# LEWIS CARROLL's DEDUCTIVE LOGIC 

Source: Lewis Carroll's Symbolic Logic Edited by W. Bartley, III

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Lewis Caroll: (1832-1898)
He was the author of Alice in Wonderland (1865) and Through The Looking Glass. (1871)
He was the foremost Victorian photographer of children
Under his real name of Charles Dodson he taught mathematics at Oxford
He published The Game of Logic in 1886 and published Symbolic Logic (Part I) in 1896.

## Symbolic Logic (1896):

Lewis Carol devised a simple, graphical technique to solve certain deductive arguments
"Symbolic" is used in an algebraic (Boolean) sense.
Carroll's logic is different from the preceding formal (classical or Aristotilean) logic.
Carroll's logic is different from the subsequent mathematical (or modern) logic.
Carroll say his logic as valuable in many ways:
He hoped it would be "of real service to the young" as a "healthful mental recreation".
"those who really try to understand it will find it more interesting than most of the games.."
"He (the accomplished Logician) can apply this skill to any and every subject of human thought; in every one of them it will help one to get clear ideas. to make orderly arrangement of knowledge and to detect and unravel the fallacies one will meet in every subject one may be interested in."

Carroll felt his logic was much easier than classical (formal) logic for three reasons:

1. "It (formal logic) is much too hard for the average intellect"
2. "Those who do succeed in mastering its principles find it hopelessly dry and uninteresting"
3. "It's results are absolutely and entirely useless"
[Carroll might hold these criticisms against modern mathematical logic]
By contrast, Carroll felt his logic was superior for similar reasons:
4. "I have taught the method of Symbolic Logic to many children, with entire success"

Children learn it easily, and take real interest in it
2. "As to Symbolic Logic being dry, I can only say try it! I have found .. none to rival it.
3. "As to its being useless, I have already said enough." (see above)

For a complete and deliberate presentation of this topic, consult Symbolic Logic by Lewis Carroll
> "If this approach can be used any place (during a lecture or while reading or talking) and if it can be recreated without reviewing a forgotten text book, and if it can be applied to many, many situations, then truly this is a tool which is highly valuable. As such it can justify your investment of time, attention and practice necessary to use its power as its master."

## Deductive arguments:

Arguments are of two kinds: deductive and inductive
Deductive arguments must have at least one universal premise
Inductive arguments have no universal premises but must have a universal conclusion
Deductive arguments are considered either valid or invalid
A valid argument means the conclusion must be true if the premises are true.
Inductive arguments are considered either strong or weak
A strong argument means the conclusion is probably true if the premises are true.

Carroll's Symbolic Logic deals strictly with deductive arguments.

Within mathematics, deductive arguments are extremely common
Arithmetic: $\mathrm{X}+\mathrm{Y}=\mathrm{Y}+\mathrm{X} \quad$ (Commutative Law). Therefore $4+6=6+4$
$(x+y)+z=x+(y+z) \quad$ (Associative Law) Therefore, $(4+3)+2=4+(3+2)$
Algebra: $\quad Z=X+Y ; \quad X=4 ; \quad Y=6 ; \quad$ Therefore, $Z=10$
Geometry: Angles of triangle add to $180^{\circ}$. An equilateral triangle has 3 equal angles, Therefore, the angles in an equilateral triangle are each $60^{\circ}$

Outside mathematics, deductive arguments are common in rules and in the physical sciences
There are several kinds of deductive arguments
Categorical: Single (non-compound), non-modal arguments All men are mortal; Plato is a Man; Therefore, Plato is mortal.
Modal: Involving impossibility, possibility and certainty
All mammals are warm-blooded; Some dinosaurs may be mammals; Therefore some dinosaurs may be warm-blooded.

Compound Involving at least one compound proposition
Conditional If a graduate has at least a 3.3 GPA, then they graduate with honors;
No graduate received honors;
Therefore, no graduate had a GPA of at least 3.3
Disjunctive:Either John makes the car payment or the bank repossesses his car;
The bank repossessed John's car;
Therefore, John did not make the car payment.
ConjunctiveIf Joe pays Jan and Jan accepts Joe's offer, then Joe gets Jan's car;
Joe did not get Jan's car and John paid Jan;
Then Jan didn't accept Joe's offer.
Dilemmas Either logicians are right or wrong.
If they are right, then they don't need logic.
If they are wrong, then logic won't help
Therefore, logicians don't need logic.
This handout deals strictly with deductive arguments which are categorical

## CATEGORICAL DEDUCTIVE ARGUMENTS

The simplest arguments involve three categorical propositions: two premises and one conclusion Example: Socrates is a Man; All men are mortal; Thus, Socrates is mortal.

1. Each categorical proposition has three variable parts and the verb to be (is or are)
a. Quantity: No (or None), Some, All or not-All
b. Subject: a class (or the opposite of a class)
c. Predicate a second class (or the opposite of a 2nd class)

Examples: No animals are rocks; Some non-living things are rocks. All rocks are not organic
2. All classes must be members of the same universe (category)

Example: people includes male/female, rich/poor and student/non-student
3 All classes (subject \& predicate) must be bilateral
"Bilateral" is Carroll's term for binary complements: two exclusive and exhaustive classes Examples: male or female, rich or non-rich (poor), full-time or not full-time (part-time)

4 Collectively these three propositions must include three bilateral classes
5 Each proposition must include exactly two classes (a pair) -- no more and no less

## 6 Each proposition must include a different pair of the three bilateral classes

7 The three classes are generally designated by $\mathrm{m}, \mathrm{x}$, and y .
a. "m" (for middle) is the class eliminated from the conclusion (the "eliminand")
b. "x" and "y" are the two classes retained in the conclusion (the "retinands").
c. x ' means non-x (or not x ); $\mathrm{y}^{\prime}$ means non-y (or not y ); m' means non-m (or not m )

Acceptable argument: an argument which does not violate any of the rules mentioned above
None of the following arguments are acceptable
a. No $x$ are m'; All $x$ are $y$; Thus, some $m$ are $y^{\prime} \quad$ [See rule 7a above]
b. No $m$ are $x^{\prime} ; \quad$ All $x$ are $m$ '; Thus, some $x$ are $y \quad$ [See rule 6 above]
c. No $x$ are $y m \quad$ All $m x$ are $m \quad$ Thus, some my are $x \quad$ [See rules 5 and 7a]

Valid argument: one where the conclusion must be true if the premises are true Strong argument: one where the conclusion is probably true if the premises are true [Good argument: one where the premises appear true and the argument is valid or strong]

SUMMARY (for No, Some and All)
There are 576 acceptable arguments (384 omitting order)
The acceptable arguments can be summarized into 20 forms.
Of the 20 forms, 12 are valid
Of the 12 valid forms, 11 have just one conclusion and one has two conclusions

Memorizing the 12 valid arguments is very difficult!!!
Lewis Carroll created a simple, graphical solution for validating arguments

## QUANTITY:

The opposite of "No" (or "None") is "Some"
No (as a measure of quantity) means nothing or none
The negative opposite of "None" or "No" is "no-none" (or "no-No") Double negatives are very difficult mentally and should be avoided
The positive opposite of "nothing" or "no" is "something" or "some"
"Some" means at least one; Something means more than nothing.
The opposite of "ALL" is "Not All"
"Not-all" means either "none" or "some" (excluding "All")
Following by Milo Schield (It can be ommited by students)
SQUARE OF OPPOSITION FOR QUANTITY
NONE Contradictories diagonally

## Three Kinds of Opposites:

## Contradictories: (diagonally related)

Rule: If either is false, the opposite is true.
Example: If No x is y is false, then Some x is y is true If Not-all $x$ are $y$ is true, then All $x$ is $y$ is false

## Contraries: (Horizontally related)

Rule (top): If one is true, then the other must be false (both could be false)
Rule (sub): If one is false, then the other must be true (both could be true)
Example If No x is y is false then All x is y may be true (but could be false) If No x is y (All x is y ) is true then All x is y (No x is y ) must be false If Some x is y is true then Not-All x is y may be true (could be false) If Some x is y (Not-all x is y ) is false then Not-all x is y (Some x is y ) is true

## Sub-alternates: (Vertically related)

Rule: If the universal is true, then the "Particular" is true (but not vice versa). If the particular is false, then the universal is false (but not vice versa)
Example: If No x is y is true then Not-all x is y is also true
If All $x$ is $y$ is true, then Some $x$ is $y$ is also true
If Not-All x is y is true, then No x is y may be true (but could be false) If Some x is y is true, then All x is y may be true (but could be false)

## PROPOSITIONS:

There are two kinds of propositions: propositions of existence and propositions of relation
Propositions of existence assert either existence or non-existence of a subject or predicate. These propositions have only two forms:
SOME: There are [some] green cars Green cars exist (do exist) Some xy
NO: There are no (aren't any) green flies Green flies do not exist No xy
Propositions of relation assert a relation between a subject and a predicate
Carroll investigated the two forms of existence plus "ALL" and "NOT-ALL"
SOME: Some cars are green Some x are y
NO: No flies are green No x are y
ALL: All cars are green All x are y
NOT-ALL: Not-all cars are green Not all x are y


For Carroll, "Some" and "All" imply existence. "None" does not. "Not-all" is ambiguous.

## Thus, for Carroll

1. "All" propositions entail two conjunctive propositions: a SOME and a NO
"All cars are green" means "Some cars are green" AND "No cars are not-green"
2. "Not-All" propositions entail two disjunctive propositions: a SOME or a NO
"Not-all cars are green" = "No cars are green" OR "Some cars are not green"
?? [Note: Carroll's presentation does not show how to handle "not-all" statements]
Thus, all these propositions of relation can be converted into propositions of existence.

## 

## NORMAL FORM OF PROPOSITIONS:

Translation of non-standard phrases into standard terms
"Each x", "Any x" means "All x" (distributive implies collective)
Individual things (John Galt, my book, this brown puppy) are expressed as "All x"
"Only x" or "None but x" are expressed as 'No x'"
NORMAL FORM:
The subject and predicate of a proposition share a common universe of discussion They are both members of a common category (species of a common genus)

Translation of non-standard clauses into Normal form.

1. "All men are mortal" => "All things that are men are things which are mortal or "All men-things are mortal things
2. "No books are read" becomes" No things which are books are things which are read" or "No book-things are things which are read" or "No books are things which are read"
3. "No one likes flies" becomes "No persons are people who like flies
4. "None but the brave find love" becomes "No non-brave people are people who find love"

How to "write" propositions involving 2 variables with Carroll's bilateral diagrams
Consider propositions involving just two bilateral classes: x and y
"Write" each proposition using the symbols of I and O as follows:


1. "Some" rule: Place an "I" in the area referenced
a. Some x are y Put I in upper-left corner
b. Some $x$ ' are y Put I in lower-left corner
c. Some x are $\mathrm{y}^{\prime}$ Put I in upper-right corner
d. Some $x^{\prime}$ are $y^{\prime}$ Put I in lower-right corner

Same as
Same as Some y are $x^{\prime}$
Same as Some y' are x
Same as Some y' are $x^{\prime}$
2. "No" rule Place an "O" in the area referenced
a. No $x$ are $y \quad$ Put $O$ in upper-left corner
b. No x' are y Put O in lower-left corner
c. No $x$ are $y^{\prime} \quad$ Put $O$ in upper-right corner
d. No $x^{\prime}$ are $y^{\prime}$ Put O in lower-right corner

Same as No y are x
Same as No y are $\mathrm{x}^{\prime}$
Same as No $y$ 'are $x$
Same as No y' are $\mathrm{x}^{\prime}$
3. "All" rule: Break into two conjunctives: a "Some" part and a "No" part.

The "Some" part has the same subject and predicate [Use the "Some" rule]
The "No" part has the same subject but a reversed predicate [Use the "No" rule]
a. All x are y

Some x are y Put I in upper-left
No $x$ are $y^{\prime} \quad$ Put $O$ in upper-right
b. All x are $\mathrm{y}^{\prime}$

Some $x$ are $y^{\prime} \quad$ Put $I$ in upper-right
No $x$ are $y \quad$ Put $O$ in upper-left
c. All x' are y

Some x' are y Put I in lower-left
No x' are y Put I in lower-right
d. All $x^{\prime}$ are $y^{\prime}$

Some x' are y' Put I in lower-right
No x' are y Put O in lower-left
e. All y are x

Some y are x Put I in upper left
No y are $x^{\prime} \quad$ Put O in lower left
f. All y are $x$ '

Some y are x' Put I in lower left
No $y$ are $x \quad$ Put $O$ in upper left
g. All $\mathrm{y}^{\prime}$ are x

Some y' are x Put I in upper right
No $y$ ' are $x^{\prime} \quad$ Put O in lower right
$\begin{array}{ll}\text { h. All y' are } x^{\prime} & \\ \text { Some y' are } x^{\prime} & \text { Put I in lower right } \\ \text { No } y^{\prime} \text { are } x & \text { Put O in upper right }\end{array}$

CONVERSION: to exchange subject and predicate in a simple proposition

|  | Original | Conversion | Representation | Conclusion |
| :--- | :--- | :--- | :--- | :--- |
| SOME | Some x are y | Some y are x | Same (upper-left) | Conversion always OK |
| NO | No x are y | No N are x | Same (upper left) | Conversion always OK |
| ALL | All x are y | All y are x | Not the same | Conversion never OK |

## How to "read" propositions involving 2 variables using Carroll's bilateral diagrams

Consider propositions involving just two bilateral classes: x and y
Each proposition is encoded using the symbols of I (Some) and O (None)
Read each proposition as follows:




Read as All x are y (No conversion possible)


Read as All x are y' (no conversion possible)



Read as All y are $\mathbf{x}$ (No conversion possible)


Read as All y are x' (no conversion possible)

Ask a friend to quiz you on these until you can read these quickly and accurately. Invite your friend to try any combination (these 6 examples do not include all the possibilities)

How to "write" propositions involving three classes using Carroll's trilateral diagrams.


1. "SOME": Place an "I" in the area referenced by a "Some" statement
a. Some y are $m$ Put I in left side inside center box across horiz. line
b. Some $\mathrm{m}^{\prime}$ are y Put I in outside box on right side across horiz. line
c. Some x are y Put I in upper left corner across center box.
d. Some $m$ are $x$ Put I in upper half inside center box across vertical line.
2. "No": Place an "O" in the area referenced by a "No" statement
a. No y are m Put two Os in left side inside center box: 1 above $\& 1$ below line
b. No $\mathrm{m}^{\prime}$ are $\mathrm{y} \quad$ Put two Os in outside box on right side: 1 above $\& 1$ below line
c. No x are y Put two Os in upper-left corner: 1 outside $\& 1$ inside center box.
d. No m are $x \quad$ Put two Os in upper-half inside center box: 1 on left $\& 1$ on right
3. "All": Break into a "Some" part and a "No" part.

The "Some" part has the same subject and predicate [Use the "Some" rule]
The "No" part has the same subject but a reversed predicate [Use the "No" rule]
a. All y are m
[Some y are m] Put I in left side inside center box across horiz. line
[No y are m'] Put two Os on left side outside center box: 1 above\& 1 below line
b. All m' are y
[Some m' are y] Put I in left side of outside box across the horizontal line
[No m' are $\left.y^{\prime}\right] \quad$ Put two Os in outer-right section: 1 above $\& 1$ below center line.
c. All $x$ are $y$
[Some $x$ are y] Put I in upper-left corner across inner box.
[No $x$ are $\left.y^{\prime}\right] \quad$ Put two Os in upper-right corner: 1 inside $\& 1$ outside center box.
d. All m are x
[Some $m$ are $x$ ] Put I in center box across both sides of the upper half
[No $m$ are $x$ ] Put two Os in center box, upper half on both sides of center.
Work with a classmate until you can write these very accurately.
Writing these takes time to learn. You must practice until it becomes familiar.
Invite your partner to try any combination (these examples do not include all the possibilities)

## How to "read" conclusions based on three terms using Carroll's trilateral diagrams

## RULES:

1. An "I" in either part ( m or m ') of a single xy quadrant means "SOME"
2. An "O" in both parts ( m and m ') of a single xy quadrant means "NO"
3. An ALL statement can be formed from the appropriate SOME and NO statements.

Note: An "I" outranks a "No" in different parts (m versus m') of the same quadrant

$\mathrm{m}=$ inside center $\mathrm{m}^{\prime}=$ outside center

m = inside center m'=outside center

Read as SOME $\mathbf{x}$ are $\mathbf{y}$ (SOME $\mathbf{y}$ are $\mathbf{x}$ )
Read as SOME x' are y (SOME y are $\mathbf{x}^{\prime}$ )

Note: An "O" must be present in both parts of a quadrant ( m and m ') to get "No" as a conclusion

m = inside center $\mathrm{m}^{\prime}=$ outside center
Read as NO $\mathbf{x}$ are $\mathbf{y}$ (or NO $\mathbf{y}$ are $\mathbf{x}$ )

$\mathrm{m}=$ inside center
$\mathrm{m}^{\prime}=$ outside center Read as NO $\mathbf{x}$ are $\mathbf{y}^{\prime}$ (or $\mathbf{N O}^{\prime} \mathbf{y}^{\prime}$ are $\mathbf{x}$ )


$\mathrm{m}=$ inside center $\mathrm{m}^{\prime}=$ outside center
Read as ALL x are y (no conversion)

$\mathrm{m}=$ inside center
m'=outside center

Read as ALL y are x (no conversion)

The "I" can be in either part ( m or m ' or both) of the quadrant for a "Some".
The O must be in both parts ( m and m ') of a quadrant for a "No" in the conclusion.
Drill with a partner until you can read these like you read a book -- easily and accurately.
With practice it becomes like a game -- a game you can play profitably for life.

## How to identify valid deductive arguments using Carroll's trilateral diagrams

NO and NO

1. No poets are rich
x is Poet ( m is rich)
2. No yuppies are poor y is Yuppie ( m ' is poor)
3. No $x$ are $m$
4. No y are m'

m is inside center box

Since both segments of the upper left corner contain an "O", this gives a valid conclusion Conclusion: "No $\mathbf{x}$ are $\mathbf{y}$ " or "No $\mathbf{y}$ are $\mathbf{x}$ " No poets are Yuppies or No Yuppies are poets
=======================================================================1
SOME and NO

1. Some poets are poor
$x$ is poet ( $m$ is poor) y is Yuppie ( $\mathrm{y}^{\prime}$ is non-Yuppie)
2. Some $x$ are $m$
3. No y are m

m is inside center box

In the xym cell, the "None" pushes the "Some xym or Some xy'm" into the xy'm cell. Conclusion: Some $x$ are $y$ ""or "Some $y^{\prime}$ are $x \quad$ or Some poets are non-yuppies A required 'Some' in any part of a quadrant is sufficient to be required of the whole quadrant Advice: Enter the "No" statements first; enter the "Some" statements last

## How to identify valid deductive arguments using Carroll's trilateral diagrams

ALL and SOME:
Single ALL statement with retinend in subject: m is poor

1. All singers are poor $\quad \mathrm{x}$ is singer ( $\mathrm{x}^{\prime}$ is non-singer) 1. All x are m
2. Some poets are rich
y is poet ( $\mathrm{y}^{\prime}$ is non-poet)
3. Some y are m'

m is inside center box

In the xym' cell, the "No" pushes the "Some" into the x'ym' cell.
Conclusion:Some $\mathbf{x}^{\prime}$ are $\mathbf{y}$ or Some $\mathbf{y}^{\prime}$ are $\mathbf{x}$ or Some non-singers are poets

ALL and NO:
Single ALL statement with retinend in predicate

1. All poets are pianists
2. No non-poets are rich
3. Thus no non-pianists are rich m is poet ( m ' is non-poet) x is Pianist ( $\mathrm{x}^{\prime}$ is non-pianist); y is rich ( $\mathrm{y}^{\prime}$ is poor);
4. Thus, no x ' are y
5. All x are m
6. No m' are y


In the x'y cells, the "No" has no competition, thus it upholds the conclusions that "No x' are y" The conclusion has two forms: "No $\mathbf{x}$ are $\mathbf{y}$ " and "No $\mathbf{y}$ are $\mathbf{x}$ '".

## How to identify valid deductive arguments using Carroll's trilateral diagrams

ALL and ALL::
Two ALL statement with retinends in subject

1. All singers are poor
x is a singer; y is a poet
2. All $x$ are $m$
3. All poets are rich m is a poor person
4. All y are m '

m is inside center box
In the xym cell, the "No" pushes the "Some" into the xy'm cell. In the xym' cell, the "No" pushes the "some" into the x'ym' cell
Conclusion: 1. All y' are x which means All non-poets are singers
Conclusion: 2. All x ' are y which means All non-singers are poets
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ALL and ALL:
Two ALL statements with retinends in predicate
5. All poets are pianists x is pianist; y is singer
6. All m are x
7. All non-poets are singers $m$ is poet
8. All m' are $y$

m is inside center box
Conclusion: No $\mathbf{x}^{\prime}$ are $\mathbf{y}^{\prime} \quad$ which means all non-pianists are non-singers

SOME and NO PROPOSITIONS: NO "ALL" PROPOSITIONS
Total of 192 styles in 6 forms: No-No, No-Some and Some-Some with like and unlike eliminands. Of the 6 forms, 2 have valid arguments

NO \& NO (64 styles in 2 forms)
Like eliminands:
No true conclusions; no valid arguments

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| No $x$ are $m$ | No $m$ are $y$ |  |
| No $x$ are $m^{\prime}$ | No $m^{\prime}$ are $y$ |  |
| No $x^{\prime}$ are $m$ | No $m$ are $y$ |  |
| No $x^{\prime}$ are $m^{\prime}$ | No $m^{\prime}$ are $y$ |  |
| No $x^{\prime}$ are $m$ | No $m$ are $y^{\prime}$ |  |
| No $x^{\prime}$ are $m^{\prime}$ | No $m^{\prime}$ are $y^{\prime}$ |  |
| No $x$ are $m$ | No $m$ are $y^{\prime}$ |  |
| No $x$ are $m^{\prime}$ | No $m^{\prime}$ are $y^{\prime}$ |  |

plus 24 conversions of "No" separate and joint:
Unlike eliminands:
True conclusion with same signs as in premises

| Premise | Premise | Conclusion |
| :---: | :---: | :---: |
| No x are m' | No m are y | No x are y |
| No $x$ are m | No m' are y | " " " " " " |
| No x' are m' | No m are y | No $\mathrm{x}^{\prime}$ are y |
| No $\mathrm{x}^{\prime}$ are m | No m' are y | " " " " " " |
| No x are m' | No m are $\mathrm{y}^{\prime}$ | No x are $\mathrm{y}^{\prime}$ |
| No x are m | No m' are $\mathrm{y}^{\prime}$ | " " |
| No $\mathrm{x}^{\prime}$ are m' | No m are $\mathrm{y}^{\prime}$ | No $\mathrm{x}^{\prime}$ are $\mathrm{y}^{\prime}$ |
| No $\mathrm{x}^{\prime}$ are m | No m' are $\mathrm{y}^{\prime}$ | " " " " " " " |

plus 24 conversions of "No" separate and joint
SOME \& SOME (64 styles in 2 forms)

Like eliminands:
No valid conclusion

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| Some $x$ are $m$ | Some $m$ are $y$ |  |
| Some $x$ are $m^{\prime}$ | Some $m^{\prime}$ are $y$ |  |
| Some $x^{\prime}$ are $m$ | Some $m$ are $y$ |  |
| Some $x^{\prime}$ are $m^{\prime}$ | Some $m^{\prime}$ are $y$ |  |
| Some $x^{\prime}$ are $m$ | Some $m$ are $y^{\prime}$ |  |
| Some $x^{\prime}$ are $m^{\prime}$ | Some $m^{\prime}$ are $y^{\prime}$ |  |
| Some $x$ are $m$ | Some $m$ are $y^{\prime}$ |  |
| Some $x$ are $m^{\prime}$ | Some $m^{\prime}$ are $y^{\prime}$ |  |

plus 24 conversions of "Some" separate and joint

Unlike eliminands:
No valid conclusion

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| Some $x$ are $m^{\prime}$ | Some $m$ are $y$ |  |
| Some $x$ are $m$ | Some $m^{\prime}$ are $y$ |  |
| Some $x^{\prime}$ are $m^{\prime}$ | Some $m$ are $y$ |  |
| Some $x^{\prime}$ are $m$ | Some $m^{\prime}$ are $y$ |  |
| Some $x$ are $m^{\prime}$ | Some $m$ are $y^{\prime}$ |  |
| Some $x$ are $m$ | Some $m^{\prime}$ are $y^{\prime}$ |  |
| Some $x^{\prime}$ are $m^{\prime}$ | Some $m$ are $y^{\prime}$ |  |
| Some $x^{\prime}$ are $m$ | Some $m^{\prime}$ are $y^{\prime}$ |  |
| plus 24 conversions of "Some" separate and joint |  |  |

SOME \& NONE (64 styles in 2 forms)
Like eliminands
Valid conclusion: Retinand in No changes sign

| Premise | Premise | Conclusion |
| :---: | :---: | :---: |
| Some x are m | No m are y | Some x are y' |
| Some x are m' | No m' are y |  |
| Some $\mathrm{x}^{\prime}$ are m | No m are y | Some x ' are y |
| Some $\mathrm{x}^{\prime}$ are m' | No m' are y | " " " " " " " |
| Some $\mathrm{x}^{\prime}$ are m | No m are $\mathrm{y}^{\prime}$ | Some x ' are y |
| Some $\mathrm{x}^{\prime}$ are m' | No m' are $\mathrm{y}^{\prime}$ | " " " " " " " |
| Some x are m | No m are $\mathrm{y}^{\prime}$ | Some x are y |
| Some x are m' | No m' are $\mathrm{y}^{\prime}$ | " " " |

plus 24 conversions of "Some", "No" and both

Unlike eliminands:
No valid conclusion

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| Some $x$ are $m^{\prime}$ | No $m$ are $y$ |  |
| Some $x$ are $m$ | No $m^{\prime}$ are $y$ |  |
| Some $x^{\prime}$ are $m^{\prime}$ | No $m$ are $y$ |  |
| Some $x^{\prime}$ are $m$ | No $m^{\prime}$ are $y$ |  |
| Some $x$ are $m^{\prime}$ | No $m$ are $y^{\prime}$ |  |
| Some $x$ are $m$ | No $m^{\prime}$ are $y^{\prime}$ |  |
| Some $x^{\prime}$ are $m^{\prime}$ | No $m$ are $y^{\prime}$ |  |
| Some $x^{\prime}$ are $m$ | No $m^{\prime}$ are $y^{\prime}$ |  |
|  |  |  |
| plus 24 conversions of "Some", "No" and both |  |  |

"ALL" STATEMENTS: Retinand always in subject of "ALL" statement
Total of 80 styles in 6 forms: All-None, All-Some and All-All with like and unlike eliminands.
Of these 6 forms, 3 have valid arguments

ALL \& NONE (32 styles in 2 forms)

Like eliminands
Valid conclusion: "No" retinand chgs sign ( $\mathrm{S}=\mathrm{S}$ )

| Premise | Premise | Conclusion |
| :---: | :---: | :---: |
| All $x$ are m | No m are y | All x are $\mathrm{y}^{\prime}$ |
| All $x$ are m' | No m' are y | " " " " " " |
| All x ' are m | No m are y | All $\mathrm{x}^{\prime}$ are $\mathrm{y}^{\prime}$ |
| All $\mathrm{x}^{\prime}$ are m' | No m' are y | " " " " " " |
| All $x^{\prime}$ are m | No m are $\mathrm{y}^{\prime}$ | All x ' are y |
| All $\mathrm{x}^{\prime}$ are m' | No m' are $\mathrm{y}^{\prime}$ | " " " " " " |
| All $x$ are m | No m are $\mathrm{y}^{\prime}$ | All x are y |
| All x are m' | No m' are $\mathrm{y}^{\prime}$ | " " " " " " |

Unlike eliminands:
No valid conclusion

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| All $x$ are $m^{\prime}$ | No $m$ are $y$ |  |
| All $x$ are $m$ | No $m^{\prime}$ are $y$ |  |
| All $x^{\prime}$ are $m^{\prime}$ | No $m$ are $y$ |  |
| All $x^{\prime}$ are $m$ | No $m^{\prime}$ are $y$ |  |
| All $x$ are $m^{\prime}$ | No $m$ are $y^{\prime}$ |  |
| All $x$ are $m$ | No $m^{\prime}$ are $y^{\prime}$ |  |
| All $x^{\prime}$ are $m^{\prime}$ | No $m$ are $y^{\prime}$ |  |
| All $x^{\prime}$ are $m$ | No $m^{\prime}$ are $y^{\prime}$ |  |
|  |  |  |
| plus 8 more by converting "No" statements |  |  |

ALL \& SOME (32 styles in 2 forms)

Like eliminands
No valid conclusion

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| All $x$ are $m$ | Some $m$ are $y$ |  |
| All $x$ are $m^{\prime}$ | Some $m^{\prime}$ are $y$ |  |
| All $x^{\prime}$ are $m$ | Some $m$ are $y$ |  |
| All $x^{\prime}$ are $m^{\prime}$ | Some $m^{\prime}$ are $y$ |  |
| All $x^{\prime}$ are $m$ | Some $m$ are $y^{\prime}$ |  |
| All $x^{\prime}$ are $m^{\prime}$ | Some $m^{\prime}$ are $y^{\prime}$ |  |
| All $x$ are $m$ | Some $m$ are $y^{\prime}$ |  |
| All $x$ are $m^{\prime}$ | Some $m^{\prime}$ are $y^{\prime}$ |  |

Unlike eliminands:
Valid conclusion: Retinand of All changes sign

| Premise | Premise | Conclusion |
| :---: | :---: | :---: |
| All $x$ are m' | Some m are y | Some x' are y |
| All $x$ are m | Some m' are y | " " " " " " " " |
| All x are m' | Some m are y | Some x are y |
| All $x^{\prime}$ are m | Some m' are y | " " " " " " " |
| All $x$ are m' | Some m are y' | Some $\mathrm{x}^{\prime}$ are $\mathrm{y}^{\prime}$ |
| All $x$ are m | Some m' are $\mathrm{y}^{\prime}$ | " " " " " " " " |
| All x ' are m' | Some m are y' | Some x are y' |
| All $x^{\prime}$ are m | Some m' are $\mathrm{y}^{\prime}$ | " " " " " " " " |

plus 8 more by converting "Some" statements

## ALL \& ALL (16 styles in 2 forms)

Like eliminands
No valid conclusion

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| All $x$ are $m$ | All $y$ are $m$ |  |
| All $x$ are $m^{\prime}$ | All $y$ are $m^{\prime}$ |  |
| All $x^{\prime}$ are $m$ | All y are $m$ |  |
| All $x^{\prime}$ are $m^{\prime}$ | All $y$ are $m^{\prime}$ |  |
| All $x^{\prime}$ are $m$ | All y are $m$ |  |
| All $x^{\prime}$ are $m^{\prime}$ | All y are $m^{\prime}$ |  |
| All $x$ are $m$ | All $y$ are $m$ |  |
| All $x$ are $m^{\prime}$ | All y are $m^{\prime}$ |  |

No conversions permitted

Unlike eliminands:
Valid conclusion: One retinand changes sign
Premise
Premise
Conclusion
All $x$ are $m \quad$ All $y$ are $m^{\prime}$ All $x$ are $y^{\prime}$; All $x$ ' are $y$
All $x$ are $m$ All y are m " " " " " " " " " " " " " " "
All $x^{\prime}$ are $m$ All $y$ are $m^{\prime}$ All $x^{\prime}$ are $y^{\prime}$; All $x$ are $y$
All x' are m' All y are m " " " " " " " " " " " " " " "
All $x$ are $m \quad$ All $y^{\prime}$ are $\mathrm{m}^{\prime}$ All $x$ are $y$; All $x^{\prime}$ are $y^{\prime}$
All $x$ are $m$ All $y$ are $m$ " " " " " " " " " " " " " " "
All $x^{\prime}$ are $m$ All $y^{\prime}$ are $m^{\prime}$ All $x^{\prime}$ are $y$; All $x$ are $y^{\prime}$
All $x^{\prime}$ are $m^{\prime}$ All $y^{\prime}$ are $m$ " " " " " " " " " $"$
All x' are m' All y' are m " " " " " " " " " " " " " " "
No conversions permitted
"ALL" STATEMENTS: Retinand always in predicate (never in subject) of "ALL"
Total of 80 styles in 6 forms: All-None, All-Some and All-All with like and unlike eliminands. Of these 6 forms, all 6 have valid arguments.

ALL \& NONE (32 styles in 2 forms)

| Like eliminands |  |  |
| :---: | :---: | :---: |
| Valid conclusion: "No" retinand changes sign |  |  |
| Premise | Premise | Conclusion |
| All m are x | No m are y | Some x are y' |
| All m' are x | No m' are y | " " " " " " " |
| All m are $\mathrm{x}^{\prime}$ | No m are y | Some $\mathrm{x}^{\prime}$ are $\mathrm{y}^{\prime}$ |
| All m' are $\mathrm{x}^{\prime}$ | No m' are y | " " " " " " " |
| All m are x | No m are $\mathrm{y}^{\prime}$ | Some x are y |
| All m' are x | No m' are $\mathrm{y}^{\prime}$ | " " " " " " |
| All m are $\mathrm{x}^{\prime}$ | No m are $\mathrm{y}^{\prime}$ | Some x ' are y |
| All m' are $\mathrm{x}^{\prime}$ | No m' are $\mathrm{y}^{\prime}$ | " " " " " " " |

plus 8 more by converting "No" statements

Unlike eliminands:
Valid conclusion: "All' retinand changes sign

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| All $m$ are $x$ | No $m^{\prime}$ are $y$ | No $x^{\prime}$ are $y$ |
| All $m^{\prime}$ are $x$ | No $m$ are $y$ | $" " " " "$ |
| All $m$ are $x^{\prime}$ | No $m^{\prime}$ are $y$ | No $x$ are $y$ |
| All $m^{\prime}$ are $x^{\prime}$ | No $m$ are $y$ | $" " " " "$ |
| All $m$ are $x$ | No $m^{\prime}$ are $y^{\prime}$ | No $x^{\prime}$ are $y^{\prime}$ |
| All $m^{\prime}$ are $x$ | No $m$ are $y^{\prime}$ | $" " " " "$ |
| All $m$ are $x^{\prime}$ | No $m^{\prime}$ are $y^{\prime}$ | No $x$ are $y^{\prime}$ |
| All $m^{\prime}$ are $x^{\prime}$ | No $m$ are $y^{\prime}$ | $" " "^{\prime \prime "}$ |

plus 8 more by converting "No" statements

ALL \& SOME (32 styles in 2 forms)

Like eliminands
Valid conclusion: Retinands retain sign

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| All $m$ are $x$ | Some $m$ are $y$ | Some $x$ are $y$ |
| All $m^{\prime}$ are $x$ | Some $m^{\prime}$ are $y$ | $" " " " "$ |
| All $m$ are $x^{\prime}$ | Some $m$ are $y$ | Some $x^{\prime}$ are $y$ |
| All $m^{\prime}$ are $x^{\prime}$ | Some $m^{\prime}$ are $y$ | $" " " " "$ |
| All $m$ are $x$ | Some $m$ are $y^{\prime}$ | Some $x$ are $y^{\prime}$ |
| All $m^{\prime}$ are $x$ | Some $m^{\prime}$ are $y^{\prime}$ | $" " " " "$ |
| All $m$ are $x^{\prime}$ | Some $m$ are $y^{\prime}$ | Some $x^{\prime}$ are $y^{\prime}$ |
| All $m^{\prime}$ are $x^{\prime}$ | Some $m^{\prime}$ are $y^{\prime}$ | $" " " "$ |

plus 8 more by converting "Some" statements

Unlike eliminands:
No valid conclusion:

| Premise | Premise | Conclusion |
| :--- | :--- | :--- |
| All $m$ are $x$ | Some $m^{\prime}$ are $y$ |  |
| All $m^{\prime}$ are $x$ | Some $m$ are $y$ |  |
| All $m$ are $x^{\prime}$ | Some $m^{\prime}$ are $y$ |  |
| All $m^{\prime}$ are $x^{\prime}$ | Some $m$ are $y$ |  |
| All $m$ are $x$ | Some $m^{\prime}$ are $y^{\prime}$ |  |
| All $m^{\prime}$ are $x$ | Some $m$ are $y^{\prime}$ |  |
| All $m$ are $x^{\prime}$ | Some $m^{\prime}$ are $y^{\prime}$ |  |
| All $m^{\prime}$ are $x^{\prime}$ | Some $m$ are $y^{\prime}$ |  |

plus 8 more by converting "Some" statements

ALL \& ALL ( 16 styles in 2 forms)

| Like eliminands |  |  |
| :---: | :---: | :---: |
| Valid conclusion: No change in sign |  |  |
| Premise | Premise | Conclusion |
| All m are x | All m are y | Some x are y |
| All m' are x | All m' are y |  |
| All m are $\mathrm{x}^{\prime}$ | All m are $\mathrm{y}^{\prime}$ | Some x ' are $\mathrm{y}^{\prime}$ |
| All m' are $\mathrm{x}^{\prime}$ | All m' are $\mathrm{y}^{\prime}$ |  |
| All m are $\mathrm{x}^{\prime}$ | All m are y | Some x ' are y |
| All m' are $\mathrm{x}^{\prime}$ | All m' are y | " " " " " " " " |
| All m are x | All $m$ are $\mathrm{y}^{\prime}$ | Some x are $\mathrm{y}^{\prime}$ |
| All m' are x | All m' are $\mathrm{y}^{\prime}$ | " " " " " " " " |

Unlike eliminands:
Valid conclusion: Both change sign
Premise Premise Conclusion
All $m$ are $x \quad$ All $m$ ' are $y \quad$ No $x$ 'are $y^{\prime}$

All m'are $\mathrm{x} \quad$ All mare $\mathrm{y} \quad$ " " " " " "

$\begin{array}{lll}\text { All } m \text { are } \mathrm{x} & \text { All } \mathrm{m} \text { are } \mathrm{y} & " " " " " " \\ \text { All } \mathrm{m} \text { are } \mathrm{x} & \text { All } \mathrm{m}^{\prime} \text { are } \mathrm{y}^{\prime} & \text { No } \mathrm{x} \text { are } \mathrm{y}\end{array}$
$\begin{array}{lll}\text { All } m \text { are } x & \text { All } m \text { are } y^{\prime} & \text { No } x \text { are } y \\ \text { All } m^{\prime} \text { are } x & \text { All } m \text { are } y^{\prime} & " " " " "\end{array}$
All $m$ are $x^{\prime} \quad$ All $m^{\prime}$ are $y^{\prime} \quad$ No $x$ are $y$
All m' are $\mathrm{x}^{\prime} \quad$ All m are $\mathrm{y}^{\prime} \quad$ " " " " " "

TWO "ALL" Statements with Retinands mixed: One in subject of ALL, other in predicate of ALL Total of 32 styles in 2 forms: All-All with like and unlike eliminands. Of the 2 forms, both have valid conclusions.

ALL \& ALL: (32 unique styles in 2 forms)

Like eliminands
Valid conclusion: No change in sign ( $\mathrm{S}=\mathrm{S} ; \mathrm{P}=\mathrm{P}$ )
Premise Premise Conclusion

All $x$ are $m \quad$ All $m$ are $y \quad$ All $x$ are $y$
All x are $\mathrm{m} \quad$ All m'are $\mathrm{y} \quad \mathrm{"} " \mathrm{"}$ " " "
All $x^{\prime}$ are $m \quad$ All $m$ are $y \quad$ All $x^{\prime}$ are $y$
All $x^{\prime}$ are $m^{\prime} \quad$ All m' are $y \quad " " " " "$ "
All $x^{\prime}$ are $m \quad$ All $m$ are $y^{\prime}$ All $x$ are $y^{\prime}$
All x' are m' All m' are y' " " " " " "
All $x$ are $m \quad$ All $m$ are $y^{\prime}$ All $x^{\prime}$ are $y^{\prime}$
All $x$ are $\mathrm{m}^{\prime} \quad$ All m'are $\mathrm{y}^{\prime}$ " " " " " "
plus 8 more by ????

Unlike eliminands:
Valid conclusion: Retinand of All changes sign
Premise Premise Conclusion
All x are $\mathrm{m}^{\prime} \quad$ All m are $\mathrm{y} \quad$ Some x ' are y

All x are $\mathrm{m} \quad$ All m'are $\mathrm{y} \quad$ " " " " " " " "
All $x^{\prime}$ are $m^{\prime} \quad$ All $m$ are $y \quad$ Some $x$ are $y$ All $x$ ' are $m \quad$ All m' are y " " " " " " " All $x$ are $\mathrm{m}^{\prime} \quad$ All $m$ are $y^{\prime} \quad$ Some $x^{\prime}$ are $y^{\prime}$ All x are m All m' are y' $\quad "+"+" " "$ $\begin{array}{lll}\text { All } x^{\prime} \text { are } m^{\prime} & \text { All } m \text { are } y^{\prime} & \text { Some } x \text { are } y^{\prime} \\ \text { All } x^{\prime} \text { are } m & \text { All } m^{\prime} \text { are } y^{\prime} & " " "+" " "^{\prime \prime}\end{array}$
plus 8 more by ?????
==============================================================================10=1

## SCHIELD GENERAL SUMMARY

STYLES OF STATEMENTS:
Each statement can have 3 signs of quantity (All, Some or No)
Each statement can have 4 possible choices in subject position (eliminating distinction $\mathrm{b} / \mathrm{t} \mathrm{x}$ and y )
Each statement can have 2 possible choices in predicate position (eliminating unworkable combos)
There are 24 meaningful variations of each statement (excluding the 24 unworkable ones: no m are m'; no x are y )
[There are 108 variations of each statement:
STYLES OF TWO PREMISES:
Number possible $=576=(3 * 4 * 2)^{\wedge 2}$ where $3=$ quantity, $4=1$ st argument and $2=2$ nd argument


## FORMS OF PREMISES

These 576 styles ( 384 excluding order) can be summarized into 20 unique forms
Of these 20 different forms of arguments, 12 are sometimes valid and 8 are never valid.
Conclusion \#1: $60 \%$ of all the types of arguments are valid ( $40 \%$ are not)
Conclusion \#2: With one exception, all valid arguments have only 1 valid conclusion,
Less than a $10 \%$ ( 1 in 12) chance a valid argument has more than one conclusion.
Conclusion \#3: Only a 5\% chance (one in 20) that an argument selected at random (by form) is valid and has more than one conclusion.

## SUMMARY OF VALID ARGUMENTS

Both premises have converses (page A-1)

1. No and No yield No if unlike
2. Some and No yield Some (reverse No) if Like
3. With retinands in subject of ALL (page A-2)
a. All and No yield All (reverse No) if like
b. All and Some yield Some (Reverse All) if unlike [See \#2]
c. All and All yield Double All (reverse 1) if unlike
4. With retinands in predicate of ALL (page A-3)
a. All and No yield Some(reverse No) if like; yield No (reverse All) if like
b. All and Some yield Some if like
c. All and All yield Some if like; yield No (reverse both signs) if unlike
5. With retinands mixed between two ALLs (page A-4)
if like, yield All ( $\mathrm{S}=\mathrm{S} ; \mathrm{P}=\mathrm{P}$ )
if unlike yield Some (Reverse subject of All)

| 6 forms NO |  SOME <br> All $x$ are $y^{\prime}$ Like: Some $x$ are $y^{\prime}$ <br> $\ldots$. Unlike: No $x^{\prime}$ are $y$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| NO $\begin{array}{ll}\text { Like: } \\ & \text { Unlike }\end{array}$ |  |  |  |  |
| SOME | Like: Some x are y Unlike: . . . |  |  |  |
| At least 1 "All" | Retinand in Subject |  | ---------- | --------------- |
| 14 forms |  |  | Retinand | in Predicate |
| NO | Like All x are $\mathrm{y}^{\prime}$ <br> Unlike: . . . |  | Like: <br> Unlike: | Some x are $\mathrm{y}^{\prime}$ No $x^{\prime}$ are $y$ |
| SOME | Like <br> Unlike | Some x' are y | Like: <br> Unlike: | Some x are y |
| ALL (Ret.match) | Like: <br> Unlike | All $x$ are $y^{\prime}$ All $x^{\prime}$ are $y$ | Like: <br> Unlike: | Some x are y No x' are $y^{\prime}$ |
| ALL (Ret. opposite) | Like All x are y <br> Unlike: Some x' are y |  | Like: <br> Unlike: | All x are y Some x 'are y |

CONCLUSION:
Too difficult to memorize 20 forms with 4 characteristics (qty, like, ret in subj if All) plus outcomes.
Must be able to reconstruct quickly (on a napkin)

SUMMARY OF RULES: \{Draft: In process $\}$

For each premise:
Cannot have two ellimnands or two retinands (exclusive)
For a pair of premises
Must both include eliminands; must each have a retinand; must each have a different retinand

First statement (48)
3 quantity
6 subject
2 predicate
Second statement (24)
3 quantity
4 subject
2 predicate

