YEARS of MATHEMATICS TEACHER

from the 2000s

Facing Facts: Achieving Balance in High School Mathematics

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It is a sign of mathematics' growing importance that debates about mathematics education have moved from the school classroom to the public arena. In the process, and not to its benefit, discourse on mathematics education has acquired some of the polarized character of politics. So when the Editorial Panel of the *Mathematics Teacher* asked me to look into my crystal ball to foresee what might lie ahead, I turned to history for evidence and insight.

YESTERDAY

One month into my freshman year in college the Soviet Union placed the world's first artificial satellite in an elliptical orbit around the earth. Students listened excitedly to Sputnik's tinny, high-pitched voice as it returned every 98 minutes during the satellite's trip around the world.[1] As mathematics students tested their estimates of orbital parameters against scanty details gleaned from the news, their subject took on unexpected relevance. We sensed that history had changed just at the moment we were launching our own personal journeys into higher learning.

The challenge Sputnik posed to the United States was felt across society and stimulated significant political, military, technological, and scientific developments. It also created an atmosphere of educational reform that enabled school mathematics to achieve a position of unprecedented favor and prominence.[2] Mathematics educators identify this period by its seminal reports[3] and signature projects such as the School Mathematics Study Group (SMSG).[4] Mathematicians joined educators to develop curricula that would offer "high quality mathematics for college-capable students, particularly those heading for technical or scientific careers."[5]

Many now also recall this as an era of strident criticism focused on the widely ridiculed "new math." Not surprisingly, mathematicians brought quite different perspectives to the Sputnik challenge[6] and forcefully argued their positions both within mathematical circles[7] and in public forums. In the widely read *Why Johnny Can't Add,* New York University mathematician Morris Kline lampooned his colleagues for designing a curriculum suitable only for the "minute fraction" of students who were like themselves.[8] Even today, a Google search for "new math" yields nearly 600,000 hits, beginning with homages to Tom Lehrer, whose popular parodies helped deal the new math a fatal blow in the court of public opinion. (Lyrics as well as recordings of these legendary parodies are readily available on the Internet.)

The discussion launched by Sputnik was far from the first serious engagement of school mathematics issues to be taken on by research mathematicians. More than half a century earlier, American Mathematical Society president Eliakim Hastings Moore had argued "as a pure mathematician" that both mathematics and society would be best served if grade-school children were trained in "observation, experiment, reflection, and deduction" that made direct connections between mathematics and matters of "thoroughly concrete character."[9]

About the same time, the great German mathematician Felix Klein delivered a series of lectures on elementary mathematics "from an advanced standpoint" to provide German schoolteachers with a rigorous development of elementary mathematics. Klein's lectures have become a classic; they were translated into English in 1932 and have recently been reissued.[10] Whereas Moore stressed concrete experiences, Klein expounded logical clarity. These differences in mathematicians' perspectives echoed loudly in the "new math" era, and still do to this day.

A quarter of a century after the Sputnik-inspired effort to transform school mathematics, headlines declared that the United States was still a "nation at risk." This time the alarm derived not from a basketball-sized satellite but from a "rising tide of mediocrity" in education.[11] Subsequent reports from international studies confirmed this assessment, describing school mathematics in the United States as the product of an "underachieving curriculum."[12]

- Twenty years after Sputnik, only fifteen states required more than one year of mathematics for graduation from high school, and only three in ten recent high school graduates had taken algebra 2.[13]
- Between 1963 and 1980, average scores on the mathematics section of the SAT fell by nearly half a standard deviation, much more than could be explained by the modest increase in numbers of test-takers during that period.[14]
- U.S. high school seniors who were enrolled in college-preparatory mathematics courses—beneficiaries of two decades of post-Sputnik reform effort—performed substantially below the average of their international peers.[15]

Once again mathematicians, scientists, and educators took up the challenge, but this time with an explicit goal of improving education for all students, not just future scientists. In *Science for All Americans,* the American Association for the Advancement of Science (AAAS) set forth specific understandings about science and mathematics that all graduates of U.S. high schools should attain.[16] As part of this effort, a team of mathematicians[17] described the outcomes of school mathematics for a "typical adult" in terms of *processes* (abstraction, representation, transformation, application, and comparison), *content* (arithmetic, algebra, geometry, analysis, discrete mathematics, logic, set theory, probability, and statistics), and *connections* (with language, emotions, science, and technology).[18]

Transformative efforts of the 1980s differed from the Sputnik-inspired reforms in both goals and strategies. First, they were intended to benefit all students, not only the college-bound—and certainly not only those intending a scientific or technical career. Second, they emphasized outcomes (performance) rather than inputs such as curriculum and teaching. Thus was born the idea of "national standards" for school mathematics—an unprecedented strategy that was enunciated only *sotto voce,* since it directly challenged the cherished U.S. tradition of local control of education. NCTM gave life to this movement by producing the nation's first disciplinary standards for education.[19] And they did it on their own dime, since in the mid-1980s no funding agency, federal or private, was willing to underwrite such a blatant challenge to local control. Leaders of other mathematical and scientific societies representing university, college, and industrial mathematicians endorsed the "vision" of these standards although not their every detail.

As the AAAS and NCTM efforts gained public recognition, others joined the campaign for excellence in mathematics and science education.[20] In 1986, the National Research Council established the Mathematical Sciences Education

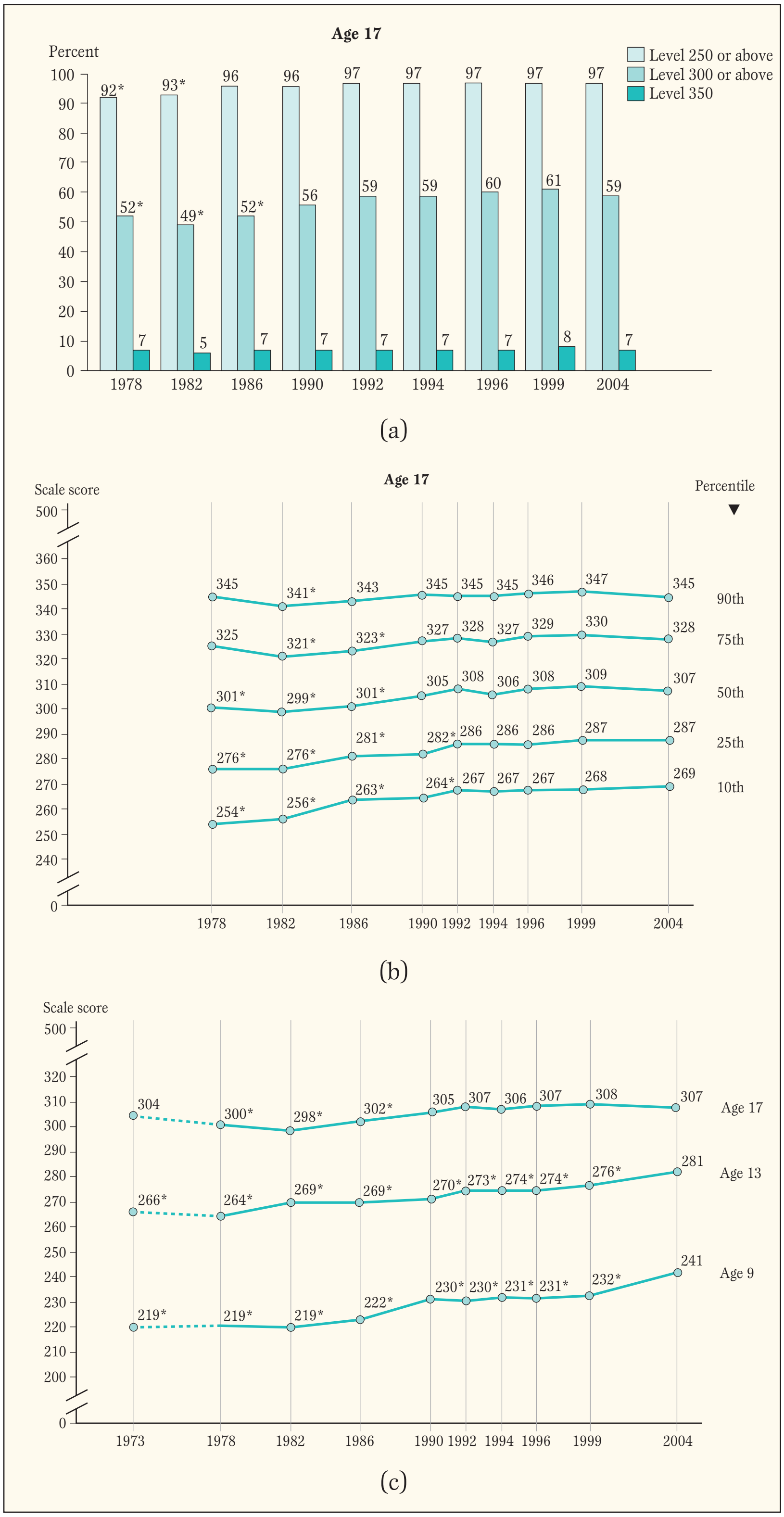

Fig. 1 NAEP mathematics trends for 17-year-old students. (a) Trends in percentages of 17-year-old students scoring at or above different mathematics performance levels; (b) Trends in mathematics scale scores at selected percentiles for 17-year-old students, 1978–2004; (c) Trends in average mathematics scale scores for students at age 9, 13, and 17, 1973–2004.

Sources: (a) nces.ed.gov/nationsreportcard/ltt/results2004/nat-math-pef.asp
(b) nces.ed.gov/nationsreportcard/ltt/results2004/nat-math-percentile.asp
(c) nces.ed.gov/nationsreportcard/ltt/results2004/nat-math-scalescore.asp

Board (MSEB) to provide a high-profile national stimulus to continuing reform. In 1990, President George H. W. Bush and state governors led by Bill Clinton took the unprecedented step of adopting national goals for education, one of which was, brazenly, that "by the year 2000, U.S. students will be the first in the world in science and mathematics achievement."[21] The U.S. Department of Labor looked at what work requires of schools and painted a distinctive nondisciplinary portrait.[22] Congress and federal agencies supported the development of standards in other disciplines and skill clusters (e.g., biotechnology, manufacturing) and provided funds for "systemic" initiatives that enlisted industry and higher education in support of improved school outcomes.

TODAY

In 2005, a half-century after Sputnik and a quarter-century after *A Nation at Risk,* nearly all states have established content standards in mathematics. Mathematical performance of 8- and 12-year-olds has slowly but steadily improved.[23] More than half the states require at least three years of high school mathematics, and many more students—including students in all racial and ethnic groups—take advanced mathematics courses.[24] Indeed, three out of four high school graduates now complete algebra 2 or an equivalent course, a completion rate that is 60 percent higher than when *A Nation at Risk* first appeared.[25]

Yet notwithstanding all these positive indicators, the overall level of mathematics that students take away from high school has not noticeably improved. Perhaps most stunning is that the average mathematics score of 17-year-olds on the long-term trend assessment of the National Assessment of Educational Progress (NAEP) has remained essentially unchanged for at least a quarter century (**fig. 1**).[26] Only 1 in 6 twelfth-grade students achieves the "proficient" level on the NAEP mathematics assessment, and only 1 in 50 performs at the "advanced" level.

Worse still, the gap between low- and high-performing students remains unconscionably large, as do the persistent differences among students from different racial, ethnic, and socio-economic backgrounds:

- The difference between the average NAEP mathematics scores of the highest and lowest quartiles for 17-year-old students is approximately the same as the difference between the average scores of 17- and 9-year-olds.[27]
- Whereas 1 in 3 Asian/Pacific Islanders and 1 in 5 white twelfth-graders scored at NAEP's "proficient" level, fewer than 1 in 25 Hispanic or black high school seniors did so.
- Low-income twelfth-grade students perform

similarly to eighth-graders and are twice as likely as their higher-income peers to score below NAEP's "basic" level in mathematics.[28]

- Although modest gains during the 1970s and 1980s narrowed gaps among racial and ethnic groups at the twelfth-grade level (**fig. 2**), since 1990 little further progress has been made.[29]

Here is the paradox: Despite significant increases in enrollments in advanced mathematics courses, today's high school graduates appear to be no more competent mathematically than their parents were in the early 1970s.

If taking more advanced mathematics does not improve students' mathematics performance, what does it do? Perhaps it encourages dropouts. About a third of U.S. students now leave high school without a diploma. Indeed, between 1975 and 2002, school completion rates declined both nationally and in four out of every five states;[30] the proportion of 18-year-olds with a high school diploma has now declined to near the level it was when Sputnik was launched. Internationally, the United States has slipped to tenth place in high school completion rates.[31] For many students, failure in mathematics is a contributing cause of not graduating from high school.

The litany of education ills continues into higher education. Even though more students than ever report earning an A average in high school, nearly half of all college and university faculty say that most of their students lack basic skills required for college-level work.[32] Not surprisingly, in the last twenty years the number of bachelor's degrees awarded in mathematics-intensive fields has fallen by about a third. Assessment of college seniors' quantitative literacy reveals that approximately 1 in 5 are not able to reliably carry out tasks such as calculating the total cost of ordering specific office supplies from a catalog.[33] Corporate leaders at a national summit on competitiveness predicted that without significant change the United States would soon lose its lead in innovation to the rapidly expanding juggernauts of China and India.[34]

This tale is very discouraging. Two waves of reform stretching across nearly half a century have rearranged the who, what, why, and how of mathematics education but have left the nation arguably no better off. Chastened by experience, we no longer aim to be first in the world but now simply hope to avoid putting children at risk.[35] Echoing the "rising tide" warning of a generation ago—but using a more ominous metaphor—the National Research Council now warns of a "gathering storm" created by declining educational performance in a world where expertise is only "a mouse click away."[36]

As in previous eras, critics lash out at what many now call the "new 'new math'," raising arguments similar to those heard two decades (not to mention a century) earlier. Challenges to the new standards are posted on Web sites[37] and printed in major newspapers.[38] In 2000, NCTM revised its standards, but criticisms continue. Congress now demands accountability, requiring states and schools to show evidence that no child is being left behind—or at least not many. Yet headlines herald a variety of dissents,[39] evasions,[40] and scholarly objections.[41] Meanwhile, four out of five middle school students report that they would rather clean their rooms or go to the dentist than do their math homework.[42]

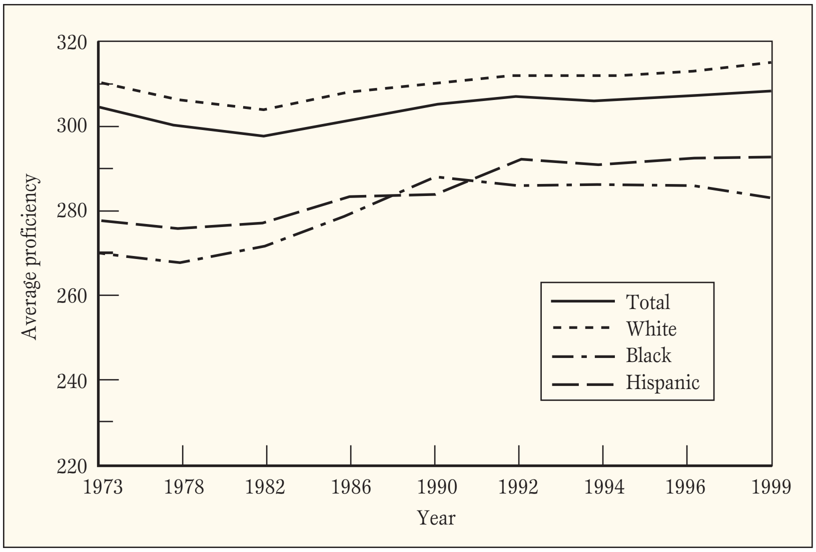

Fig. 2 NAEP mathematics proficiency for 17-year-olds by race-ethnicity
Source: www.rand.org/pubs/monographs/2005/RAND_MG255.pdf

Like so many other aspects of mathematics education, the present echoes the past. Apart from the role of calculators (which did not exist fifty years ago), many of the issues that led to what the media calls "math wars" are the same now as then: ensuring appropriate preparation for scientists and engineers; engaging mathematicians to help define the school curriculum; balancing traditional skills in arithmetic and algebra with other valuable topics (then it was sets, now it is data); and ensuring an appropriate level of mathematical rigor.[43] Even the term *math wars* is not new: It is the title of a fascinating post-Sputnik essay in the *American Scholar,* published in 1962.[44]

Yet some things are different. Whereas the dissenters in the new math era were concerned that the reforms slighted the needs of the majority in favor of a minority,[45] today's dissenters often argue that reformers slight the needs of the mathematically talented minority.[46] (To be fair, in both eras advocates of reform reject these critiques.) This reversal makes today's debates more delicate, since critics of current reform efforts may appear to be arguing from self-interest when in their view they are expressing an imperative in the nation's interest.

Another difference is in rhetoric. The mathematician authors of a widely publicized joint letter challenging the "new math" nonetheless conclude, senatorially, "we fervently wish much success to the workers on the new curricula." Even as they vigorously challenged ideas, they publicly praised the efforts of reformers. In today's contentious climate, however, many mathematics teachers and educators feel disparaged by their higher-status university critics.[47] Fitting the age in which we live, much of the discourse that is perceived as disrespectful takes place in unpublished but relatively public e-mail, blogs, or Web sites. Even as some groups seek common ground by bringing into dialogue advocates of many perspectives,[48] others have concluded that disagreements now "are just too deep to allow any non-trivial consensus."[49]

One of the deep divides concerns the relative balance of two important but different goals: to improve the mathematical knowledge and skills of all high school graduates and to increase the numbers of students prepared for scientific careers that require a strong mathematical anchor. The first goal requires a curriculum of sufficient flexibility and variety to interest and challenge all students, whereas the second appears to require a strategy to identify and advance specially able students to mathematically intensive programs. As noted above, many mathematicians and educators have argued that accomplishing the first goal would also accomplish the second, and would be the best way to do so. Studies of "second tier" students[50] and "switchers"—those who drop out of a potential science career—tend to confirm this hypothesis.[51]

One conclusion seems incontestable: Almost everyone significantly underestimates the intensity of effort and depth of understanding required to teach mathematics effectively.[52] Even professional mathematicians, for example, do not agree on how such an important and common term as *fraction* should be defined.[53] **Figure 3** offers a sampler of common opinions about what ails mathematics education (apart from major societal issues that affect all aspects of students' education). Clearly, there is no consensus.

The history of unfulfilled goals during the last half-century strongly suggests that there is no single cause, nor even any small number of causes, which if corrected would bring about significant improvement. Some failures are due to lack of rigor, others to excess of rigor; some to lack of mathematical consultation, others to excesses prescribed by mathematicians; some to neglect of computation, others to mindless drill on skills; some to narrow focus on algebra, others to lack of algebraic proficiency. For every conjectured cause there are examples that support the opposite. That is the nature of complexity: Simple analyses will always be wrong.

Mathematician Edward Begle, founder of SMSG, is reported to have concluded that "mathematics education is much more complicated than you expected, even though you expected it to be more complicated than you expected."[54] Apparently this lesson needs to be learned afresh by every generation and every individual. It flashes a bright yellow light of caution before anyone who peers into a crystal ball.

TOMORROW

The Sputnik challenge ushered in a paradigm in which fluency in the algebraic skills needed for calculus became the measure of quality for high school mathematics. Yet despite extraordinary efforts by mathematicians, educators, and government policy experts, today's typical 17-year-old knows no more mathematics than his or her grandparents did when they were that age. It is probably about time that we face facts: Aiming school mathematics for calculus is not an effective strategy to achieve the goal of improving all students' mathematical competence.

Good alternatives exist. They can be found by looking carefully at all the ways in which mathematics appears in postsecondary contexts. Notwithstanding other purposes and pressures, secondary education does respond to the demands of higher education. If colleges say that calculus is what everyone needs, or that good students are those who can quickly manipulate algebraically intricate expressions, then parents will demand and schools will focus on this type of mathematics. But programs with these mathematical requirements represent only the one-third of postsecondary education encompassed by STEM disciplines (science, technology, engineering, and mathematics). Moreover, these kinds of courses, which rely on very specific skills, have the effect of filtering out many otherwise interested and able students. As political pressure mounts to increase the flow of students in the STEM pipeline, these courses—and with them their prerequisites—will change.

That pressure is already building. *Science* editor Donald Kennedy, Howard Hughes Medical Institute president Thomas Cech, and former National Academy of Science president Bruce Alberts have all argued recently that introductory college STEM courses drive good students out of science[55] and must change in order to encourage improvement in secondary school courses.[56] If these courses continue to stress facts and skills, parents and politicians will continue to expect high school courses to convey nothing more than a string of facts and skills. Instead of asking students to absorb what is already discovered, these leading scientists argue, introductory college courses should encourage students to explore the world "the way working scientists do."

A sampler of commonly heard observations concerning the problems of mathematics education in the United States. Some contain grains of truth, others grains of falsehood; some are supported by wisps of anecdotal evidence, others are unsubstantiated opinions. All are expressed in one context or another; none is universally accepted.

Curriculum

- Compared with mathematics curricula in other higher performing nations, U.S. curricula are excessively repetitive, crammed with too many topics, and lacking any distinction between important and incidental topics.
- The organization of secondary school mathematics into separate layers of algebra and geometry makes it difficult for students to recognize the coherence of mathematics.
- Secondary school mathematics focuses too narrowly on preparation for calculus, ignoring other parts of mathematics that are more important and meaningful for many students.
- Many new curricula slight little-used algorithms of arithmetic (e.g., long division, addition of mixed fractions) under the mistaken belief that they are of no mathematical significance.
- Too many students enter high school without strong introductions to algebra, and too many leave high school without sufficient fluency in algebraic techniques necessary for calculus.

Standards

- Standards and textbooks often contain sloppy writing and ambiguous (or incorrect) mathematics which, too often, teachers either don't notice or don't know how to correct.
- Teams that write state standards and school texts do not have sufficient representation by active research mathematicians who are, arguably, the experts in the subject of mathematics.
- Most mathematics programs, especially those based on newer standards or textbooks, fail to pay careful attention to mathematical substance and rigor.

Classroom

- Profligate use of calculators in elementary grades enables many students to advance without mastering arithmetic to the point of automaticity.
- Frequent high stakes standardized tests force U.S. teachers to focus on routine skills and deny students sufficient opportunities to engage in high-level mathematical thinking.
- Too many mathematics teachers are poorly prepared in mathematics: Some have studied very little mathematics, some never really understood the mathematics they studied, and some have forgotten much of what they once knew.
- Teachers need but typically lack the "pedagogical content knowledge" that is difficult to acquire from standard undergraduate mathematics courses.

Relevance

- Too little school time and energy is focused on the special needs of mathematically able students who will become tomorrow's scientists and engineers.
- Little in the current secondary school curriculum is relevant to the personal and citizenship needs of students who will live in a data-intensive, computer-dominated society.
- Homogenous programs where all students study the same curriculum bore the strongest and overwhelm the weakest, leaving students in both groups frustrated and undereducated.
- Notwithstanding the significant expansion of quantitative methods in many jobs and careers, secondary school mathematics includes almost nothing that directly links mathematics with the real needs of the technical workforce.

Fig. 3 Explanations, excuses, conjectures, accusations

Mathematics is no different. It is no secret that uninviting introductory college courses drive students out of mathematics—and thus out of science. Significant efforts are under way to revitalize undergraduate mathematics,[57] not least in relation to courses below calculus,[58] courses taken by prospective teachers,[59] and courses taken by students in various "partner disciplines."[60] Because of its extraordinary power, mathematics at the postsecondary level is used and taught in programs as diverse as farming and linguistics, forensics and genomics, finance and epidemiology. The number of such programs is growing rapidly as the applications of mathematics radiate outward from the physical sciences through the biological, social, behavioral, and applied sciences, to the more distant humanities and fine arts.[61] Courses in which students learn how mathematics is used constitute a stealth curriculum that thrives outside the confining boundaries of college and university departments of mathematics.[62]

A different but equally important perspective on what higher education expects of high school graduates can be found under the amorphous label of numeracy or quantitative literacy.[63] Orbiting well beyond the STEM disciplines that are under mathematics' gravitational influence, numeracy addresses issues such as investments, energy, health, taxes, global warming, and potential pandemics that confront citizens in the daily news or in their ordinary lives. Along with writing, quantitative literacy is of central importance to critical thinking, itself a major goal of general education.[64]

Thus, my first recommendation: *Curriculum and policy leaders in secondary education should consider equally the needs of all disciplines and careers in which mathematical tools are used, as well as the quantitative aspects of general education. Preparing all students to squeeze through the calculus filter is neither appropriate nor effective as a way to meet the mathematical expectations of higher education.*

By working with practitioners of mathematics as well as with the *dramatis personae* of mathematics departments, secondary educators can address two of the most common critiques of school mathematics, namely, that its curriculum is tuned primarily to the needs of a minority and that it is an isolated subject employing ideas, language, and procedures found only in mathematics class. As mathematics colonizes diverse fields, it develops dialects that diverge from the "King's English" of functions, equations, definitions, and theorems. These newly important dialects employ the language of search strategies, data structures, confidence intervals, and decision trees, and routinely arise in a wide variety of programs and departments. By paying greater attention to the contemporary practice of mathematics, schools can develop varied programs that teach students to recognize and speak the many dialects of mathematics.

The overall level of mathematics that students take away from high school has not noticeably improved

Higher education is not alone in feeling mathematics' increased power. Riding the wave of Internet commerce, mathematical entrepreneurs are spreading their influence into virtually all aspects of business. "Top mathematicians are becoming a new global elite," reported *Business Week* in a cover story on the rising influence of mathematics.[65] "Quants," as these entrepreneurial mathematicians are sometimes called, do this by focusing more on algorithms than on equations, more on data than on theorems.[66] Once our bodies, minds, arts, and crafts are digitized—think DNA, brain scans, photography, architecture—they become subject to mathematical analysis ("data mining") that can significantly influence our society, economy, and quality of life. Almost none of the tools used in these new data-driven analyses is found in the typical high school mathematics program.

Instead, students get arithmetic, algebra, geometry, trigonometry, and calculus—a "hurried, rigid, linear sequence of courses that sifts through millions of students to produce thousands of mathematicians, scientists, and engineers."[67] By focusing the mathematics curriculum on subjects that are familiar to students' parents, this tradition makes parents less receptive to curricular change: When their children study what they studied, parents see little need for reform—not even in areas such as mathematics and science where political leaders demand improvement.[68] Well established though it may be, this sequence of courses misrepresents the nature of mathematics, denies students the motivation and excitement of mathematics' many new applications, and diminishes one of mathematics' primary strengths, namely, the powerful logical connection that weave its many threads into a whole cloth.

Countless mathematicians have decried practices that separate mathematics into discrete parts. Among the most recent is Michael Atiyah, winner of the 2004 Abel Prize for mathematics, who called such efforts "artificial."[69] The United States is virtually alone in the world in organizing secondary school mathematics as a ladder of separate subjects. Unfortunately, the recent movement toward state standards and high-stakes tests entrenches this tradition even more firmly. It is a pity that good intentions—high expectations for all students—should reinforce the status quo ante.

Hence my second recommendation: *To sustain student interest in mathematics—thus to serve both their and the nation's needs—the secondary school curriculum should offer a coherent, balanced introduction to the most widely used parts of the mathematical sciences in a manner that regularly connects each part with several others.*

To be clear, I am not arguing for or against so-called "integrated" curricula. The issue is not integration but breadth, balance, utility, coherence, and connectedness.[70] Understating the obvious, there is no one right way to approach the teaching of mathematics that will work well for all teachers and all students. (To those who believe otherwise, I suggest that they read Frank McCourt's account of teaching grammar—not so different from algebra—to vocational students.[71]) There are many ways to organize curricula; the challenge, now rarely met, is to avoid those that distort mathematics and turn off students.

Breadth and balance is hardly a new or radical suggestion. Twenty-five years ago, well before computers had elevated algorithms and data mining to near-Newtonian importance, the authors of *A Nation at Risk* urged that secondary school mathematics equip graduates not just with geometry and algebra but also with probability, statistics, and the capability to estimate, approximate, and confirm.[72] Twenty years earlier, the mathematician authors of the 1962 joint letter on the high school mathematics curriculum asserted that it should provide for the needs of all students by contributing to the "cultural background of the general student" and by offering preparation for future users of mathematics, including those studying the social sciences, "which may need progressively more mathematics in the future."[73] Our current fixation on the mathematics needed for calculus hardly meets these forty-year-old criteria, much less the needs of today.

A key factor that helps calculus retain its star status is its reputation for difficulty or "rigor," to use the currently fashionable cliché.[74] To be sure, understanding continuous and infinite processes poses a special intellectual challenge, but not every calculus course demands that much. Moreover, many other mathematical topics can be just as challenging—or just as mechanical. Some school treatments of, say, statistics or graph theory do fail to create a solid platform of results on which students can build. But when this happens, it is because of contingent rather than essential reasons, many of which are consequences of the priority afforded topics that lead toward the goal of calculus.

I am sure you have heard the argument: The reason 4 million fifth graders need to learn how to add 3/7 and 5/13 to get 74/91 (instead of 0.813…) is that the algorithm for adding fractions recurs in calculus when integrating functions by partial fractions. Students could develop equally rigorous thinking about fractions by exploring the subtleties of percentages (how to define graduation rate?), round-off errors (how to fairly apportion a legislature?), or constrained optimization (how to redistrict a state?) that are under current debate. Rigorous thinking exists everywhere in mathematics, not just on the road from algebra to calculus.

Thus my final recommendation: *All areas of mathematics should be used to advance students' rigorous thinking and their capacity to create compelling arguments. It is this capacity, not the particular topics studied, that will serve students well in the postsecondary world.*

Summing up, evidence from a half-century of reform efforts shows that the mainstream tradition of focusing school mathematics on preparation for a calculus-based postsecondary curriculum is not capable of achieving urgent national goals, and that no amount of further tinkering is likely to change that to any substantial degree. It also shows that no particular group—neither mathematicians nor educators, neither reformers nor traditionalists—has expressed a unified or effective alternative.

> Along with writing, quantitative literacy is of central importance to critical thinking

Fortunately, the extraordinary recent expansion of mathematical applications opens the door to a new strategy: Aim school mathematics, especially at the secondary level, at today's diverse and ever-expanding frontier of mathematical practice. The unique power of mathematics that the current curriculum provides for a minority of calculus-bound students—e.g., reasoning, abstraction, generalization—can reach a substantial majority of students through a more diverse curriculum designed to offer breadth, balance, utility, and coherence.

ENDNOTES

Note: The purpose of this extensive list of notes is not just to fulfill a scholarly obligation to credit sources but to provide readers with specific resources that may be of help in clarifying issues that arise in local discussions about mathematics education.

1. This historic signal is recorded at www.hq.nasa.gov/office/pao/History/sputnik/sputnik.wav.

2. Conference Board of the Mathematical Sciences, National Advisory Committee on Mathematics Education (NACOME), *Overview and Analysis of School Mathematics, Grades K–12* (Washington, DC: Conference Board of the Mathematical Sciences, 1975).

3. College Entrance Examination Board (CEEB), Commission on Mathematics, *Program for College Preparatory Mathematics* (New York: CEEB, 1959); Cambridge Conference on School Mathematics, *Goals for School Mathematics* (Boston: Houghton Mifflin, 1963).

4. A Guide to the School Mathematics Study Group Records, 1958–1977, Archives of American Mathematics, Center for American History, University of Texas at Austin. Available at www.lib.utexas.edu/taro/utcah/00284/cah-00284.html.

5. NACOME, *Overview and Analysis of School Mathematics,* 23.

6. Senior mathematicians both led and opposed this post-Sputnik reform effort. Leaders included Max Beberman, Edward Begle, Andrew Gleason, and Henry Pollak, while opponents included Lipman Bers, Morris Kline, and George Pólya.

7. Lars Ahlfors et al., "On the Mathematics Curriculum of the High School," *American Mathematical Monthly* 69

(May 1962): 189–93; *Mathematics Teacher* 55 (March 1962): 191–95.

8. Morris Kline, *Why Johnny Can't Add: The Failure of the New Math* (New York: St. Martin's Press, 1973).

9. Eliakim Hastings Moore, "On the Foundations of Mathematics," *Science,* n.s., 17 (March 13, 1903): 401–16.

10. Felix Klein, *Elementarmathematik vom höheren standpunkte aus,* Teil I–II (Leipzig: G. B. Teubner, 1908–1909). English: *Elementary Mathematics from an Advanced Standpoint* (2 vols.), translated by E. R. Hedrick and C. A. Noble (New York: MacMillan, 1932; repr., Mineola, NY: Dover, 2004).

11. National Commission on Excellence in Education (NCEE), *A Nation at Risk: The Imperative for Educational Reform: A Report to the Nation and the Secretary of Education, U.S. Department of Education* (Washington, DC: NCEE, 1983).

12. Curtis C. McKnight et al., *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (Champaign, IL: Stipes Publishing, 1987).

13. NCEE, *Nation at Risk,* 20.

14. Ibid., 9.

15. McKnight, *Underachieving Curriculum,* vii.

16. *Science for All Americans: A Project 2061 Report on Literacy Goals in Science, Mathematics, and Technology* (Washington, DC: American Association for the Advancement of Science, 1989).

17. The mathematics team for Project 2061 consisted of David Blackwell, Lenore Blum, Paul Garabedian, Paul Halmos, Leon Henkin, Harvey Keynes, R. Duncan Luce, Ingram Olkin, James Sethian, Audrey Terras, and P. Emery Thomas.

18. David Blackwell and Leon Henkin, *Mathematics: Report of the Project 2061 Phase I Mathematics Panel* (Washington, DC: American Association for the Advancement of Science, 1989).

19. National Council of Teachers of Mathematics (NCTM), *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: NCTM, 1989).

20. Lynn Arthur Steen, "Out from Underachievement," *Issues in Science and Technology* 5 (Fall 1988): 88–93.

21. *America 2000: An Education Strategy* (Washington, DC: U.S. Department of Education, 1991).

22. U.S. Department of Labor, Secretary's Commission on Achieving Necessary Skills (SCANS), *What Work Requires of Schools: A SCANS Report for America 2000* (Washington, DC: SCANS, 1991).

23. National Center for Education Statistics, *The Nation's Report Card: Long-Term Trend* (Washington, DC: U.S. Department of Education, 2005). Available at nces.ed.gov/nationsreportcard/ltt/results2004/nat-math-perf.asp.

24. Council of Chief State School Officers, State Education Assessment Center, *State Indicators of Science and Mathematics Education 2005* (Washington, DC: Council of Chief State School Officers, 2005).

25. National Center for Education Statistics, table ED4-A, *America's Children: Key National Indicators of Well-Being* (Washington, DC: U.S. Department of Education, 2005). childstats.ed.gov/americaschildren/tables/ed4a.asp.

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