On Being a Mathematical Citizen: The Natural NExT Step

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I am truly honored to join the distinguished list of speakers in this lecture series dedicated to the memory of my good friend Jim Leitzel. Most of you probably knew Jim through his leadership of Project NExT. Before that, Jim chaired many MAA committees dealing with mathematics education and guided development of *A Call for Change*, MAA's pioneering recommendations for preparing teachers of mathematics. A builder of mathematical communities, Jim was a model mathematical citizen and my inspiration for this talk.

China, 1983

In June 1983 my wife and I spent three weeks with Jim and his wife Joan visiting universities and secondary schools in eastern China. We were part of a delegation of mathematics educators opening connections five years after the end of the Cultural Revolution when China was just beginning to awaken from its long national nightmare.

Even back then Jim worried about preparing teachers. In one conversation after another, he explained to our Chinese hosts details of a program he had helped develop at Ohio State for teachers of mathematics who wanted to learn more about the applications of mathematics to areas such as business, economics, and science.

At that time the concept of "applications" of the mathematics taught in school or college caused great confusion among our Chinese translators. For Chinese mathematicians, applied mathematics was a strictly postgraduate research endeavor.

What U. S. educators think of as "applications" of secondary or undergraduate mathematics the Chinese called "practical" or "popular" mathematics. They associated these problems—what someone described as "potted applications"—with the excesses of the Cultural Revolution when scholars such as Hua Luo-Geng were assigned to teach factory and farm workers how to solve practical problems.

Jim spent many patient hours trying to bridge this gulf between our cultures: he listened, learned, engaged, and encouraged. In thinking about my topic of being a mathematical citizen, I am reminded of how Jim's efforts to bridge cultures serves as an example for us all.

America, 2007

Today we are awash in anxiety about mathematic education. Many of the issues seem little changed from those that motivated the Ohio State program that Jim Leitzel was explaining 24 years ago to our Chinese colleagues.

In addition to worrying about mathematics education, Americans are increasingly concerned about the overall quality of our educational systems at all levels. Here's a sample of recent alarms, ranging from concerns about mathematics in particular, to all STEM disciplines (science, technology, engineering, mathematics), to the full scope of education:[1]

- Tom Friedman, in *The World is Flat*, argues that our national welfare is threatened by a "numbers gap" in mathematically trained workers.[2]
- In *Rising Above the Gathering Storm*, the National Research Council urges increased effort to recruit and enhance the capabilities of our nation's mathematics and science teachers.[3]
- Dennis Bartels, executive director of San Francisco's famed Exploratorium, argues in a recent commentary that our most important priority should be what he calls the "democratization of scientific knowledge," beginning with the education of teachers.[4]
- "Recruitment, retention, renewal" are the three "imperatives" required to elevate the status of the teaching profession, according to a new report by the Business-Higher Education Forum (BHEF).[5]
- The report of the commission on higher education appointed by Secretary of Education Margaret Spellings stresses important needs for access, learning, transparency, and accountability.[6]
- A recent report from the Council of Graduate Schools urges doctoral universities "to encourage scholars to use their knowledge and skills in a real-world setting in service to community, the state, the nation, and the world."[7]

My thesis today is that by virtue of our training, mathematicians have distinctive habits of mind that can enhance public discussion of public issues. Moreover, and most importantly, we have a professional obligation to move beyond the boundaries of our own discipline to bring our special skills of analysis and clarification to bear on important public policy discussions.

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[1] As an aid to anyone who wants to pursue the issues discussed in this talk, extensive references are given in footnotes with hyperlinks. Actual web site addresses are also provided in square brackets. All sites were verified on August 10, 2007; some sites require registration or subscription.

[2] Thomas Friedman. "It's a Flat World, After All." *New York Times Magazine*, April 3, 2005. [www.nytimes.com/2005/04/03/magazine/03DOMINANCE.html?ex=1186804800&en=e64ad2a8334593e2&ei=5070]

[3] Committee on Science, Engineering, and Public Policy. Rising Above the Gathering Storm. Wasington, DC. National Academies Press, 2007. [www.nap.edu/openbook.php?isbn=0309100399]

[4] Dennis M. Bartels. The Democratization of Scientific Knowledge. February 5, 2007. [www.edweek.org/ew/articles/2007/02/05/22bartels.h26.html?qs=democratization]

[5] Business Higher Education Forum (BHEF). *An American Imperative*. June 11, 2007. [www.bhef.com/solutions/anamericanimperative.asp]

[6] U.S. Department of Education. *A Test of Leadership: Charting the Future of U.S. Higher Education*. Washington, D.C., September 2006. [www.ed.gov/about/bdscomm/list/hiedfuture/reports/final-report.pdf]

[7] Council of Graduate Schools. *Graduate Education: The Backbone of American Competitiveness*. April, 2007. [www.cgsnet.org/portals/0/pdf/GR_GradEdAmComp_0407.pdf]

As evidence for this proposition, I have selected a few current issues in educational policy and practice that can benefit from mathematicians' insights. By selecting examples from education I do not mean to imply that education is the only arena that can benefit from mathematical outreach; it just happens to be the area I am most familiar with. Others of you may find equally compelling issues in environmental policy, health insurance, energy resources, or international relations. Mathematics can contribute to all these areas, and many more.

I don't need to tell you that mathematics is ubiquitous and pervasive. What I would like to convince you is that to be a mathematical citizen, you need to use your mathematics for more than mathematics itself.

College Outcomes

I begin with something close to all our hearts: undergraduate education. Specifically, how should we measure its value?

The increasing cost of higher education, and its increasing importance, has generated ever-increasing calls for greater public accountability. A few years ago assessment guru Peter Ewell and I wrote a brief survey of this new environment for *Focus* with the alliterative title "The Four A's: Accountability, Accreditation, Assessment, and Articulation."[8] One result of this public concern is the growing influence of (and related controversy about) college ranking systems such as the one created by *U.S. News & World Report*. Faculty and administrators often argue that the work of higher education is too complex and too varied to be accurately judged by simple output measures. Nonetheless, we live in a world in which simple measures thrive, whether or not they measure anything important, or anything at all.

One could spend a full semester plumbing the depths of the challenge posed by assessment of higher education. Here I want to touch on just three particulars to illustrate my argument about the value of mathematical thinking. One concerns measures of quantity (graduation rates), another measures of quality (general education), and a third measures of readiness (alignment).

College Graduation Rates

The graduation rate offers a simple measure that is widely accepted as a primary quantitative yardstick of accountability in higher education—whether of an entire institution, or of colleges within a university, or of different athletic teams. The public accepts graduation rate as a meaningful and relatively reliable indicator of a college's success because it is a simple ratio that they think they understand, and it matches student aspirations to earn a degree. Moreover, to those who pay the costs of higher education, graduation rates seems a good way to hold colleges accountable for educating those whom they admit.

For example, Education Trust—a Washington-based educational equity advocacy group—uses the large variation in graduation rates among otherwise comparable institutions as evidence that

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[8] Peter Ewell and Lynn Arthur Steen. "The Four A's: Accountability, Accreditation, Assessment, and Articulation." *Focus*, May 2003. [www.maa.org/features/fourAs.html]

something is fundamentally wrong with much of higher education. To help the public judge for themselves, Education Trust has created a web search engine that makes it easy to compare graduation rates for different institutions with similar characteristics.[9]

Anyone who thinks carefully about the definition and calculation of a graduation rate sees trouble. And mathematicians are among society's most expert advisors on matters of definition and calculation.

First, official graduation rates are based only on students who enter in the fall term as full time degree-seeking students. Second, the rate counts as graduates only those who finish at the institution where they first enroll. Students who meet these conditions are now a minority in American higher education.

This raises an interesting mathematical challenge: how best to define graduation rate?

Alexander McCormick, Senior Scholar at the Carnegie Foundation for the Advancement of Teaching, recently proposed replacing the graduation rate with what he calls a "success rate"— the proportion of students who, six years after first entering, have either graduated from college somewhere or are still enrolled and making progress towards a degree.[10] In his proposal, success is defined as not dropping out.

Clifford Adelman, an experienced analyst of education data, argues that the measure of success should be attainment of a degree, not perpetual enrollment.[11] He proposes to include all entering students who enroll for more than one course anytime during a twelve-month academic year and to track rates for four different kinds of students: dependent, traditional age students; independent adults; transfer-in students; and transfer-out students.

Alexander Astin, former director of the Higher Education Research Institute at UCLA, notes that two-thirds of the institutional variation in degree completion is attributable to differences in characteristics of entering students. Therefore, he suggests, instead of looking only at graduation rates we should look at the differences between actual rates and the rate that might be expected based on the kinds of students a college enrolls.[12]

The definition of graduation rate is no small matter. Graduation rates influence the flow of federal and state money to higher education, students' perception of institutional quality, and the ground rules for intercollegiate athletics. A misleading indicator can create significant inefficiency when resources are withheld from effective programs whose successes are not captured by the particular definition in use.

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[9] Education Trust. College Results Online. [www.collegeresults.org]

[10] Alexander McCormack. "First, Do No Harm." *Carnegie Conversations*. Carnegie Foundation for the Advancement of Teaching, April, 2007. [www.carnegiefoundation.org/perspectives/sub.asp?key=245&subkey=2349]

[11] Clifford Adelman. "Making Graduation Rates Matter." *Inside Higher Education*, March 12, 2007. [www.insidehighered.com/views/2007/03/12/adelman]

[12] Alexander W. Astin. "To Use Graduation Rates to Measure Excellence, You Have to Do Your Homework." *Chronicle of Higher Education*, October 22, 2004, p. B20. [chronicle.com/weekly/v51/i09/09b02001.htm]

Output Assessments

In addition to indicators of quantity, parents and taxpayers also want evidence of quality. The recent report of the Commission on the Future of Higher Education urges colleges and universities to measure and report meaningful student learning outcomes.[13] Here "meaningful" refers both to internal and external objectives that reflect the complex and subtle goals of higher education while at the same time using a yardstick that the public can understand and that is relatively consistent.

Several relatively new instruments have been developed that claim to assess the broad outcomes of higher education independent of major. These include:

- CAAP: Collegiate Assessment of Academic Progress (from ACT)[14]
- MAPP: Measure of Academic Proficiency and Progress (from ETS)[15]
- CLA: Collegiate Learning Assessment (from the Council on Aid to Education)[16]
- NSSE: National Survey of Student Engagement (from Indiana University)[17]

A recent study from the University of California raises questions that should interest a mathematical mind about the potential use of such instruments to compare colleges.[18] They found that undergraduates studying the same disciplines on different campuses have academic experiences that are more similar to each other than to students studying different subjects on the same campus.

For example, students who majored in the social sciences and humanities report higher levels of satisfaction with their undergraduate education as well as better skills in critical thinking, communication, cultural appreciation, and social awareness. But students majoring in engineering, business, mathematics, and computer science report more collaborative learning, while those majoring in engineering and natural science studied much harder than their peers with other majors.

So, under circumstances in which variation within institutions exceeds variation across institutions, what mischief might emerge if these instruments are used to compare institutions? Can one honestly say that such results are both meaningful and useful for members of the public? My hunch is that results from these kinds of assessments will take a lot of careful analysis and interpretation by people who know how to make and explain fine distinctions.

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[13] U.S. Department of Education. *A Test of Leadership: Charting the Future of U.S. Higher Education*. Washington, D.C., September 2006. [www.ed.gov/about/bdscomm/list/hiedfuture/reports/final-report.pdf]

[14] American College Testing. CAAP: Collegiate Assessment of Academic Progress. [www.act.org/caap]

[15] Educational Testing Service. MAPP: Measure of Academic Performance and Proficiency. [www.ets.org/portal/site/ets/menuitem.1488512ecfd5b8849a77b13bc3921509/?vgnextoid=ff3aaf5e44df4010VgnVCM10000022f95190RCRD&vgnextchannel=f98546f1674f4010VgnVCM10000022f95190RCRD]

[16] Council on Aid to Education. CLA: Collegiate Learning Assessment. [www.cae.org/content/pro_collegiate.htm]

[17] Indiana University. NSSE: National Survey of Student Engagement. [nsse.iub.edu/index.cfm]

[18] Steve Chatman. *Institutional Versus Academic Discipline Measures of Student Experience: A Matter of Relative Validity*. University of California Undergraduate Experience Survey, Center for Studies in Higher Education at Berkeley, May 2007. [cshe.berkeley.edu/publications/docs/ROP.Chatman.AcadDisciplines.6.5.07.pdf]

Alignment

Of course, the results of college education depend in part on student preparation for college. Here too, in the transition from high school to college, lie widespread confusion and occasional contradictions. In state after state, political and educational leaders are trying to improve the alignment of their separate educational systems, especially K-12 with higher education. These efforts, laudable though their intentions may be, face some significant hurdles.

One recent study shows that neither admissions nor placement tests in mathematics give sufficient weight to the higher-level cognitive skills that are critical to success in college.[19] The same study shows significant discrepancies between the portfolio of skills assessed in the emerging state-required high school exit exams and those assessed by mathematics departments as part of their placement procedures.

A related study in California is even more explicit. It shows that many areas of mathematics addressed in community college placement exams are rarely tested on high school exit exams because they are thought of as part of middle school mathematics (e.g., whole numbers, fractions, decimals, percents, tables, graphs).[20]

Another paradox can be seen in the mathematics scale used by ACT for its widely used college admissions test. Based on empirical evidence drawn from nearly one hundred thousand test-takers, ACT identifies a score of 22 as a "college readiness benchmark" on the ACT mathematics test indicating that students achieving this score have "a high probability" of earning a C or higher and a 50/50 chance of earning a B or higher in college algebra.[21] Yet the most advanced problems routinely solved by typical students who score in the range of 20-23 include: solve routine first-degree equations; multiply two binomials; exhibit knowledge of slope; evaluate quadratic functions at integer values. Calculating a slope is one level higher (24-27); solving quadratic equations is higher still (28-32).[22]

The chasm between these mathematical skills and those that standards-writers claim is expected by colleges is striking, and in need of considerable dialogue to resolve. I suspect that part of the gap is created by the difference between wish lists of skills that mathematicians claim are necessary for college success and the reality of many college programs in which math avoidance is common, anticipated and perhaps even enabled.

As you may suspect, I have no intention of answering any of these tough questions. Indeed, the whole point of this talk is that working on problems such as these is *your* job—the next challenge that NExT fellows should take up—as well as, of course, all other mathematicians.

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[19] Achieve. Aligned Expectations? A Closer Look at College Admissions and Placement Tests. Washington, DC: Achieve, Inc., April, 2007. [www.achieve.org/files/Admissions_and_Placement_FINAL2.pdf]

[20] Richard S. Brown and David N. Niemi. "Investigating the Alignment of High School and Community College Assessments in California." National Center for Public Policy and Higher Education, June 2007. [www.highereducation.org/reports/brown_niemi/BROWN_NIEMI.pdf]

[21] American College Testing. *What Are ACT's College Readiness Benchmarks*? [www.act.org/path/policy/pdf/benchmarks.pdf]

[22] American College Testing. College Readiness Standards for Mathematics. [www.act.org/standard/planact/math/index.html]

Instead, I want to downshift to a set of similar challenges at the secondary level. As before, I begin with graduation rates.

High School Graduation Rates

Until very recently, the American public tended to think that nearly every American graduated from high school. Remember all the excuses in the press twenty years ago about why our SIMSS and TIMSS scores were lower than other countries? Whereas other nations with higher twelfth grade scores educated only their elite, U.S. editorialists argued, we educate everyone.

It wasn't true then, and is even less true today. In fact, the national on-time high school graduation rate peaked in 1969 at about 77% and has been falling ever since.[23] It is now apparently, a few points under 70%. In short, only two out of three students who begin ninth grade graduate four years later.

I say "apparently" since graduation rates are not as simple as dividing one number by another. In contrast to colleges that report data in conformity with federal law in order for their students to qualify for federal student aid, in the absence of a federal standard each state is free to adopt its own definition.[24] The box below shows several of the simpler and more common methods as described in a report by The Urban Institute;[25] additional more complicated versions are outlined in a comprehensive report on graduation rates by the Alliance for Excellence in Education.[26] Not surprisingly, the rates produced by these different methods vary widely, even with the same data.[27] "Calculating an apparently simple value—the percent of students who graduate from high school—is anything but simple," concludes The Urban Institute report, "and the best way to go about it is anything but apparent."

Recently state governors agreed to adopt a single method—the so-called adjusted cohort graduation rate (ACGR)—that, apart from the adjustments, is similar to the one used in higher education: divide the number of freshman in one year by the number of graduates four years later, adjusting for students who transfer in or out.[28] The result has been a series of headlines warning citizens that many of the official high school graduation rates are actually lower than had been previously reported. This makes officials squirm, but it is a good opportunity for mathematically-minded folks to help the public understand why such rates are so complicated.[29]

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[23] Paul E. Barton. *One Third of a Nation: Rising Dropout Rates and Declining Opportunities*. Princeton, NJ: Educational Testing Service, February 2005. [www.ets.org/Media/Research/pdf/PICONETHIRD.pdf]

[24] Daria Hall. Graduation Matters: Improving Accountability for High School Graduation. Education Trust, August 1, 2007. [www2.edtrust.org/NR/rdonlyres/5AEDABBC-79B7-47E5-9C66-7403BF76C3E2/0/GradMatters.pdf]

[25] Christopher B. Swanson. State Approaches for Calculating High School Graduation Rates. Education Policy Center, The Urban Institute, 2003. [www.urban.org/publications/410848.html]

[26] Lyndsay Pinkus. Who's Counted? Who's Counting? Understanding High School Graduation Rates. Washington, DC: Alliance for Excellent Education, 2006. [www.all4ed.org/publications/WhosCounting/WhosCounting.pdf]

[27] Jennifer Medina. "A Study Finds Some States Lagging on Graduation Rates." *New York Times*, August 2, 2007. [select.nytimes.com/search/restricted/article?res=F10917F83E540C718CDDA10894DF404482]

[28] National Governors Association. Graduation Counts: Redesigning the American High School. 2005. [www.nga.org/Files/pdf/0507GRAD.PDF]

[29] Alliance for Excellence in Education. Understanding High School Graduation Rates. [www.all4ed.org/publications/wcwc/National_wc.pdf]

Basic Completion Ratio (BCR):
Numerator: Total diplomas awarded in year N.
Denominator: Total 9th grade enrollment in year N-4.

Adjusted Completion Ratio (ACR):
Numerator: Total diplomas awarded in year N.
Denominator: Average enrollment in grades 8, 9 and 10 in year N-4 adjusted to reflect changes in total enrollments in grade 9-12 between year N-4 and N.

National Center on Educational Statistics (NCES):
Numerator: Total diplomas awarded in year N.
Denominator: Total diplomas awarded in year N plus all students who dropped out during each of the previous four years.

Longitudinal Graduation Rate (LGR):
Numerator: Total diplomas awarded in year N to members of the entering cohort in year N-4.
Denominator: Size of entering cohort in year N-4 students less those who transferred to another high school or who died.

Cumulative Promotion Index (CPI):
The product of the four transition ratios of enrollments in grades 10, 11, 12, and diploma awards to those in grades 9, 10, 11 and 12, respectively.

Some Common Formulas for Calculating Graduation Rates

High School Mathematics

As the national push for enhanced STEM education increases, some are now asking, as a recent headline in *Education Week* put it, what kind of mathematics really matters?[30] So far, the canonical answer is: the math you'll need for college. That's the way to keep options open. Anything else, people argue, exemplifies what President Bush calls "the soft bigotry of low expectations."

In high school, more advanced tends to mean more abstract, not more applicable. That's because the academic curriculum aims at college. Employers, however, see a frustrating paradox: even though students have studied more mathematics in high school, they graduate deficient in middle school skills such as percentages, decimals, and ratios that are prerequisites to successful employment.

Anthony Carnevale, a labor economist who studies the link between education and jobs, argues that Algebra II is the "threshold course"—the high school course that most clearly distinguishes those who go on to jobs with high earning trajectories from those who do not.[31] Because of the

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[30] Sean Cavanagh. "What Kind of Math Matters?" *Education Week*, June 12, 2007, pp. 21-23. [www.edweek.org/ew/articles/2007/06/12/40math.h26.html]

[31] Anthony P. Carnevale and Donna M. Desrochers. *Standards for What? The Economic Roots of K-16 Reform*. Princeton, NJ: Educational Testing Service, 2003, p. 55. [www.transitionmathproject.org/assets/docs/resources/standards_for_what.pdf]

power of this argument, enrollments in Algebra II have more than doubled in the last decade and roughly two-thirds of the states now require Algebra II for graduation.

But scores on the 12th grade NAEP mathematics test have hardly budged during this same period.[32, 33] Neither has there been a dramatic decline in the need for remediation in college mathematics. Moreover, according to a recent article in *Science*, the proportion of underrepresented minorities that demonstrate proficiency on the NAEP mathematics tests has slipped in each ethnic group.[34]

So if neither employers nor academics see noticeable results from the significantly greater emphasis on Algebra II, what's wrong?

Educational philosopher Nel Noddings suggests that the problem is a proliferation of what she calls fake academic courses: no proofs, no word problems, no brain teasers, no arguments—only a steady routine of drill on the discrete skills enumerated in the frameworks for state tests. "I've observed such classes," she writes. "They have pseudo-algebra and pseudo- geometry. This is pedagogical fraud, and students are doubly cheated: they do poorly in the required courses and they are deprived of courses in which they might have done well."[35]

Noddings argument is that in the interest of high expectations—where high equates with academic—schools have dropped low-status practical courses. But then to help their students pass state exams in the newly required academic courses, they eviscerated their intellectual content.

In other words, people argued that applied courses have no intellectual content, so everyone should take academic courses. As a consequence, many of *these* courses have lost their intellectual content. We've downshifted from cookbook calculus to automated algebra where over-emphasis on lists of learning objectives promote shallow learning and swift forgetting.

Social scientists recognize this result as a consequence of Campbell's law—a kind of uncertainty principle for public policy enunciated in 1976 by social psychologist Donald Campbell:

> The more any quantitative social indicator is used for social decision-making, the more subject it will be to corruption pressures and the more apt it will be to distort and corrupt the social processes it is intended to monitor.[36]

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[32] Kathleen Kennedy Manzo. "Students Taking Tougher Courses." *Education Week,* 22 February, 1997. [www.edweek.org/ew/articles/2007/02/22/25naep_web.h26.html?print=1]

[33] Peter Schmidt. "High-School Students Aim Higher Without Learning More, Federal Studies Find." *Chronicle of Higher Education*, March 9, 2007. [chronicle.com/weekly/v53/i27/27a03201.htm]

[34] Jeffrey Mervis. "U.S. Math Tests Don't Line Up." *Science*, 315 (16 March 2007) 1485. [www.sciencemag.org/cgi/content/full/315/5818/1485]

[35] Nel Noddings. "The New Anti-Intellectualism in America: When Curricular Rigor and 'Pedagogical Fraud' Go Hand in Hand." *Education Week*, March 20, 2007. [www.edweek.org/ew/articles/2007/03/20/28 noddings.h26.html?print=1]

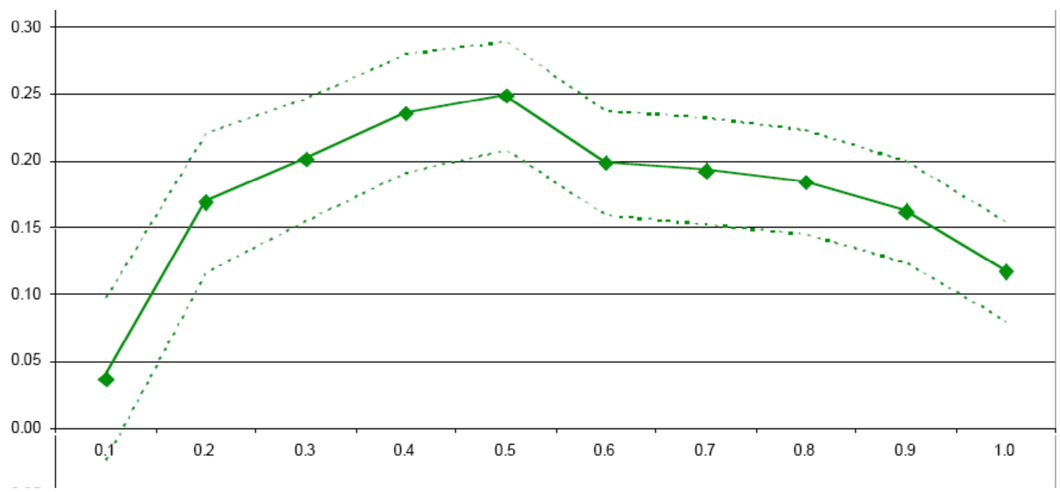
[36] Campbell, Donald T. "Assessing the Impact of Planned Social Change." Paper #8, Occasional Paper Series, The Public Affairs Center, Dartmouth College, December, 1976, p. 49. [www.wmich.edu/evalctr/pubs/ops/ops08.pdf]

In education, I think of this more as a Perversity Principle: the more importance we place on specific results, the less likely we are to achieve them in the form we intend.

Proficiency Counts

A good example of the Perversity Principle is the effect on education of the way schools are judged under the No Child Left Behind law: by the percent of students who are proficient. A few months ago the *Washington Post* quoted a middle school teacher as reporting "We were told to cross off the kids who would never pass. We were told to cross off the kids who, if we handed them the test tomorrow, would pass. And then the kids who were left over, those were the kids we were supposed to focus on."[37]

Two economists at the University of Chicago used data from the Chicago Public Schools to test whether this teacher's comment described typical behavior. They used data from dozens of schools in Chicago to test the hypothesis that when proficiency counts are used as the primary standard for judgment, teachers gradually focus most of their effort on students near the middle of the achievement spectrum, to the neglect of those at either end.



Impact of Proficiency Tests on Sixth Grade Mathematics Achievement
The increase in average grade level in relation to achievement score percentiles after two years of instruction under a proficiency count regime.

Empirical data from Chicago conform to the predictions of this model.[38] Usually graphs of learning growth are slightly exponential: the more you know, the more you learn. But the graph of learning growth after two years of assessment under a system of proficiency counts looks like an upside down U: the most learning took place for kids in the middle of the knowledge spectrum, the least for kids at each end.

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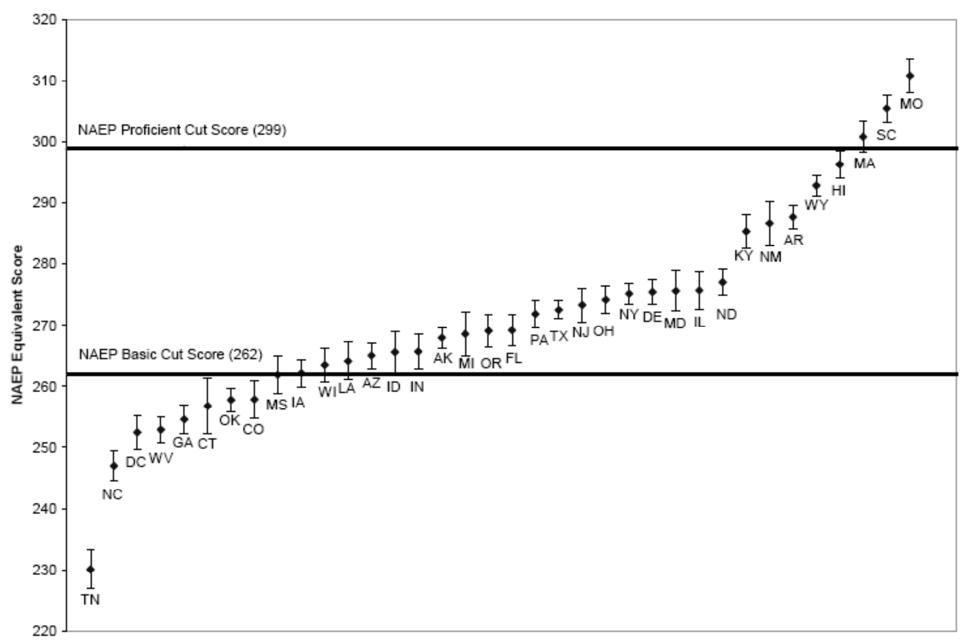
[37] Daniel de Vise. "A Concentrated Approach to Exams: Rockville School's Efforts Raise Questions of Test-Prep Ethics." *Washington Post*, Sunday, March 4, 2007; C01. [www.washingtonpost.com/wp-dyn/content/article/2007/03/03/AR2007030301372_pf.html]

[38] Derek Neal and Diane Whitmore Schanzenbach. *Left Behind By Design: Proficiency Counts and Test-Based Accountability*. University of Chicago, July, 2007. [harrisschool.uchicago.edu/faculty/articles/diane_left.pdf]

Variation in Standards

Our system of local control of education not only permits states to define graduation rates any way they please, but also to set standards for high school graduation at any level they please. The No Child Left Behind law—President Bush's signature effort to both raise and equalize educational accomplishment—required each state to report publicly on the percentage of students in different categories who are proficient according to the state's own standards. The law contains significant penalties for schools that do not make adequate yearly progress towards these goals.

But to accommodate our tradition of local control, each state remained free to set its own level of proficiency. When researchers matched states' proficiency reports against those of the randomized National Assessment of Educational Progress (NAEP), they found (a) great variation in the definitions of proficiency among the states and (b) a strong negative correlation (.83) between the percent of students deemed proficient and the level of accomplishment that the state required for proficiency.[39] Indeed, what many states call "proficient" is closer to what the national test rates as merely "basic" (see graph below). The differences between state proficiency standards were sometimes more than double the national gap between minority and white students.



State vs. National Proficiency Levels for 8th Grade Mathematics
Comparison of proficiency levels on state tests with the proficiency (upper line) and basic (lower line) levels established by the National Assessment of Educational Progress (NAEP). Vertical bars indicate uncertainty ranges for state estimates.

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[39] Institute of Education Sciences. *Mapping 2005 State Proficiency Standards Onto the NAEP Scales*. NCES 2007–482. Washington, DC: National Center for Educational Statistics, U.S. Department of Education, June, 2007. [nces.ed.gov/nationsreportcard/pubs/studies/2007482.asp]

One consequence is that a child who is considered to be proficient in one state (Connecticut, for example) may fall far short of expectations if the family moves across state lines (say, to Massachusetts). Some now argue that this data demonstrates the need for national standards; Indeed, early this year Senator Christopher Dodd (D-CT) and Representative Vern Ehlers (R-MI) introduced a bill to create and implement voluntary national standards in mathematics and science by synthesizing existing national standards.[40] Not surprisingly, representatives of state governments oppose this move.[41] (Many mathematicians will recall similar arguments a decade or more ago during a failed effort to produce a voluntary national assessment of 8th grade mathematics.)

Would mathematicians produce standards with such huge variation from state to state? I rather doubt it. As mathematical citizens, MAA members and NExT alumni should be active participants in setting these state proficiency levels, as well as in debating the pros and cons of national standards. I'm sure that's what Jim Leitzel would be doing.

Determining Proficiency

Partly because of the public scrutiny over whether or not schools are making adequate yearly progress under the NCLB law, and also because in some states too many students are failing tests required for graduation, one state after another is arguing over the passing scores that determine whether or not a student can graduate.[42]

For example, parents in Washington state were upset because too many students failed the state exams. So they hired a consultant who just reported that the tests, and the standards on which they were based, were not too hard but too lax![43]

People think state tests are scored like the tests they took when they were in school: your score is the total of all points earned on all the items that you answered correctly. Few know just how misleading this image is.

A few years ago Alan Tucker (of SUNY-Stony Brook) got involved in this debate when he was appointed to a commission in New York to investigate why so many students failed the 2003 Math A Regents test. His eyes were opened, and he wrote several papers about what he learned from this experience.[44, 45] The summary below is based on these papers.

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[40] Yudhijit Bhattacharjee. "States Urged to Sign Up for a Higher Standard of Learning." Science, 315 (2 February 2007) 595. [www.sciencemag.org/cgi/content/full/315/5812/595]

[41] Michele McNeil. "NCSL Declares Opposition to National Standards, Citing Flaws in NCLB." *Education Week*, August 6, 2007. [www.edweek.org/ew/articles/2007/08/06/45ncsl_web.h26.html?print=1]

[42] Ford Fessenden. "How to Measure Student Proficiency? States Disagree on Setting Standards and Tests." *New York Times*, December 31, 2003. [select.nytimes.com/search/restricted/article?res=F60613F93A5A0C728FDDAB0994DB404482]

[43] Linda Shaw. "State Math Standards Too Low, New Review Says." *The Seattle Times*, July 25m 2007. [seattletimes.nwsource.com/html/localnews/2003795680_math19m.html]

[44] Alan Tucker. Problems with Standards-Based Mathematics Tests. [www.ams.sunysb.edu/~tucker/StandardsProb.pdf]

[45] Alan Tucker. The New York Regents Math Test Problems. [www.ams.sunysb.edu/~tucker/MathAPaper.pdf]

Item Response Theory

Scoring of state tests is based on a psychometric methodology called Item Response Theory[46] whose purpose is to maintain a constant performance standard from year to year. It was errors in the application of this methodology that created the mess in New York: The commission on which Alan served concluded that the passing score of 51 that was used on the 2003 test should actually have been about 30. Quite a difference!

Item Response Theory relies on two major (and highly questionable) assumptions:

- First, that the mathematical ability of students and the difficulty of test items can be placed on a common one-dimensional scale.
- Second, that for each item the probability of a correct answer for a student of ability x is given by a family of *item response curves* that is controlled by a small number of parameters.

A typical family of item response curves is give by the probability function $p_{\alpha,\beta}(x) = 1/(1+e^{-\alpha(x-\beta)})$ where α is a scale parameter that controls the slope of the response curve and β corresponds to the difficulty level of the item. (Each such curve is shaped like a lazy "S" with inflection point at $x = \beta$ corresponding to $p = ½$. Some versions of item response theory use additional parameters.)

Assessment using IRT is a two stage process: first set a performance standard, and then score tests in relation to this standard. The catch is that each item on a test has a different performance characteristic that is supposedly reflected by its item response curve.

To set the performance standard, a sample of questions is given to a sample of students and then ranked by the percentage of students getting them right. Then a committee of teachers or other experts reviews the ranked list of questions, seeking to bookmark items that mark the boundary of each performance level (e.g., "advanced," "proficient," or "basic"). In one version of this process, the bookmark is supposed to be the hardest item that 2/3rds of those who meet the desired performance standard would get right.

Values of the item response curve parameters α and β are determined from field test data. The resulting curve is then "read backwards" to determine the ability level of a hypothetical student who is will get the bookmarked items correct with probability 2/3. Then the performance standard for the test—the so-called "cut score"—becomes the expected score of a student at this (marginal) ability level, that is, the sum of the expected number of points earned on each test item according to the item response curve of each item.

Policy Concerns

As we have seen, performance standards vary greatly from state to state. The IRT process may well be one of the reasons. Certainly, there are many opportunities for arbitrariness. Some of those identified by Alan in his analysis include:

- The process that matches bookmark items to proficiency standards is quite subjective.

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[46] Frank Baker. The Basics of Item Response Theory. College Park, MD: ERIC Clearinghouse on Assessment and Evaluation, University of Maryland, 2001. [edres.org/irt/baker/]

- Student performance varies unpredictably depending on which items they have practiced. (For this reason, teachers' judgment of item difficulties is frequently inconsistent with student performance.)
- The assumption that student abilities and item difficulties can be placed on the same scale is highly simplistic.
- Items designed to assess understanding and creative problem solving generate relatively unreliable psychometric statistics, which leads test developers to favor more routine questions.
- Field test data fit item response curves too imperfectly to determine item parameters and cut scores with a high degree of accuracy.
- Different vendors use slightly different versions of IRT theories (e.g., response curves with three rather than two parameters).

One conclusion is that passing scores on standards-based tests are very unlikely to be comparable at a level of precision that justifies high-stakes consequences.

Another is that the very theory underlying this testing protocol biases tests against the sort of mathematical reasoning that a high-quality K-12 mathematical education should develop in future citizens. The more the questions probe complex thought, the less well the scoring theory fits the student performance data and "the more likely it is that equating methods will misperform."[47]

If you need further evidence of the need for mathematicians to lend their minds to this kind of policy debate, I submit in Appendix A one page from a recent 18 page research paper whose purpose is to enhance the ability of item response theory to produce estimates of individual performance from matrix-designed assessments (such as NAEP).[48]

Liberal Learning

So far all my examples could be thought of as variations on a theme of putting mathematics to use in the particular sphere of education policy, that is, of fulfilling one part of the challenge posed by the Council on Graduate Education to encourage scholars to use their knowledge and skills "in service to community, the state, the nation, and the world."

I close by calling your attention to a brand new report that poses a different kind of challenge. It is *Beyond the Basics: Achieving a Liberal Education for All Children*, edited by Chester Finn, president of the Thomas B. Fordham Foundation and Diane Ravitch, a former Assistant Secretary of Education.[49]

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[47] Alan Tucker. "In Testing, How Reliable Are Year-to-Year Comparisons?" *Education Week*, August 11, 2004. [www.edweek.org/ew/articles/2004/08/11/44tucker.h23.html?qs=Alan_Tucker]

[48] Tamás Antal. On the Latent Regression Model of Item Response Theory. ETS Research Report, RR-07-12, 2007. [www.ets.org/portal/site/ets/menuitem.c988ba0e5dd572bada20bc47c3921509/?vgnextoid=5781d824 bee 91110VgnVCM10000022f95190RCRD&vgnextchannel=dcb3be3a864f4010VgnVCM10000022f95190RCRD]

[49] Chester Finn and Diane Ravitch, editors. *Beyond the Basics: Achieving a Liberal Education for All Children*. Thomas B. Fordham Institute, July, 2007. [www.edexcellence.net/institute/publication/publication.cfm?id=372]

For those who do not follow such things, I should mention that Finn and Ravitch and the Fordham Foundation have been among the most forceful advocates for aggressive state standards monitored by high stakes assessments. Fordham publishes biannual reports grading state standards, and very few get above a C.

Finn and Ravitch, it seems, have just discovered the Perversity Principle. It turns out that if you test only reading and mathematics, only reading and mathematics get taught. I quote (with slight paraphrasing) from their introduction:

> Pressure to pass basic skills tests leads teachers—often against their better judgment—to substitute “drill and kill” for “problem solving” … . “Rich content” doesn’t have many forms of self-defense, not in the face of external demands to hoist more kids over a specific bar to be determined by their scores on standardized tests. …
>
> We should have seen this coming. We and others who have pressed for higher academic standards in recent years … should have anticipated … that more emphasis on some things would inevitably mean less attention to others. …
>
> We were wrong. We didn’t see how completely standards-based reform would turn into a basic-skills testing frenzy or the negative impact that it would have on educational quality. …
>
> Those who see K-12 education as the solution to [shortages of STEM workers] are pointing America toward yet another round of unintended consequences: STEMs without flowers.
>
> Too much STEM may mean too few leaves and flowers. If children are deprived of the full richness of liberal education, they will face unmanageable challenges on many fronts:
> - The gradual death of liberal learning in higher education.
> - An accountability movement increasingly focused only on “basic skills.”
> - Growing support for math and science at the expense of the rest of the curriculum.
> - Widening gaps and accelerating advantage of the have-a-lots over the have-littles.
>
> If this dire scenario plays out, the American vision of a democratic education system nourishing a democratic society will perish.

Finn and Ravitch's call for putting the flowers back on the STEMs is also a dialogue in which mathematicians should participate—not by *applying* mathematics, but by *unfolding* mathematics as part of, rather than in opposition to, the goals of liberal education. Many whose own mathematics education never revealed this face of mathematics have a hard time seeing our discipline that way. It is out responsibility to help them do so now.

If Jim were here I'm sure he would eagerly take up this new challenge as a natural extension of that long ago dialogue in China about the nature and role of applications in teacher education. STEM *with* flowers offers us an excellent opportunity to engage the world as mathematical citizens.

Appendix

One page from Tamás Antal's 18 page report "On the Latent Regression Model of Item Response Theory" (ETS Research Report, RR-07-12, March, 2007).

Then,

$$\begin{aligned}
&\frac{\partial \mathcal{E}((\theta-\tilde{\theta}_i)\otimes(\theta-\tilde{\theta}_i))}{\partial \sigma_{\kappa\kappa'}} \\
= \;& 2\int_{\mathbb{R}^K}\left(\theta - \frac{\partial \tilde{\theta}_i}{\partial \sigma_{\kappa\kappa'}}\right)\otimes_s(\theta-\tilde{\theta}_i)\mathrm{d}\mu_i \\
& - \int_{\mathbb{R}^K}(\theta-\tilde{\theta}_i)\otimes(\theta-\tilde{\theta}_i)\frac{1}{\mathcal{N}(L_i)}\frac{\partial \mathcal{N}(L_i)}{\partial \sigma_{\kappa\kappa'}}\mathrm{d}\mu_i \\
& + \int_{\mathbb{R}^K}(\theta-\tilde{\theta}_i)\otimes(\theta-\tilde{\theta}_i)\frac{\partial \varphi(\theta;\Gamma x_i,\Sigma)}{\partial \sigma_{\kappa\kappa'}}\frac{1}{\varphi(\theta;\Gamma x_i,\Sigma)}\mathrm{d}\mu_i.
\end{aligned}$$

This, using

$$\left(\theta - \frac{\partial \tilde{\theta}_i}{\partial \sigma_{\kappa\kappa'}}\right)\otimes(\theta-\tilde{\theta}_i) = (\theta-\tilde{\theta}_i)^{\otimes 2} + \left(\tilde{\theta}_i - \frac{\partial \tilde{\theta}_i}{\partial \sigma_{\kappa\kappa'}}\right)\otimes(\theta-\tilde{\theta}_i),$$

yields

$$\begin{aligned}
&\frac{\partial \mathcal{E}((\theta-\tilde{\theta}_i)\otimes(\theta-\tilde{\theta}_i))}{\partial \sigma_{\kappa\kappa'}} \\
= \;& \breve{\Sigma}^{(i)} + \int_{\mathbb{R}^K}\left((\theta-\tilde{\theta}_i)^{\otimes 2} - \breve{\Sigma}^{(i)}\right)\frac{\partial \varphi(\theta;\Gamma x_i,\Sigma)}{\partial \sigma_{\kappa\kappa'}}\frac{1}{\varphi(\theta;\Gamma x_i,\Sigma)}\mathrm{d}\mu_i \\
= \;& \breve{\Sigma}^{(i)} - \frac{1}{2}\int_{\mathbb{R}^K}\left((\theta-\tilde{\theta}_i)^{\otimes 2} - \breve{\Sigma}^{(i)}\right)\frac{\partial \langle \theta-\Gamma x_i \,|\, \Sigma^{-1} \,|\, \theta - \Gamma x_i\rangle}{\partial \sigma_{\kappa\kappa'}}\mathrm{d}\mu_i \\
= \;& \breve{\Sigma}^{(i)} + \frac{1}{2}(2-\delta_{\kappa\kappa'})\Big[\big\langle \mathcal{E}\left((\theta-\tilde{\theta})_i^{\otimes 4}\right) - \breve{\Sigma}^{(i)\otimes 2} \,\big|\, (\Sigma^{-1}\otimes_s\Sigma^{-1})_{\kappa\kappa'}\big\rangle_{34} \\
& + \big\langle \mathcal{E}\left((\theta-\tilde{\theta})_i^{\otimes 3}\right)\otimes(\tilde{\theta}_i - \Gamma x_i) \,\big|\, (\Sigma^{-1}\otimes_s\Sigma^{-1})_{\kappa\kappa'}\big\rangle_{34} \\
& + \big\langle \mathcal{E}\left((\theta-\tilde{\theta})_i^{\otimes 2}\otimes(\tilde{\theta}_i-\Gamma x_i)\otimes(\theta-\tilde{\theta})_i\right) \,\big|\, (\Sigma^{-1}\otimes_s\Sigma^{-1})_{\kappa\kappa'}\big\rangle_{34}\Big]. \qquad (62)
\end{aligned}$$

Finally, putting all the above together yields

$$\begin{aligned}
&\frac{\partial^2 L}{\partial \sigma_{\kappa\kappa'}\partial \sigma_{kk'}} \\
= \;& -\frac{N}{2}(2-\delta_{kk'})(2-\delta_{\kappa\kappa'})(\Sigma^{-1}\otimes_s\Sigma^{-1})_{kk'\kappa\kappa'} \\
& + \frac{1}{2}(2-\delta_{kk'})\sum_{i=1}^{N}\Big\langle \frac{\partial \mathcal{E}((\theta-\tilde{\theta}_i)\otimes(\theta-\tilde{\theta}_i))}{\partial \sigma_{\kappa\kappa'}} \,\Big|\, (\Sigma^{-1}\otimes_s\Sigma^{-1})_{kk'}\Big\rangle \\
& - \frac{1}{2}(2-\delta_{kk'})(2-\delta_{\kappa\kappa'})\sum_{i=1}^{N}\big\langle \breve{\Sigma}^{(i)} \,\big|\, (\Sigma^{-1}\otimes_s\Sigma^{-1}\otimes_s\Sigma^{-1})_{kk'\kappa\kappa'}\big\rangle \\
& + (2-\delta_{kk'})\sum_{i=1}^{N}\Big\langle \left(\frac{\partial \tilde{\theta}_i}{\partial \sigma_{\kappa\kappa'}} - \Gamma x_i\right)\otimes_s(\tilde{\theta}_i - \Gamma x_i) \,\Big|\, (\Sigma^{-1}\otimes_s\Sigma^{-1})_{kk'}\Big\rangle \\
& - \frac{1}{2}(2-\delta_{kk'})(2-\delta_{\kappa\kappa'})\sum_{i=1}^{N}\big\langle (\tilde{\theta}_i - \Gamma x_i)^{\otimes 2} \,\big|\, (\Sigma^{-1}\otimes_s\Sigma^{-1}\otimes_s\Sigma^{-1})_{kk'\kappa\kappa'}\big\rangle. \qquad (63)
\end{aligned}$$