Statistical Tools and Statistical Literacy: The Case of The Average

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Statistical tools and statistical literacy: The case of the average

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The simple arithmetic mean ("average") is a core statistical concept that is widely used in a variety of non-school and school contexts, and is arguably the most frequently used statistical term that people are exposed to. Yet, rarely do we see a thorough discussion aimed at *teachers* of statistics regarding what we would like students to know or understand about averages. This in turn causes lack of clarity regarding how to *assess* students' knowledge of averages, and is possibly one of the causes for the frequent use of simplistic, computationally oriented items testing knowledge of averages (e.g., "what is the average of the following set of numbers..."). This paper is intended to serve as a starting point for a discussion regarding the goals of teaching students about averages and how to assess their emerging knowledge.

Averages vs. averaging: concepts as tools

I argue that teachers should aim not only for knowledge of averages, but of *averaging*. An analogy to the art of painting will clarify the distinction between knowing a concept and understanding how to use it. Think, say, of a brush as one of several tools painters have at their disposal. What do painters need to know about their brushes? Beyond a few physical properties (e.g., shape, size), probably not much. What is more important for a painter is to know the *usages* of brushes, and to develop skill in using brushes.

Helpful knowledge may involve, e.g.: What a brush is good for; how to use it to achieve desired effects given different types of surfaces or paints; how to recognize and prevent possible damage to a painting due to incorrect usage of a brush (e.g., brushing surfaces that shouldn't be treated with a certain brush); or knowing when a tool other than a brush is better for a certain task. Skill with "brushing" should thus extend well beyond the ability to indiscriminately spread paint on things (which I compare to students' proficiency in blindly averaging any set of numbers in sight, regardless of their origin, nature, or context).

Likewise, knowing what an average *is* or what its mathematical properties are should not be the primary goal of statistics educators who teach about averages. (After all, the average of a univariate distribution has little meaning in and of itself.) Rather, a computed average gains meaning and turns into a useful piece of knowledge when it is used in a meaningful context, and when it is genuinely needed, e.g. to compare and study possible differences between groups, detect trends, etc.

Like painters who use brushes, students should recognize that 'average' is a *tool* (mental and mathematical) that can be invoked in certain situations. As with other kinds of tools, basic requirements are that a student understands (1) What the average can be used for; (2) In what ways is it different than, or similar to, other tools; (3) Under what conditions it makes sense to use an average, and why; and (4) What might happen if an average is used when in fact it shouldn't be, or vice versa. Further, we want students to know not only about the averaging tool itself, but also about the averaging tool *in the context of other tools in the toolbox*. Students should recognize that the average may not necessarily be the first tool to be used, the only tool needed, or the most appropriate one. Students should also know that there might be different ways to use it (e.g. for exploratory or confirmatory purposes), depending on the task to be accomplished.

An analogy to painting or art would not be complete without considering that, in addition to those painters who create or generate art, there are "consumers" of art, who are in the position of wanting or needing to appreciate and comprehend the work of painters, or who use the work of painters as a basis for their own work (e.g., interior designers). While "art appreciation" by such consumers may not require the same intimate knowledge of brushes and how to use them (as described above), it may nevertheless demand some general knowledge of painting techniques and tools, and how painters use their tools to convey messages and images in an attempt to leave a certain impression on the consumers of their work.

In a similar vein, averages or averaging are often used as part of *communicative* acts, and serve to convey information to *passive* listeners who might form a general impression of a phenomenon (e.g., when a TV newsperson says "researchers found that the average American..."). Averages are also used to convey information to people who need to *interpret* statistical statements in order to form opinions and make decisions (e.g., in workplace contexts), but who do not themselves generate the data or do the research.

It is thus important to develop students' ability to deal with averages in both the passive and active, or interpretive and generative, contexts. Unfortunately, even though the teaching of statistics is often justified by referring to demands on citizens and consumers who live in an "information society," many teachers do little to confront the actual "demands". Rather, they end up using in instruction (and assessments) mostly situations that require students to *compute* rather than interpret averages. This may not be sufficient to develop students' statistical literacy skills and make sure they can come up with reasonable interpretations of statistical statements and arguments encountered in newspaper articles, TV news, advertisements, or on the job.

More about statistical literacy

As future consumers of statistical information, students should be aware of the various *legitimate* interpretations for the word average, and how these differ from or overlap with the statistical meaning(s) for average. As with other mathematical and statistical concepts (e.g., table, product, sample, representative), students may have difficulty distinguishing everyday meanings from mathematical meanings in classroom discourse (see, e.g., Gal et al., 1992; Laborde, 1990).

One useful exercise for developing statistical literacy regarding averages could begin by asking students to find and compare definitions of averages in several dictionaries, including those aimed at school students. (You may be surprised at the range of "official" interpretations for average other than 'what you get from adding-and-dividing'; Depending on the source examined, average may be defined as: typical, usual, normal, regular, common, ordinary, or middle, among others.)

In addition, one could ask students to *write down* and then discuss and compare their interpretations for plain sentences in which the word average is embedded.

Examples for sentences students might encounter in the media as passive listeners are:

1. "The average American adult eats 40 pounds of ice-cream per year".
2. "On average, American adults eat 40 pounds of ice-cream per year".
3. "American adults eat an average of 40.5 pounds of ice-cream per year".

Examples for sentences students might use when conducting their own surveys are:

4. "On average, how much ice-cream do you eat each week?"
5. "What is the average amount of ice-cream you eat each week?"
6. "In an average week, how much ice-cream do you eat?"

A discussion of the meaning or intention behind "average" in the above sentences (including the extent to which the add-and-divide algorithm is implied in each) can be effective in getting students to practice and develop written and oral communication skills. Specifically, it can confront students with the range of interpretations different people have for the same statement, raise to the surface the ambiguity inherent in the usage of the word average (given its various interpretations), sensitize students to misinterpretations of questionnaire items as a possible source of error during data collection (questions 4-6 above), and illustrate possible misinterpretations that may emerge when reporting results (questions 1-3 above).

The discussion can also help students realize that full interpretation of "average" (or other statistical terms) when used in data-collection or data-reporting contexts may require a broader knowledge of the context, purpose and design of a study, and the intentions of those communicating any statistical data. Without such information, it may be difficult to unambiguously determine if average in the above six sentences indicates:

(a) an exact figure - whatever you get from the add-and-divide algorithm;

(b) a modal value - e.g., typical/ usual/ middle;

(c) a ballpark zone - a rough estimate for where the center or "bulk" of the data may be, and a permission to not be precise (this is what "an average" usually suggests for many people).

Statements about averages such as 1-3 above can also serve as a basis for discussing with students the nature of the data from which the average was generated. As pointed out by Mokros & Russell (1992), people should be able to hypothesize about the range, dispersion, or shape of the underlying data from which a reported average was derived, including the recognition that there may be no real 'center' to the data, or that a reported average provides a distorted estimate for the center of a distribution because of outliers or skewness.

One particular context in which students' hypotheses about the data underlying an average may prove critical is when two or more averages are being compared to each other. In this case, students have to realize that the significance, meaning and interpretation of the difference between reported averages depends on *assumptions* they can make about the data distribution in each sample. Further, students should also be made sensitive to the fragility or possible error inherent in comparing sample-based averages (because of the size, variability, attributes, or representativeness of the samples), and its implications for the degree of confidence they should attach to any conclusions they draw from averages.

Assessing knowledge of averages in context

It is difficult to fully judge what a learner knows about a statistical tool without providing a *context* for using the tool (i.e., a problem). It's here that most math textbooks and math teachers fail; they routinely provide *no* context or only a minimal one for averaging questions, or present *real data*, but still without any *real problem* behind it that can motivate learners to evaluate the applicability and usefulness of the average in this context.

Asking students to compute or estimate averages from data presented in several different modes (e.g., a series of numbers, from data in a table, scatterplot or histogram, from aggregated or grouped figures, or by weighting other averages), can often reveal certain strengths and flaws in students' knowledge. Yet, assessments should not focus too much attention on the ability to *find* the average per se.

To learn about students' understanding of the average, teachers of statistics should create realistic contexts which present to students a genuine need to *use* an average as opposed to other statistical tools or techniques that may also be appropriate. This is similar to the case of painters, where brushing may be only one of several techniques for covering an area with paint. Unless the conditions favor brushing, there is no guarantee that the painter will not opt for other techniques.

(In statistics, students may prefer to use, e.g., a median.) As suggested earlier, an integral part of the knowledge of "average" (or of other measures of center) involves recognition of the scope and conditions for its application, and an understanding of the tool's role in the toolbox of the statistician. If students are forced to use an average, as many teachers do, there is no assurance that students will know how and when to use it when they are not being specifically told to do so.

Knowledge of averaging should also be reflected in students' ability to provide reasonable interpretations of statements or arguments that involve averages, and their ability to critique how others use and report averages in the larger context of reporting results from a statistical investigation. However, our research with students from "regular" or below-average schools (Gal, 1992) shows that many of them have great difficulties writing coherently about statistical issues, usually due to underdeveloped vocabulary and other linguistic skills, and possibly due to little prior opportunity to practice writing complete arguments about quantitative issues.

Thus, to learn what students know about averages requires that students are first coached in written and oral communication around statistical issues, and that their communications are assessed qualitatively, with special attention to whether their comprehension of the problem and its larger context is the same as that of the teacher.

In all, while the average may appear to be a relatively simple concept from a mathematical point of view, its varied uses for statistical and communicative purposes require that we carefully examine how and in what contexts we teach students to use and interpret averages, how assessment contexts are set up, and to what extent students' knowledge of other statistical issues, such as sampling or experimental design, as well as their personal world knowledge, is allowed to inform their reasoning about averages in the chosen assessment problems.

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