Numeracy

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Illiteracy and innumeracy are social ills created in part by increased demand for words and numbers. As printing brought words to the masses and made literacy a prerequisite for productive life, so now computing has made numeracy an essential feature of today's society. But it is innumeracy, not numeracy, that dominates the headlines: ignorance of basic quantitative tools is endemic in American society and is approaching epidemic levels among many subcultures of the American mosaic. Innumeracy thus leads to inequity in opportunity and threatens to undermine America's capacity for productive work. Today's schools must accept the daunting challenge of achieving appropriate levels of numeracy for all students.

Numeracy is to mathematics as literacy is to language. Each represents a distinctive means of communication that is indispensable to civilized life. Nevertheless, if persistent news reports are to be believed, both numeracy and literacy are in serious decline in contemporary U. S. society.

Despite great differences in structure and form, both mathematical language and natural language are powerful tools for description, communication, and representation. Numeracy is especially important for a nation expecting to compete in a global economy fueled by information technology. Whereas natural language is redundant,

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ambiguous, and concrete, mathematical language is concise, precise, and abstract. Full expression of our thoughts and visions requires the richness of both natural and mathematical language. Like the yin and the yang, numeracy and literacy are the entwined complements of human communication.

A RISING TIDE OF NUMBERS

The term *numeracy* (and its adjectival form *numerate*) is more widely used in England than in the United States. It was described in a report of an official British government committee of inquiry as comprising those mathematical skills that "enable an individual to cope with the practical demands of everyday life."[1] Literacy is often defined in similar terms, for example, as "using printed and written information to function in society."[2]

This tradition of practical purpose has had the effect of equating both literacy and numeracy with the scope of the elementary school curriculum. It is what is meant by "reading, 'riting, and 'rithmetic." Indeed, in countries all over the world, the principal purpose of primary education is to achieve a minimal acceptable level of literacy and numeracy. Yet it is only in the last century that this goal has become widely accepted. So whatever levels of literacy and numeracy we may have achieved are not standards steeped in ancient tradition. Today's vision of a literate and numerate society is a rather recent ideal.

Fifty years ago literacy was defined in the United States as fourth-grade education. Even today this standard is common in developing nations as a minimum goal for educational policy. But as society has become more complex, with global communication and worldwide markets penetrating local societies, minimal levels of both literacy and numeracy have risen relentlessly. Today we read regularly about "functional literacy," "cultural literacy," "scientific literacy," "quantitative literacy," and "environmental literacy." These terms, however defined, represent diverse attempts to express the higher demands of literacy imposed by contemporary society.

Expectations for numeracy have risen at least as fast as have the demands for literacy. Daily news is filled with statistics and graphs, with data and percentages. From home finance to sports, from tax policy to state lotteries, and from health insurance to new drug approvals, citizens are bombarded with information expressed in numbers, rates, and percentages.

A glance at any newspaper reveals how common numbers, graphs, and percentages have become. "Consumer prices rise 0.3 percent. But 5 of index's 7 components slowed rate of increase in November." This actual four-column headline in a typical daily newspaper assumes a reader who understands that 0.3 percent is a monthly (rather than an annual) increase, who can mentally translate 0.3 percent to the more common annual rate of 3.6 percent, who understands that the consumer price index is a weighted composite of several components, and who recognizes what a "slowed rate of increase" of the CPI might mean for the future. More sophisticated readers might be expected to understand the variability inherent in the single digit 0.3 percent (which could suggest anything between 3.0 percent and 4.2 percent annual rates, a 40 percent margin of error); or the impact of compounding on monthly rates (which in itself leads to a 4.3 percent annual rate, or to a range of 3.4 percent to 5.1 percent); or perhaps the relation between changing rates of increase and second derivatives.

In the workplace, numeracy has become the gatekeeper of many desirable jobs. According to one recent study, the fraction of new jobs needing mathematical skills that correspond to a full four-year high school curriculum will be 60 percent higher in the 1990s than in the 1970s, whereas the fraction of new jobs requiring the lowest levels of mathematics skills is projected to decline by 50 percent in the next fifteen to twenty years.[3] Already, three-fourths of all majors available at colleges and universities now require some college-level mathematics.

HISTORICAL PERSPECTIVE

Although arithmetic and geometry arose as instruments of commerce in ancient times, numeracy as a common demand of everyday life is a distinctive product of the scientific age. Just five hundred years ago the merchants of Venice began for the first time to teach addition, subtraction, multiplication, and division as means of expanding their commercial influence.[4] Three hundred years later, great universities began to require this "vulgar arithmetic" as a requirement for entrance, alongside Homer and Cicero. Today universities expect students to be ready to learn calculus—which itself was just discovered three hundred years ago—and newspapers expect readers well versed in compound interest, weighted averages, and statistical margins of error.

In our age, the rapid emergence of computers has spawned an unprecedented explosion of data. Thus, what sufficed for numeracy just four decades ago is no longer sufficient today. Today's ordinary business vocabulary includes terms such as *bit* and *spreadsheet*; librarians talk about "Boolean searches" of catalogue data; and graphic artists use "spline curves" for smooth models. Ordinary calculators have keys for functions that only a decade ago were unheard of outside scientific and engineering circles. The extensive efforts by business to reeducate workers to use computers effectively and willingly show just how rapidly standards of numeracy have changed in our lifetime. Numeracy is not a fixed entity to be earned and possessed once and for all.

Although the definition of numeracy—whatever suffices for the practical necessities of life—continually changes, it does not simply expand. Few people any longer need to take square roots by hand, even though such methods were emphasized in school arithmetic for nearly four centuries. Long division, which began its rise in fourteenth-century Venice, has likely passed its prime as hand calculators become as ubiquitous as pencils. By the turn of the century even algebra may be performed more often by machine than by human hand.

This continually changing backdrop makes it difficult to establish reliable standards for numeracy. Indeed, the increasing gap that many observers have noted between the average performance of contemporary U. S. citizens and the implicit expectations of society may be due as much to increased expectations as to decreased performance. It is very difficult to separate these two variables, since today's students are products of today's society, whereas yesterday's students were products of their society. In such a context, talk about decline in numeracy remains more speculation than fact.

CONTEMPORARY EVIDENCE

Today's numeracy should be compared with requirements of today's society. The "nation's report card," which samples the 80 percent of seventeen-year-olds who are still in school, provides a fair measure of what passes for numeracy.[5] Most students in this sample can perform simple one-step arithmetic problems such as comparing six dimes and eleven nickels, or reading a bar graph. However, only half of these students—that is, about 40 percent of the nation's seventeen-year-olds—can solve moderately more sophisticated problems such as finding 87 percent of 10, or computing the area of a rectangle. And only 6.4 percent of these students—representing only one in twenty of young U.S. adults—can perform simple multistep problems such as calculating total repayment (principal plus interest) on a loan, or locating the square root of 17 between two consecutive integers.

These recent results in the United States confirm evidence gathered a decade earlier by the Cockcroft commission in England.[6] Instead of relying only on written tests (as is typical in the United States), the British commission interviewed hundreds of adults to determine just how they used mathematics on the job and in everyday life. Interviewers in this study discovered a common perception of mathematics as such a "daunting subject" that more than half of those approached simply refused to take part in the study.

The Cockcroft study revealed intense apprehension in the face of simple mathematical problems: the extent of anxiety, helplessness, fear, and guilt was the "most striking feature" of this study. It documented widespread inability to understand percentages, even those as simple as tips or sales tax. Many thought, for example, that a fall in the rate of inflation should cause a fall in prices.

Two other features of this study are worth noting, for they are undoubtedly as true today in the United States as they were twelve years ago in England. Many people manage to organize their lives so that they make virtually no use of mathematics. By relying on others for what needs to be done or by resorting to coping strategies (for example, writing checks rather than estimating change), they successfully evade the mathematics that confronts them. In the real world, many people survive without ever using any quantitative skills.

On the job, the Cockcroft study discovered a surprising pattern. Most workers who needed to use specific job-related mathematics did so by methods and tricks passed on by fellow workers that had little connection (certainly none that they understood) with methods taught in school. Tradesmen frequently dealt in fractions with limited sets of denominators (e.g., halves, quarters, and eighths), so calculation within this domain could be done by special methods rather than by the general-purpose "common denominator" strategies taught in school. In another example, a worker who had frequent reason to multiply numbers by 7 did so by multiplying by 3, adding the result to itself, and then adding the original number.

The paradox of workers learning (oftentimes inventing) new mathematics instead of using what they have been taught in school is the result of insecurity brought on by their school experience with mathematics. Many otherwise well-educated persons are virtually innumerate; others become "mathophobic," avoiding tasks or careers that require any use of mathematics.[7] Unless the mathematics studied in school is understood with confidence—and all data show that only a minority of students achieves this type of understanding—it will not be used in any situation where the results really matter.

The most important result of school mathematics is the confidence to make effective use of whatever mathematics was learned, whether it be arithmetic or geometry, statistics or calculus. When apprehension, uncertainty, and fear become associated with fractions, percentages, and averages, avoidance is sure to follow. The consequences of innumeracy—an inability to cope with common quantitative tasks—are magnified by the very insecurity that it creates.

AN INVISIBLE CULTURE

Mathematics is often called the "invisible culture" of our age. Although surface features such as numbers and graphs can be seen in every newspaper, deeper insights are frequently hidden from public view. Mathematical and statistical ideas are embedded deeply and subtly in the world around us. The ideas of mathematics—of numbers and shapes, of change and chance—influence both the way we live and the way we work.[8]

Consideration of numeracy is often submerged in discussions of literacy, exposing only the traditional tip of basic skills (" 'rithmetic") for public scrutiny and comparative assessment. Strategies to improve numeracy will never be effective if they fail to recognize that arithmetical skills comprise only a small part of the mathematical power appropriate to today's world. Approaches to numeracy must reflect the different dimensions in which mathematical and statistical ideas operate.

Practical Numeracy

Many mathematical and statistical skills can be put to immediate use in the routine tasks of daily life. The ability to compare loans, to calculate risks, to estimate unit prices, to understand scale drawings, and to appreciate the effects of various rates of inflation bring immediate real benefit. Regardless of one's work or standard of living, confident application of practical numeracy provides an edge in many decisions of daily life.

Those who lack either confidence or skills to employ basic arithmetic, statistics, and geometry lead their economic lives at the mercy of others. Advertisers prey on those who shirk from thinking through the implications of exaggerated quantitative claims. Lotteries take in disproportionate revenue from less well-educated citizens in part because few people with minimal education understand chance.[9] Without practical numeracy, a person is left defenseless against those who would take advantage of their goodwill and resources.

Civic Numeracy

Whereas practical numeracy primarily benefits the individual, the focus of civic numeracy is on benefits to society. Discussions of important health and environment issues (for example, acid rain, the greenhouse effect, and waste management) are often vapid or deceitful if conducted without appropriate use of mathematical or statistical language. Inferences drawn from data about crime or AIDS, economic and geographic planning based on population projections, and arguments about the federal budget depend in essential ways on subtle aspects of statistical or econometric analyses. Civic numeracy seeks to ensure that citizens are capable of understanding mathematically based concepts that arise in major public policy issues.

Civic issues requiring a numerate citizenry arise on many different occasions. Much of the confusion—and near panic—surrounding the 1979 nuclear emergency at the Three Mile Island power plant was due, according to the investigative commission, to reporting by both public officials and journalists that omitted or mixed up important units of measurement. As the commission said, it was like reporting the score of a baseball game as 5 to 3 without saying who had won.

The continuing debate over mandatory AIDS testing provides a good example of quantitative issues hidden just beneath the surface of many public debates. Since no test for the AIDS antibody is perfect, there will always be a small number (perhaps 2 percent) of errors that may produce either false positive or false negative results. The innumerate public infers from this that testing is 98 percent accurate. But since the actual incidence of AIDS in the general population is less than the error in the test, any widespread test administered to a random sample of citizens will produce more results indicative of AIDS because of errors in the test than because of actual AIDS in the population. The personal consequences of these erroneous messages in psychological, economic, and emotional grief are rarely recognized by a public which naively assumes that any accurate test will produce accurate results when put into widespread use.

A public unable to reason with figures is an electorate unable to discriminate between rational and reckless claims in public policy. Debates about acceptable levels of suspected carcinogens, about the efficacy of high-risk medical procedures, and about regulation of hazardous waste all hinge on sophisticated understanding of quantitative issues involving data, chance, and statistical inference. Just as Thomas Jefferson viewed "an enlightened citizenry" as the only proper foundation for democracy, so in today's society we depend on "a numerate citizenry" for informed and productive debate of public issues.

Professional Numeracy

Many jobs require mathematical skills. Today's jobs, on average, require more mathematical skills than yesterday's jobs.[10] Leaders of business and industry repeatedly emphasize the role of mathematics education in providing the analytical skills necessary for employment. One measure of the seriousness that business attaches to mathematics is that American industry spends nearly as much each year on the mathematical education of its employees as is spent on mathematics education in public schools.[11]

Everyone knows that science depends on mathematics. Fewer recognize that mathematical or statistical methods are now indispensable in most professional areas of study. Computer packages, themselves based on mathematical models of scientific or economic phenomena, are widely used to simulate hypothetical situations in areas ranging from medicine to investment banking, from social planning to aircraft design. From medical technology (CAT scanners) to economic planning (projecting tax revenues), from genetics (decoding DNA) to geology (locating oil reserves), mathematical methods have made an indelible imprint on every part of science and industry.

On assembly lines statistical process control is regularly used to ensure quality: workers, most often high school graduates, must learn to use control charts and other statistical tools in the routine operation of manufacturing processes. Bank clerks must be able to interpret to customers the complexities of mortgage rates and investment risks. Doctors need to interpret to patients the uncertainties of diagnoses and the comparative risks of different treatments.

Professional numeracy provides important yet very different benefits to individuals, to businesses, and to nations. For students, mathematics opens doors to careers. For companies, a mathematically competent work force paves the way for new products and competitive production. For nations, mathematics provides the power of innovation to compete in a global technological economy. All benefit when professional numeracy is high; all are hurt when it is low.

Numeracy for Leisure

No observer of American culture can fail to notice the immense amount of time, energy, and money devoted to various types of leisure activity. Paradoxically, a very large number of adults seems to enjoy mathematical and logical challenges as part of their leisure activities. The popularity of puzzles, games of strategy, lotteries, and sport wagers reveals a deep vein of amateur mathematics lying just beneath the public's surface indifference.

Chance and strategy underlie all games of chance, from illegal numbers games to state lotteries, from casino gambling to horse racing. Millions of individuals who are innumerate by school standards thrive in the environment of gambling by relying on specialized home-grown methods, just as the workers in the Cockcroft study relied on special tricks to carry out the mathematical requirements of their trade.

Games and puzzles, ranging from solitaire to chess and from board games to bridge, reveal a different vein of public empathy with mathematical thinking. Many people in widely different professions harbor nostalgic dreams, often well-hidden, of the "Aha" experience they once enjoyed in school mathematics. The feeling of success that comes with the solution of a challenging problem is part of mathematical experience, a part that many persons miss in their regular lives. The popularity of magazine columns on mathematical and computer recreations attests to the broad appeal of recreational mathematics.

So strong is this drive that thousands of amateur mathematicians have devoted millions of hours trying to trisect the angle because they have heard in some remote geometry course that this problem, so simple to state, has defied solution since the time of ancient Greece. What they have failed to hear, or failed to grasp, is that nineteenth-century mathematicians proved that this problem and others like it are impossible to solve. The evidence of the Don Quixotes of mathematics shows that the capacity and drive for mathematics cannot be totally eradicated by unpleasant school experiences.

Many adults romanticize the aura of certainty afforded by the school caricature of arithmetic and geometry: they seek security against a threatening, changing culture by invoking the power of a mathematics they have never learned. Down this road lie numerology, astrology, and pseudoscience. It is truly alarming to discover how many adults trust astrology more than astronomy, numerology more than mathematics, and creationism more than molecular biology.

Both John Allen Paulos[12] and Martin Gardner[13] have documented with convincing examples the deep links between innumeracy and numerology, between scientific illiteracy and pseudoscience. The human need for explanation fills the vacuum of quantitative and scientific illiteracy with beguiling nonsense. Too often, the price of innumeracy is not ignorance, but delusion.

Cultural Numeracy

Like language, religion, and music, mathematics is a universal part of human culture. For many, albeit not for the majority, it is a subject appreciated as much for its beauty as for its power. The enduring qualities of abstract ideas such as symmetry and proof can be understood best as part of the legacy of human culture which is passed on from generation to generation.

Jacob Bronowski documented with superb insight the historical convolutions that blended mathematics, art, religion, and science into a single strand in the story of human culture.[14] Just as the expression of patterns flowered in Moslem art, so the search for pattern drove Renaissance science. Even today mathematicians and scientists commonly employ elegance as a standard by which to judge competing theories.

Two famous twentieth-century essays capture at opposite poles the abiding counterpoint among mathematics, art, and science. G. H. Hardy, the great British number theorist, wrote in his apologia of pure mathematics that "beauty is the first test: there is no permanent place in the world for ugly mathematics."[15] A quarter of a century later, mathematical physicist Eugene Wigner wrote of the "unreasonable effectiveness" of mathematics in the natural sciences as a "wonderful gift which we neither deserve nor understand."[16] Thus the mystery: beauty determines truth, and truth reflects reality.

Although it may sound to some like an oxymoron, mathematics appreciation has always been an important part of cultural literacy. To understand why so many of the greatest thinkers—from Plato to Pascal, from Archimedes to Einstein—rooted their work in principles of mathematics, to comprehend the *sui generis* nature of mathematical knowledge, to witness the surprising effectiveness of mathematics in the natural sciences, to explore the role of mathematical models in the great new scientific quest to understand the mind, to understand how order begets chaos and chance produces regularity—these and countless other facets of mathematical activity reveal their power and significance only on the level of philosophy, history, and epistemology.

The rationale for cultural numeracy parallels that advanced by E. D. Hirsch[17] for cultural literacy: to provide a common background fabric on which to weave the tapestry of civilization. Mathematics is part of this tapestry. Even young children can learn from mathematics the power of thought as distinct from the power of authority. For those with the ears to hear, echoes of Euclid sound in the words of Jefferson: "We hold these truths to be self-evident . . . . " Numeracy in this sense is an intrinsic part of our cultural heritage.

EDUCATIONAL IMPLICATIONS

Traditional school mathematics curricula do not deal uniformly with all aspects of numeracy. A pragmatic public supports two facets (e.g., practical and professional) virtually to the exclusion of the others, although within the two areas that are emphasized, the classroom treatment is often inappropriate to the objectives. Indeed, school mathematics is simultaneously society's main provider of numeracy and its principle source of innumeracy.

The skills required for practical numeracy can be taught to most students during the years of universal primary education through grades six, seven, and eight. Unfortunately, traditional elementary school curricula have concentrated on arithmetic to the exclusion of most other topics. Contemporary recommendations wisely suggest a broader curriculum, including practical geometry, data analysis, calculator skills, chance behavior, measurement, and estimation.[18] In the broadest sense, mathematics is not just about numbers and shapes, but is also a science of patterns.[19]

Beyond grades nine or ten, school and college study of mathematics has traditionally focused on a few very limited parts of professional numeracy. High school courses prepare students for calculus, which is the traditional mathematical standard for the natural sciences. College courses in elementary statistics—which could just as well be taught in high school—provide similar introductions to the quantitative prerequisites for the social and human sciences. However, computer methods have so significantly altered the role of mathematical and statistical methods on the job that most traditional school courses fail today's challenge of providing appropriate professional numeracy.

Civic, leisure, and cultural features are rarely developed in school mathematics, except perhaps in occasional enrichment topics that are never tested and hence never learned well. These aspects of numeracy are slighted because neither teachers nor administrators embrace a broad vision of numeracy. All too often, schools teach mathematics primarily as a set of skills needed to earn a living, not as a general approach to understanding patterns and solving problems. The disconnection of mathematical study from other school subjects—from history and sports, from language, and even from science—is one of the major impediments to numeracy in today's schools.

Students learn chiefly what they are motivated to learn. The evidence of mathematical methods learned out of school—on the job, in the street—shows that when numerical or geometrical methods are reinforced by use, they are both learned and remembered. In this respect, the language of mathematics is just like natural language: effective learning requires immersion in a culture that is speaking and using the language. Children learn to read and write not solely because of their language arts instruction in school, but equally because of the reinforcement provided by other school subjects, and by their environment at home. Where reading and writing are not reinforced at home, the progress of learning is much slower.

Numeracy is rarely reinforced, either in school or at home. Parents, coaches, and teachers of other subjects seldom make the effort to engage children in activities that would use mathematical or statistical methods—perhaps because the adults themselves tend to avoid such methods. No matter how effective mathematics instruction may be in school—and to be honest one must admit that it often is quite ineffective—it will have little lasting value unless student motivation and expertise is reinforced by extensive contact with mathematical, geometric, and statistical ideas in other environments.

LESS OR MORE?

While scientists, educators, and business leaders press for increased levels of numeracy, several social critics have raised questions about the basic premise that more effort leads to better results. Columnist William Raspberry, echoing social scientist Paul Burke, has argued that the wisest social policy is to focus required school study only on what I have called practical numeracy, leaving all other facets to elective study.[20] Within the range of potential meanings of *numeracy*, they adopt the minimalist position—numeracy for survival, not numeracy for civilization.

The issue is partly philosophical—involving the role of numeracy in cultural literacy—and partly strategic and economic: how best to deploy scarce resources (notably, excellent mathematics teachers) to meet necessary obligations of government. If not everybody needs mathematics beyond percentages and simple logic, why strive to teach more to all? Decades of evidence show that we fail both in the larger goal of developing multifaceted numeracy in all students and in the limited goal of developing practical numeracy. Raspberry and Burke argue, in effect, that by trying to achieve the former, we ensure failure even in the latter.

Their analysis rests not only on limited resources, but also on the evidence of hostility, frustration, and failure that mounts rapidly in required mathematics courses in grades eight through ten. As some argue that the laws against drugs are in part responsible for the high incidence of drug-related crime, so Raspberry and Burke argue that requirements in mathematics beyond the level of practical numeracy are themselves the cause of much of the nation's problem with innumeracy. Contrary to current national trends for increased high school requirements, they would reduce required secondary school mathematics to a one-year ninth-grade capstone course in practical numeracy.

One can hardly dispute this analysis because of evidence of current schooling. The typical mode of instruction in mathematics is almost exclusively catechetical:[21] standard texts are discussed bit by bit; standard questions accompany the text; standard answers are taught; and students are expected to recite standard answers with minimum variation or interpretation. Such teaching, developed centuries ago to provide mass religious education, is ill suited as a medium for teaching analytical thinking, creative problem solving, and the art of reasoning. What it produces, more often than not, is just "inert knowledge." Less of this sterile, rigid mathematics would certainly be a net gain for the nation.

The proper question is not whether to have more or less of an outmoded and ineffective tradition, but whether it is possible to do better with more effective school practice. Most experienced teachers and scholars believe that improvement is possible, and indeed under way. In that case, if school instruction does rise to the challenge of numeracy for all, one must still ask whether requirements or electives are the best strategy for public education in mathematics.

When it comes to civic, leisure, and cultural numeracy, the issue is no different than for cultural literacy. Shakespeare and Euclid share parallel pedestals in the architecture of core curricula. For each, schools must struggle to balance the motivation intrinsic to elective courses against the certain exposure of requirements. There is no simple answer.

However, professional numeracy arises from a different motivation—jobs—and requires a separate analysis. Mathematical knowledge is required in two ways in the arena of careers and employment: to get a job and to perform job duties. As the Cockcroft study documents so well, people cope with routine, on-the-job mathematics whether it is learned in school or not. They do not, however, cope well with nonroutine issues: one analysis of the Challenger disaster revealed that managers might have made different decisions had they had better understanding of basic statistics.

It is in securing jobs that mathematics functions as a "critical filter," being required for licensing examinations, college entrance, and course prerequisites. Because they are often less well prepared in mathematics than white males, minorities and women are filtered out in disproportionate numbers from many desirable jobs. Some argue that the role of numeracy on standard exams and course prerequisites should be reduced to match typical job requirements; others, including the president of the National Academy of Engineering,[22] argue that mathematics should become a pump rather than a filter in the educational pipeline from school to job.

The debate about mathematics as a filter, perhaps unnecessary, is a variation on the long-standing educational idea that subjects like mathematics and Latin train the mind. Many trades and professions keep their numeracy standards high in order to select individuals with a certain quality of mind (or, critics charge, of a certain socioeconomic status). So long as this is the case, prudent educators will require young students to continue their study of mathematics not because they *will* need it but because they *may* need it. The consequences for a student's economic future are too serious, and the temptation to opt out of a difficult course too great, to justify electives as wise mathematics policy for students who are still in the required years of schooling.

DIVERSITY

Many manifestations—practical, civic, professional, leisure, cultural—reveal diversity as the norm for numeracy. Just as static descriptions locked in the past are insufficient for today's needs, so narrow one-dimensional descriptors (e.g., " 'rithmetic") are inadequate for the panorama of mathematics in contemporary society.

Diversity in kind is matched—indeed, probably overwhelmed—by diversity in accomplishment. For example, pre- and post-tests of eighth-grade students show that each of the four major tracks (remedial, regular, enriched, algebra) ends the year less well prepared than the next highest class did as it began the year.[23] Enormous variation exists, even at that level, among students who study mathematics. In the eighth grade alone, the four-year spread in entering skills was increased, as a consequence of one year of educational effort, to nearly seven years.

These data are typical of the way students learn mathematics: they learn at very different speeds. Moreover, the more students know, the more they can learn: learning now enables learning later. This is one of the important intrinsic arguments for improving literacy and numeracy. Because of the sequential nature of mathematical knowledge, innumeracy inherited from early years becomes an insurmountable obstacle to subsequent study of any mathematics-related field.

Mathematical learning progresses in proportion to what one already knows. Hence, the range of student learning grows exponentially. The farther one moves up the educational ladder, the farther apart students become. It is not uncommon for the mathematical performance of students entering large universities to be spread across the entire educational spectrum, from third or fourth grade to junior or senior year in college. In no other discipline is the range of achievement as large as it is in mathematics.

One measure of the spread is provided by the mathematical performance of U. S. students as they enter adulthood. We know that on average they do poorly. The weakest leave school, usually as dropouts, with the numeracy level of an average third grader. Solving problems that would stump most college teachers of mathematics, the strongest compete successfully in an international mathematical Olympiad. The gap between these extremes is immense, and filled with students.

The wide variability in mathematical achievement of students, together with the varied types and purposes of numeracy, suggests the futility of any explicit definition of *numerate*. It is neither efficient nor possible for everyone to know the same thing. Reality dictates a continuum of types and levels of numeracy distinguished by purpose, accomplishment, and style.

Demand for civic and professional numeracy—for mathematical skills of citizen and worker—leads directly to increased mathematical diversity of the population. Because students who know more learn faster, increasing educational effort in school mathematics often increases the gap between the strongest and the weakest students more than it raises the average performance of students. Increase in the variance of quantitative skills, not just relative weakness in average performance, is perhaps the most important debilitating social consequence of society's increased demand for mathematical and statistical power.

EQUITY AND EXCELLENCE

Increased variance leads to inequity. In jobs based on mathematics, inequity translates into severe underrepresentation of women and minorities. Concern about this issue has traditionally been based on issues of equity—that all Americans deserve equal opportunity for access to mathematically based careers. Demographic reality now shows that inadequate mathematical preparation of major parts of our work force will produce an America unprepared to function effectively in the twenty-first century. Equity has joined economic reality as a compelling factor in mathematics education.[24]

National projections make the case in stark terms. At the beginning of the twenty-first century—just one decade from now—only 15 percent of net new workers will be white males; the other 85 percent will be women, minorities, and immigrants.[25] Yet advanced mathematics remains primarily an enclave of white males. Without significant (and unprecedented) increase in the proportion of underrepresented populations who take advanced degrees in science, the flow of new scientific and engineering personnel will be well below national need by the early part of the next century.[26]

Strategies for increasing participation of underrepresented groups must encourage both equity and excellence. Equity requires mathematical expectations of *all* students; its focus is to ensure that all students receive a mathematical background sufficient to compete for decent employment and to function as effective citizens in the information age. Excellence focuses primarily on what are often called "pipeline" issues: the need for vast increase in the number of scientific professionals (including teachers, engineers, and technical workers) from underrepresented groups.

Although the goals of equity and excellence sometimes appear to clash, in mathematics education they converge on a single issue: heightened expectations. Equity for all requires appropriate challenge for all—both for those who learn mathematics slowly as well as for those who show special talent for mathematics. Excellence demands that students achieve all they are capable of accomplishing, since nothing less will be sufficient to sustain our national economic and scientific aspirations.

Public emphasis on literacy and numeracy can too easily lead to specifications for minimum performance, which in turn lead to minimum accomplishment. Sometimes such campaigns feature a "back-to-basics" approach which shortchanges all students. Useful numeracy should entail equity and excellence for all. Hence, school mathematics—in curriculum, in pedagogy, and in assessment—should reflect a commitment to equity that simultaneously fosters excellence. In such programs there would be no ceiling on a child's aspiration.

TWO LITERACIES

C. P. Snow introduced the term *two cultures* to describe the schism he found between the scientific and the humanistic, between the world of nature and the world of people.[27] In interviews of M.I.T. alumni Benson Snyder documented two similar "modes" that represent "distinct yet complementary ways of knowing."[28] Numeracy and literacy—the language of nature and the language of people—are the two literacies of our age. Snow's schism and his label remain a reminder that this duality represents a truly fundamental dichotomy.

Despite the gap between them, literacy and numeracy have much in common. In each there is tension between narrow and broad interpretations—between practical benefits and cultural effects. As each is a language, each must be taught in a context of realistic use both to sustain motivation and to ensure mastery. Moreover, the way we use these two languages determines the way we think.

Nevertheless, numeracy remains the more daunting challenge. For each person who never learned to read, there must be a hundred who boast that they were never any good at math. That imbalance is especially troublesome in an age of data and measurement, of computers and statistics. Changing school mathematics is an important ingredient in any program for reform, but one must also look to society beyond the schools for serious change of lasting benefit. Here are some small but important changes that would make great improvements in numeracy:

- *Don't teach just arithmetic.* Numeracy requires a rich blend of statistics, geometry, and arithmetic, catalyzed by careful reasoning rooted in common sense.
- *Don't rely on worksheets.* Students learn best in active contexts featuring discussion, writing, debate, investigation, and cooperation. Isolated facts on artificial worksheets reinforce the image of school mathematics as totally artificial, unrelated to real life.
- *Don't ignore calculators.* Children must learn many ways to calculate—manually, mentally, electronically—in realistic contexts that reflect the world around them. Calculators are part of that world and should be part of school mathematics.
- *Don't rely only on school.* Children are influenced as much by the entertainment and sports industries as by formal school instruction. There is much that those industries could do to promote both numeracy and literacy.
- *Don't use just short-answer tests.* Assessment instruments strongly influence the shape of instruction and learning. In numeracy as in literacy, formulation and expression are more important than simple answers. Tests should reveal how students think, not just what they know.
- *Don't depend only on mathematics.* Although numeracy may be taught in mathematics classes, to be learned effectively it must be used widely in other contexts, both in school and at home, in entertainment and in sports.

Although we can neither precisely define nor measure numeracy, we can improve it. Especially in an age of computers, we really must take steps to improve the level of numeracy in all segments of society. With numeracy comes increased confidence for individuals to gain control over their lives and their jobs. Numeracy provides the ability to plan, to challenge, and to predict; it reveals the power of reason and unlocks the language of nature.

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