Statistical Literacy, Quantitative Reasoning and Quantitative Literacy.

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BACKGROUND:
At the K-12 level, the NCTM has issued curriculum standards to develop mathematically literate citizens. To help mathematics teachers in the middle and high school grade levels, the ASA has initiated annual Quantitative Literacy workshops. At the college and university level, the CUPM has made four recommendations concerning quantitative literacy. See Appendix I.

At the college level, the focus is sometimes referenced as quantitative reasoning. See Appendix III.

ESSAY:
There is a fundamental ambiguity in the basic terms used in specifying these goals. The terms 'quantitative literacy' and 'quantitative reasoning' are not defined in terms of essentials. They are indicated in terms of operational aspects such as "appreciation" or "awareness". Even when they specify "skills" or "concepts", they are circular; they refer back to "quantitative reasoning" and "numerical arguments".

The ambiguity is most in evidenced in the terms "problem solving" and "quantitative reasoning". Problems can be either ways of practicing a given method (the particular problems are incidental) or problem solving can be the central activity (the methods used are relatively incidental). Reasoning can be either creative (formulate a relationship or hypothesis) or critical (validate or support that thesis). Generally, the former is termed inductive and the latter deductive. But this effectively overlooks the support for an inductive argument and makes deductive argumentation the ideal – which it is in all of mathematics except for some parts of statistics.

What students desperately need in college are tools and techniques for identifying disputable claims, for identifying arguments supporting these claims and for assessing the strengths and weaknesses of these arguments. Students need the most help in dealing with those claims and arguments involving numerical summaries of data. Students are so accustomed to treating numbers as facts that they have trouble envisioning numerical claims as being disputable. And since mathematicians typically focus on deductive arguments, they are seldom disposed to help students reason with inductive arguments (aside from those involving mathematical induction).

The one part of mathematics that has always focused on inductive argumentation is statistics. Here we find the movement from particular (sample) to universal (population). Here we find terms reflecting uncertainty: probability, confidence and error. If students are to learn how to reason inductively with numerical summaries of data, they must study statistics. But they have no general need to understand the aspects dealing with chance. That is a more specialized need. Their general need is on when to look for a confounding factor or bias in making a prediction or in explaining an observed relationship. They need practice in seeing that a mathematical relationship is ambiguous. For example, suppose home price = $50K plus $40k per bathroom. Most students conclude that if your house was located in this area and had one bathroom valued at $90k, one could expect – on average -- to make $25k by installing a bathroom costing $15k.

RECOMMENDATION:
We must distinguish statistical literacy from quantitative literacy. We must break reasoning into two dimensions: object and method: Under object, we must separate arguments about the natures and causes of things from arguments about models and descriptions of such things. Under method, we must recognize the presence of two different kinds of arguments (deductive and inductive). Specifically, we our goal must be to focus on claims about reality as the object and the inductive as being the appropriate method. Our methods of achieving these goals will be to use models and numerical summaries of relationships and to use deduction to build valid arguments, but these are means and not ends.

PROBLEM:
It will be extremely difficult to educate traditional teachers of mathematics who are schooled in abstracting variables from properties, in abstracting relationships from entities and in abstracting form from substance. It will be extremely difficult to educate teachers of mathematics who are well-schooled in deductive arguments while avoiding inductive reasoning.

SOLUTION:
A good place to begin is with those who are teaching statistics. Here the focus is at least a mixture between form and substance, model and reality, deduction and induction. Statisticians must clearly identify the difference between statistics and probability, between hypothetical deductive reasoning and inductive reasoning. Statisticians must focus on helping students use statistics to make arguments about the natures and causes of things. They cannot simply say "correlation is not causation". *Students are required by faculty in other departments to study statistics as a tool -- to learn about the natures and causes of things.* And when students are not offered anything positive in this regard, they conclude that statistics might be interesting in some cases, but it is not a fundamental value in their lives as thinkers or decision-makers.

But having the right teachers is not sufficient. They must have materials that focus on the task of helping students reason critically about predictions and explanations. College students should be able to read and evaluate a complex argument involving statistical relationships such as that presented in the Bell Curve. They don't need the details of logistic regression; they do need the idea of controlling for confounding factors. They don't need the idea of confidence interval (in a sample of 12,000 people) for the mean; they do need the idea of practical significance and a prediction interval.

ACTION:
We must create a forum for discussing these matters and enlarging the circle of those who share this focus. We must identify those who are willing to learn more about applying critical thinking to quantitative claims. We need those who are willing to test out new materials and to make constructive suggestions. We need those who are willing to argue that our current Gen-Ed requirements are missing the mark by their ambiguity.

ADDITIONAL RESOURCES:

- APPENDIX I: NCTM Committee on Quantitative Literacy
- APPENDIX II NCTM Goals, Curriculum Standards, Evaluation Standards and specific standards for quantitative reasoning and for statistics.
- APPENDIX III: College-level Gen-Ed requirements and Q/R courses:

APPENDIX I: NCTM Committee on Quantitative Literacy

At the primary and secondary school levels, the National Council of Teachers of Mathematics (NCTM at www://sdannual.nctm.org/index.htm/) issued its celebrated Curriculum and Evaluation Standards for School Mathematics in 1989. "The Standards represent a major effort to develop mathematically literate citizens." The five overall goals and the curriculum standards at the three levels (K-4, 5-8 and 9-12) are listed in Appendix II along with the standards for evaluation. In addition, the detailed standards for reasoning and for statistics are also shown.

To assist mathematics teachers in the middle and high school grade levels, the American Statistical Association (www.amstat.org/education) has initiated annual Quantitative Literacy workshops. Here are found links to QL projects (ql-projects.html), teacher qualifications (ql-teachers.html) and on fellowships (seaql-fellowships.html) as well as on ASA publications (ql-publications.html) and Dale Seymour publications (dale-seymour.html).

At the college and university levels, the Committee on the Undergraduate Program in Mathematics (C.U.P.M.) has had a committee on Quantitative Literacy since 1985 (See /past/ql/committee.html under www.maa.org). In 1989, this subcommittee made four recommendations to remedy the sorry state of general mathematical knowledge among the American people. (See /past/ql/ql_toc.html under www.maa.org). Their four recommendations were stated as conclusions. These four conclusions were:

1. Colleges and universities should treat quantitative literacy as a thoroughly legitimate and even necessary goal for baccalaureate graduates.
2. College and universities should expect every college graduate to be able to apply simple mathematical methods to the solution of real-world problems.
3. Colleges and universities should devise and establish quantitative literacy programs each consisting of foundation experience and a continuation experience, and mathematics departments should provide leadership in the development of such programs.
4. Colleges and universities should accept responsibility for overseeing their quantitative literacy programs through regular assessments.

In addition, there is a complement to the standards (past/ql/ql_summary.html). Unfortunately, the links to the four parts and the four appendices are not currently operational (6/12/97 - 9/27/97).

For others sources, consult North Central Regional Educational Laboratory (www.ncrel.org) under Math (sdrs/areas/ma0cont.htm). There is a nice discussion of a "Critical Issue: Implementing Curriculum, Instruction and Assessment Standards in Mathematics." They carry a review of the Quantitative Literacy Series (sdrs/areas/issues/content/cntareas/math/ma6quant.htm). "The Quantitative Literacy Series is a four-part series of books written through a collaborative effort of teachers and statisticians and is designed for use with middle and high school students. The books are a part of the American Statistical Association Project-Quantitative Literacy, which was funded in part by the National Science Foundation."

APPENDIX II NCTM Goals, Curriculum Standards, Evaluation Standards and specific standards for quantitative reasoning and for statistics.

Five overall goals:

1. *Learn to value mathematics.* Students should have numerous, varied learning experiences that illuminate the cultural, historical, and scientific evolution of mathematics. These experiences should be designed to evoke students' appreciation of mathematics' role in the development of contemporary society and to promote their understanding of relationships among the fields of mathematics and the disciplines it serves: the humanities and the physical, social, and life sciences.

2. *Learn to reason mathematically.* Skills in making conjectures, gathering evidence, and building an argument to support a theory are fundamental to doing mathematics. Therefore, sound reasoning should be valued as much as students' ability to find correct answers.

3. *Learn to communicate mathematically.* To express and expand their understanding of mathematical ideas, students need to learn the symbols and terms of mathematics. This goal is best accomplished in the context of problem solving that involves students in reading, writing, and talking in the language of mathematics. As students strive to communicate their ideas, they will learn to clarify, refine and consolidate their thinking.

4. *Become confident of their mathematical abilities.* Study that relates to everyday life and builds students' sense of self-reliance will lead them to trust their thinking skills and apply their growing mathematical power. School mathematics should prompt students to realize that doing mathematics should prompt students to realize that doing mathematics is a common, familiar human activity.

5. *Become mathematical problem solvers.* Problem solving is the process through which students discover and apply the power and utility of mathematics. Skill in problem solving is essential to productive citizenship.

Curriculum Standards:

- At the K-4 levels, this involved 13 standards: mathematics as problem solving, mathematics as communications, mathematics as reasoning, mathematics as connections, estimation, number sense and numeration, concepts of whole number operations, whole number computation, geometry and spatial sense, measurement, statistics and probability, fractions and decimals, patterns and relationships.
- At the 4-8 level, this involved 13 standards: mathematics as problem solving, mathematics as communications, mathematics as reasoning, mathematics as connections, number and number relationships, number systems and number theory, computation and estimation, patterns and functions, algebra, statistics, probability, geometry, and measurements.
- At the 9-12 levels, this involved 14 standards: mathematics as problem solving, mathematics as communications, mathematics as reasoning, mathematics as connections, algebra, functions, geometry from a synthetic perspective, geometry from an algebraic perspective, trigonometry, statistics, probability, discrete mathematics, conceptual understanding of calculus, and mathematical structure.

Evaluation Standards:
In addition, it listed 14 evaluation standards: alignment, multiple sources of information, appropriate assessment methods and uses, mathematical power, problem solving, communication, reasoning, mathematical concepts, mathematical procedures, mathematical disposition, indicators for program evaluation, curriculum and instructional resources, instruction, and evaluation team.

The following illustrates the standards for mathematical reasoning:

> "In grades K--4, the study of mathematics should emphasize reasoning so that students can draw logical conclusions about mathematics, use models, known facts, properties, and relationships to explain their thinking, justify their answers and solution processes, use patterns and relationships to analyze mathematical situations, and believe that mathematics makes sense."

> "In grades 5-8, reasoning shall permeate the mathematics curriculum so that students can recognize and apply deductive and inductive reasoning, understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs, make and evaluate mathematical conjectures and arguments; validate their own thinking, appreciate the pervasive use and power of reasoning as a part of mathematics."
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> "In grades 9-12, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can make and test conjectures, formulate counterexamples, follow logical arguments, judge the validity of arguments, and construct simple valid arguments." In addition, college-intending students can construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

The following illustrates the standards for statistics:

> "In grades K-4, the mathematics curriculum should include experiences with data analysis and probability so that students can collect, organize, and describe data, construct, read, and interpret displays of data, formulate and solve problems that involve collecting and analyzing data, and explore concepts of chance."
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> "In grades 5-8, the mathematics curriculum should include exploration of statistics in real-world situations so that students can systematically collect, organize, and describe data, construct, read, and interpret tables, charts, and graphs, make inferences and convincing arguments that are based on data analysis, evaluate arguments that are based on data analysis, and develop an appreciation for statistical methods as powerful means for decision making."
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> "In grades 9-12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations, use curve fitting to predict from data, understand and apply measures of central tendency, variability, and correlation, understand sampling and recognize its role in statistical claims, design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes, analyze the effects of data transformations on measures of central tendency and variability, and so that, in addition, college-intending students can transform data to aid in data interpretation and prediction, and test hypotheses using appropriate statistics."

APPENDIX III: College-level Gen-ed requirements and Q/R courses:

Individual articles include:

- Wolfe, C. R. (1993). Quantitative reasoning across a college curriculum. College Teaching, 41: 3-9. (See ~crwolfe/quant_abst.html under www.muohio.edu). "Quantitative Reasoning Across a College Curriculum:A plan for integrating quantitative reasoning throughout an undergraduate core curriculum including the natural sciences, social sciences, and humanities is presented. Four interrelated aspects of quantitative reasoning are discussed: learning from data, quantitative expression, evidence & assertions, and quantitative intuition. Learning from data refers to the skills associated with collecting and analyzing data. Quantitative expression is the ability to use and comprehend quantitative language in a variety of contexts. Facility with evidence and assertions allows one to comprehend which conclusions may be reasonably drawn from a body of evidence. Quantitative intuition refers to heuristics that lead to a "feel" for numbers and other quantitative concepts. Several quantitative exercises are described. The general pedagogical orientation of the plan is student-centered and experiential."

Individual courses include:

- University of San Francisco Math 101: "This course will introduce students to the processes by which valid statistical inferences may be drawn from quantitative data. The emphasis throughout will be on statistical ideas and concepts, rather than on numerical or algebraic manipulation. Topics to be covered will include design of experiments, sample surveys, measurement, summary and presentation of data, regression and correlation, elementary probability, the law of averages, the normal distribution, confidence intervals, hypothesis testing. A computer laboratory component will introduce the student to spreadsheets and statistical applications with an emphasis on graphical presentation of data. Offered every semester." See /math/Curriculum/Courses/Math101/index.html under www.usfca.edu.
- Dickinson University (See /~Rossman/m120/120syl.html under www.dickinson.edu). "This course aims to help you to develop both the operational skills and the conceptual understanding necessary to interpret and assess quantitative arguments confidently and competently. Presented in the context of practical and important applications, course material will emphasize critical thinking with regard to quantitative information rather than numerical computations or symbolic manipulations. The pedagogical style of the course will involve you directly in your own learning by asking you to spend class time working collaboratively on activities designed to help you discover fundamental quantitative principles for yourself. My goals for this course are to help you to develop:
  - an appreciation for the applicability and relevance of quantitative reasoning to your everyday life and to your particular field of study or interest,
  - a degree of critical awareness with which to assess the merits of numerical arguments that you encounter in the popular media,
  - abilities to apply a variety of skills related to quantitative reasoning, an understanding of some of the fundamental concepts involved in quantitative reasoning,
  - improved ability for writing about numerical arguments, and
  - increased familiarity and confidence with using the computer."