful was this course in developing your critical thinking skills?
Not helpful (0), Neutral (5%), Somewhat (21%), Very (42%), Extremely (32%)

Most students (83%) found this Statistical Literacy course very valuable (fair value or highly valuable) in helping them "read and interpret everyday statistics".

Most students (74%) found this Statistical Literacy course very valuable (very or extremely valuable) in developing their critical thinking skills.

I challenge anyone to get results like these from any other statistics course that satisfies a mathematics or statistics requirement in a college general education curriculum.

[1] Schield (2021). Student Anonymous Comments: Math 1300 Statistical Literacy (Nov '21). Copy at http://www.statlit.org/pdf/2021-Fall-UNM-Math1300-S1.pdf

CONCLUSION

Students need to be able to read and interpret social statistics in order to evaluate today's arguments. Faculty in the humanities need to persuade their math-stat colleagues to offer a statistical literacy course for those students in non-quantitative majors.

ACKNOWLEDGMENTS

Thanks to the Critical Thinking Foundation for accepting my proposed guest lecture for their 2022 Conference on Critical Thinking. Two lectures were prepared and recorded. This one involves critical thinking about confounding. It has more mathematical content. It shows what it means to 'take something into account.' The other involves critical thinking about statistics. It covers all the influences on a statistic and is more general.

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Statistical Literacy: Critical Thinking about Confounding.

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Paper at www.StatLit.org/pdf/2022-Schield-CCT.pdf

Video at https://drive.google.com/file/d/1RCu7ZC81IWdQGR4RpU0LIhWbuFN0ggx4/view?usp=sharing

Statistical Literacy: Critical Thinking about Statistics.

Slides at www.StatLit.org/pdf/2022-Schield-CCT2-Slides.pdf

Paper at www.StatLit.org/pdf/2022-Schield-CCT2.pdf

Video at https://drive.google.com/file/d/14QFCPm_WijQUZiU7IN9iCuYptVAwOHs3/view

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