Achieving Mathematical Proficiency for All
Moving Beyond the Math Wars

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In 1957 Sputnik startled a complacent America with evidence that Russia's engineering and education appeared to be ahead of our own. In a typical American reflex, we immediately decided to catch up, to make our accomplishments match our ideals. In engineering we succeeded spectacularly, but in education we have clearly failed. Where is the educational equivalent of a moon landing or space station, of the personal computer or decoded genome? First we tried curriculum reform (remember the New Math, followed by Back to Basics?) and summer institutes for teachers. In the 1990s we launched national standards, increased graduation requirements, and created new textbook series. Now we are trying high-stakes high school exit exams, and in the near future, annual testing of all elementary school students.

What has all this accomplished? U.S. students still rank well below average on international tests of mathematics, just as they did forty years ago in the first such multi-nation comparison. Three out of every four mathematics enrollments in higher education are in courses that are normally taught in high school, and 40 percent of these are clearly remedial. Fewer than one in ten mathematics enrollments in higher education are in courses above the level of AP calculus. Despite increased rhetoric about high school preparing all students for college, the level of mathematics required to pass the various state high school exit exams leaves weaker students as much as two years behind university mathematics expectations for credit-bearing courses such as college algebra or calculus. In addition, most adults appear functionally innumerate when dealing with quantitative matters such as estimating risks, managing personal finances, and comprehending daily news.

Twenty years ago, *A Nation at Risk* famously observed that if an unfriendly foreign power had attempted to impose on America the mediocre educational system we created for ourselves, we might view it as an "act of war." In a reversal of this dramatic image, we have instead turned to foreign countries -- friendly and unfriendly -- to remedy our own educational deficiencies. For lack of well-qualified U.S. citizens, graduate programs in mathematically-intensive fields rely on foreign students to fill their classes and laboratories while U.S. companies fill hundreds of thousands of technical positions with foreign nationals. Only by importing human talent from around the world has the U.S. been able to thrive. Two generations after Sputnik, we are still not educationally self-sufficient.

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The continuing poor mathematical performance of our high school graduates across four decades and countless educational reforms suggests that the problem of mathematics education is far more intractable than the challenges of landing on the moon or decoding the human genome. Alternatively, it may be that we have focused our efforts in the wrong direction. Some things certainly have improved. The number of students entering college with advanced placement in mathematics has soared during the last forty years, from a few hundred to a hundred thousand. Many U.S. students and school districts have clearly demonstrated that they are among the best in the world in international comparisons. Yet more than a million college students enroll in courses that repeat elementary parts of high school mathematics, and another half-million take mathematics courses that every college-bound student should have taken in high school.

In short, while high schools are teaching more college-level mathematics, colleges are teaching enormous amounts of high school mathematics. Instead of focusing on their core responsibility—to help all students meet high standards—schools accelerate some students into college-level work and neglect others, who emerge functionally innumerate and unprepared for college-level mathematics. With neglected students outnumbering accelerated students by a factor of 10:1, long-term poor average performance is not surprising.

This pattern of acceleration mixed with neglect continues in college, where departments of mathematics work most energetically on courses intended for well prepared students who pursue mathematically intensive majors. Few departments apply as much effort to improve the quantitative literacy of all students as they do to enrich the mathematical education of their majors. Strangely, selective focus on the best and brightest seems entirely natural in education—especially in mathematics—while it would be intolerable in other professions. Can one imagine a medical system that focused most of its effort on the healthiest patients?

At the time of Sputnik it was still fashionable to think of the ability to do mathematics as a special gift, perhaps the result of a special gene. Parents, children, and teachers all assumed that some students could do higher mathematics and others could not. For these others, higher arithmetic was about all that it was reasonable to expect. In those days it was the job of teachers and tests to separate children who could "do math" from those who couldn't. Educational data expert Anthony Carnavale observed at a recent National Academy of Sciences forum[1] that mathematics has always been about separation—of rich from poor, of boys from girls, of elites from plebeians. Historically, he observed, failure in mathematics has been the biggest barrier to upward mobility in education.

The standards movement in mathematics sought to change all that. Its theme, echoing similar campaigns in countries around the world, is "mathematics for all"; its goal is to make mathematics "a pump, not a filter." It is no accident that mathematics is the discipline that launched the standards movement in the United States. Mathematics is central to the global movement to democratize education: it undergirds the increased need for postsecondary education, technical demands of the high performance workplace, and conceptual skills required to live and work in the information age. Mathematics as separator is no longer, if it ever was, a suitable goal for education.

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The breakdown of the old consensus of mathematics as a filter whose job was first to separate and then to educate has spawned confusion, anger, and "math wars" in schools, districts, and universities. Difficult questions about the new order abound:

- Will new textbooks designed for a high-tech society leave students unprepared for the intensity of university calculus?

- Must students master basic skills before applying them, or do they best learn skills through application?
- How can curricula designed primarily to motivate students with little interest in mathematics challenge advanced students and ignite their intellectual curiosity?
- What evidence shows that group learning is more effective than direct instruction?
- How dare high school teachers change the pre-college mathematics curriculum without first consulting university mathematicians?

Debates on these issues spill into school board meetings like sibling rivalries. Almost without exception, the fundamental division is over strategies for achieving "mathematics for all." Reformers, seeing widespread innumeracy and apathy towards mathematics among average school graduates, advocate new pedagogies that are context-rich and inquiry-based; alarmed by deficiency in college students' basic skills, traditionalists—often university mathematicians—advocate established, formal, structured programs, especially for college-bound students.

These debates take on many flavors, but primarily they are about the relative merits of two different approaches to teaching mathematics: directed instruction covering a traditional sequence of topics—the kind of instruction that virtually everyone over the age of 40 experienced in school—and a newer, more flexible, activity- and discussion-centered pedagogy using inquiry-based methods to help students explore for themselves a somewhat broader spectrum of mathematical topics. To the extent that scores on high-stakes college entrance tests (e.g., SAT, AP) depend on the kind of mathematics that has traditionally been in the pre-science and engineering curriculum, parents and minority advocates join the debate, fearing that separate will again be unequal. What was good enough for the elite in the past, they say, is what should be available to everyone today.

Unfortunately, many who become invested in this debate engage in a form of educational profiling that devalues the professionalism of those with whom they disagree. Backed by research showing that most students learn mathematics best when they encounter it in familiar contexts, advocates of the new programs argue that traditional curricula ill serve the majority of students, and are largely responsible for the well-documented quantitative weakness of high school school graduates. At the same time, many mathematicians draw on their experience teaching undergraduate mathematics to argue that students who have not had the benefit of directed instruction with traditional topics will be greatly handicapped when they take mathematically intensive courses in college. From these positions some who advocate reform infer that others are only interested in "the few," while some who prefer traditional methods infer that reformers ignore the needs of future scientists and mathematicians.

In fact, all parties to these debates agree that all students should be given the challenge and opportunity to learn mathematics that is rich, important, and powerful. There is relatively little disagreement about what that mathematics should be, although more debate about substance would be valuable. The constraint of meeting current college expectations greatly limits the degree of flexibility available to any school curriculum. Disagreements center mostly on pacing of curriculum and approaches to teaching, and even more on whether pedagogy should be the subject of standards in the same way that mathematics is. Neither content nor pedagogy was much in dispute when, not long ago, high school mathematics was being taught only to one in four students; namely, those who thrived under traditional methods of instruction. Now that "mathematics for all" is a national goal, the mission of school mathematics has changed. With a new mission, we need new methods.

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The current political silver bullet for addressing the problem of chronic educational underperformance is high standards enforced by high-stakes tests. The futility of this strategy, however, is already becoming apparent in those states that are leading this wave of reform. High standards in mathematics, adopted in many states, call for three years of substantial mathematics in high school. Some states reflect college entrance guidelines by requiring two years of algebra and one of geometry. Others permit more useful courses such as statistics, modeling, probability, or discrete mathematics as substitutes for the second course in algebra. But when states create tests that are required for high school graduation, they examine only the most elementary third of their own standards. Even with this low target they discover that many students must take the exam several times to pass and thus to graduate. A similar regimen will now come into play in grades 3-8 as districts begin to comply with the new federal requirement for annual testing.

The backwash of high-stakes testing in the classroom is immense, foreseeable, and almost entirely bad. As advanced content specified in the standards for grades 10-12 is squeezed out of state exit tests-- to ensure politically acceptable passing rates -- so this content will diminish in the overall curriculum. As schools and teachers are judged by the passing rates of students, instruction will focus more on passing tests than understanding mathematics. Students who understand mathematics can easily pass these new tests. But from the fact that students can pass the tests, it does not follow that they understand the mathematics. The economic success of test-preparation schools proves that instruction directed at passing tests can accomplish that goal. Such instruction does not, however, achieve the kind of deep understanding that is required if students are to be able to use mathematics in higher education or in the high-performance workplace. High-stakes standardized testing will neither reveal nor increase students' understanding of mathematics.

Mathematical understanding, it turns out, is a theme that can engage all parties to the math wars in an enterprise worthy of their passion for mathematics. This won't be easy, but it is possible. A common understanding of what it means to understand mathematics has been one of the casualties of the shift in goals for school mathematics from "for few" to "for all." However, mathematicians and educators of every persuasion have recently united on two statements that bridge this schism: the meaning of mathematical proficiency[2] and the understanding that is required to teach it.[3] At every grade level, proficiency and understanding are far more complex than many—even mathematicians—have realized.

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Several strategic directions follow from the goal of "mathematics for all," many of which are already underway. Foremost is a need to help students experience the richness of mathematics as a tool for their lives. Although it is possible to race through the traditional school curriculum from arithmetic to calculus—finish algebra by grade eight, complete calculus by grade twelve—

students who do so wind up mathematically impoverished.[4] Calculus can easily (and for most, beneficially) wait until college. In high school students should take time to explore not only algebra and geometry, but also such rich and useful subjects as data analysis and discrete mathematics, probability and modeling, statistics and computing. The numerous and often unexpected connections among these subjects will promote deep understanding of mathematics, even as they equip students with the quantitative skills they will need as employees, consumers, parents, and citizens.[5]

Much of this richness occurs when patterns amenable to quantitative or logical analysis arise in other subjects—in history or agriculture, in carpentry or economics. For students to develop mathematical habits of mind, they need to see and do mathematics everywhere, not just in math class. As writing is now accepted as part of the entire curriculum, so should mathematics. Every teacher teaches something about mathematics—by using it or abusing it, by teaching it or ignoring it. Mathematicians and mathematics teachers can help all students understand mathematics by working with teachers in other subjects to encourage widespread reinforcement of quantitative and logical analyses across the curriculum.

If we really mean for all students to learn mathematics, we need to greatly increase the number of teachers who understand mathematics deeply and are able to effectively guide student learning. Low salaries, competition from industry, and poor working conditions make recruiting mathematics teachers an unusually difficult challenge. We make it even harder by continuing to insist on rules for licensure that bear uncertain relation to a candidate's understanding of mathematics or ability to teach. Fortunately, responding to a clear national need, a broad spectrum of the mathematics community has recently agreed on important principles for preparing teachers of mathematics.[6] States need to encourage colleges to develop new programs that implement the spirit of these new guidelines--especially their challenge to departments of mathematics to develop in prospective teachers a robust and flexible understanding of mathematics.

The increased focus on preparing teachers of mathematics has awakened many college and university mathematicians to their responsibilities for educating future teachers. It has become clear that two assumptions underlying current practice in teacher preparation are not at all well founded. First is the belief entrenched in state certification requirements that elementary school teachers need only a cursory knowledge of mathematics, often compressed into six credit hours or less. Second is the belief that secondary school teachers will be mathematically well-prepared if they complete a major in mathematics. Both beliefs ignore the crucial importance of pedagogical content knowledge, the translation of mathematical ideas into pedagogical strategies (problems, lessons, tasks) that will help students gain understanding of mathematics.

What teachers need most is to understand elementary mathematics from an advanced perspective, not (as is more customary) elementary concepts of advanced mathematics. The logical and psychological chasm separating advanced mathematics from its foundations in school are immense and cannot be easily bridged. Why is the product of two negative numbers positive? What is the meaning of a fraction whose numerator or denominator are not whole numbers? What is an imaginary number? The deep answers to these kinds of questions that students learn in college mathematics are not appropriate for younger students, so different answers are needed. Even prospective teachers who understand higher mathematics well often have little idea how to translate that understanding into a form suitable for younger students. These are issues that university mathematicians and mathematics educators must jointly address.

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These strategies to help all students learn mathematics—broadening mathematical preparation of students and improving mathematical understanding of teachers—are but two aspects of a more general strategy that is crucial to mathematics education: to view mathematics in a K-16 or even K-20 context. Mathematics and English are the only subjects required every year in school and tested at every transition—in middle grades, to leave high school, upon entering college, and sometimes to gain junior standing. Continuity of expectations and benchmarks is vitally important to ensure student progress. Continuity also guarantees that mathematics really serves all students, not just those who can anticipate and navigate the discontinuities. Continuity and coordination require cooperation, whereas traditions in both school and college encourage fierce independence.

For example, reflecting the increased quantitative demands of life and work, most new school curricula and many state high school exit exams place significant emphasis on data analysis, probability, statistics, computing, and modeling. But university admissions offices continue to stress primarily algebra and geometry, as do mathematics placement exams. Ironically, many departments within the university would welcome students with broader mathematical backgrounds. A similar inconsistency appears in the way schools and colleges approach the role of calculators: most high school courses and major national exams such as SAT and AP encourage mathematics students to use calculators to maximum advantage, yet universities often prohibit calculators on their placement and course exams. Finally, even as most new state standards for K-12 mathematics stress the importance of broadly-based mathematical proficiency, college mathematics requirements remain narrowly focused on intermediate and college algebra, as if nothing in the world of mathematics education has changed in decades.

These are but a few of many examples of thoughtless incoherence between school and college mathematics. In earlier years when the job of school mathematics was to filter students and of colleges to educate those who survived, the need for articulation was less urgent. But now, when most students aspire to college and when most of college mathematics is a rerun of high school mathematics, the case for better articulation of expectations and benchmarks in a K-16 setting is far more compelling.

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The challenge of achieving mathematical proficiency for all students is monumental, but if we fail to accept this challenge our record of underperformance will continue unabated. Some things have changed that provide legitimate cause for optimism.

- Mathematics in practice is broader and richer than ever, penetrating almost every aspect of life and work. So students have greater opportunities to see how mathematics is used and to recognize its value.

- Because the heated rhetoric of the "math wars" was widely seen as unproductive, mathematicians and educators are beginning to work together to build on common ground and clarify remaining areas of difference.
- Finally, many educators and more than a few mathematicians are beginning to see the value in looking at mathematics education as a single enterprise from kindergarten to college (or graduate school).

The challenge of mathematical proficiency is not easy, and not soon achievable. But we know what it will take to make progress: curricula with rich mathematics, teachers with deep mathematical understanding, and programs with intelligent articulation. It's time to get started.

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[1] Presentation at the forum "Quantitative Literacy: Why Numeracy Matters for Schools and Colleges" held at the National Academy of Sciences in Washingtron, DC, Dec. 1-2, 2001.

[2] Jeremy Kilpatrick, *et al*, editors. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press, 2001.

[3] James Lewis, *et al.*, editors. *The Mathematical Education of Teachers*. Washington, DC: Conference Board of the Mathematical Sciences, 2001.

[4] Alan Schoenfeld, "Reflections on an Impoverished Education" in *Mathematics and Democracy: The Case for Quantitative Literacy*. Lynn A. Steen, editor. Princeton, N.J., National Council on Education and the Disciplines, 2001.

[5] Lynn Arthur Steen, editor. *Mathematics and Democracy: The Case for Quantitative Literacy*.. Princeton, N.J., National Council on Education and the Disciplines, 2001.

[6] See Lewis above.