Bayes' Comparisons

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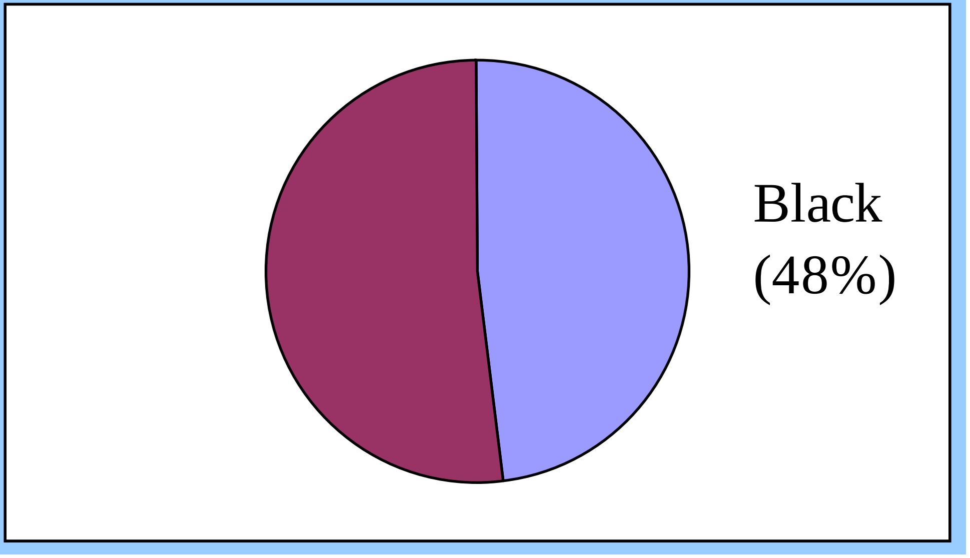
GAISE: Statistical Literacy: Reading Charts and Graphs

GAISE College Report: introductory courses in statistics should… strive to emphasize statistical literacy. We define statistical literacy as understanding the basic language of statistics: being able to read statistical graphs

MURDER VICTIMS

48% of US murder victims are black [True]

vs. 48% of US blacks are murder victims [False]



Statistical Literacy: Comparing Ratios

Comparisons of ratios in ordinary English often use “likely” or “prevalent.” Note that in 2005, 86% of US DOD army personnel were men.

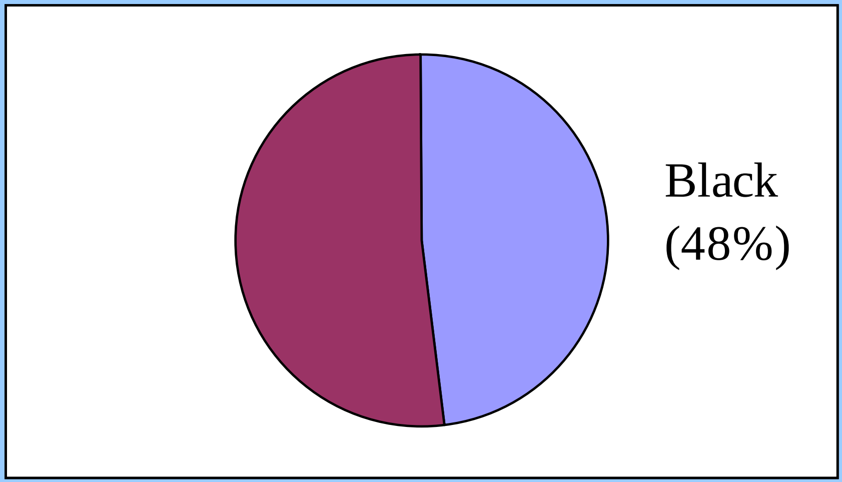
There are 2 forms of likely compares:

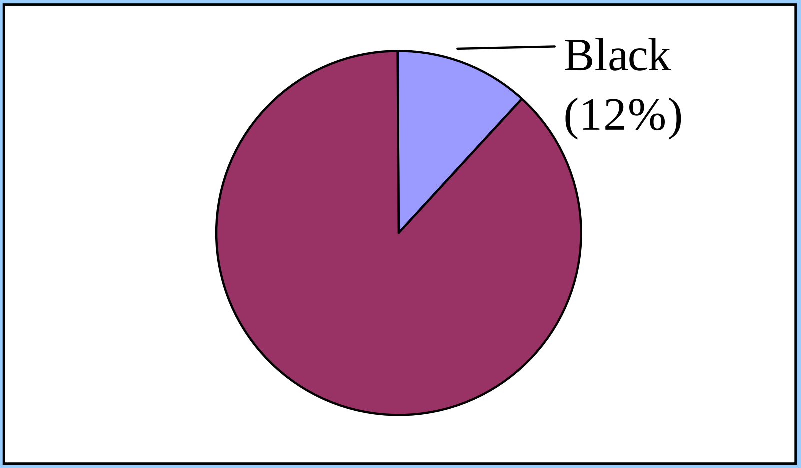
1) Military personnel are 1.7 times as likely *to be* **men** as are those in the US population.

2) **Men** are 1.7 times as prevalent *among* the military as *among* the US population.

Comparing Pie Charts
Blacks are the part

Blacks: 48% of US murder victims, 12% of US population.



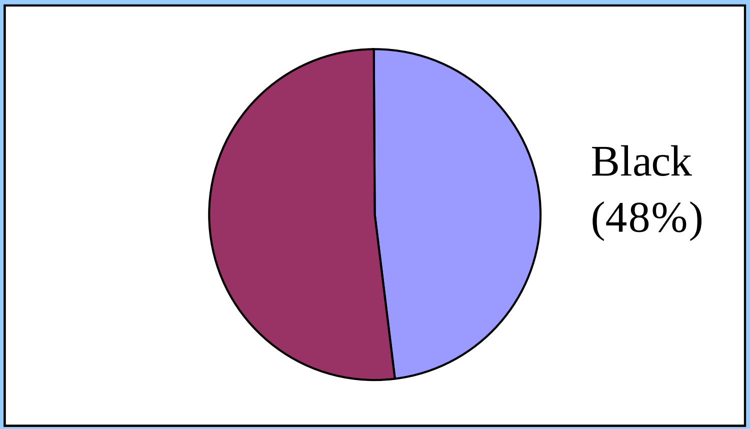


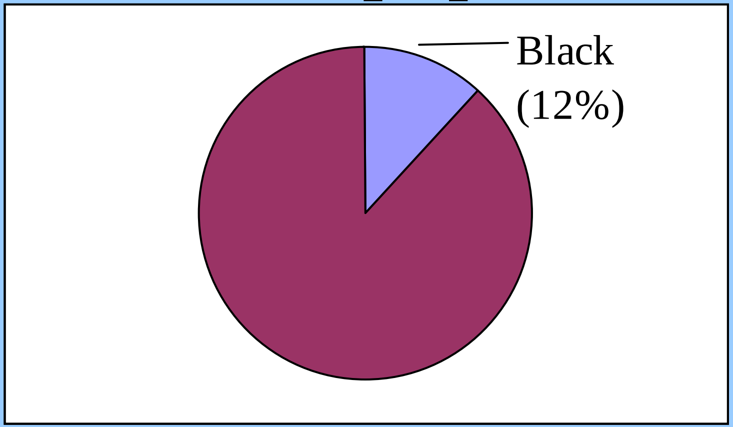
In the US in 2005, **blacks** were 4 times as prevalent *among* murder victims as [they were] *among* the population.

Alternatively: In the US in 2005, murder victims were 4 times as likely *to be* **blacks** as those in the population.

Over-Involvement Ratios

Blacks: 48% of murder victims: 12% of population.



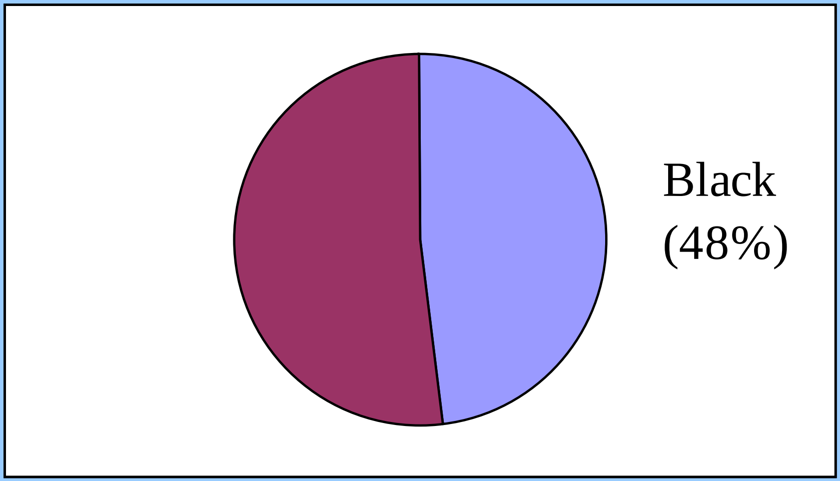


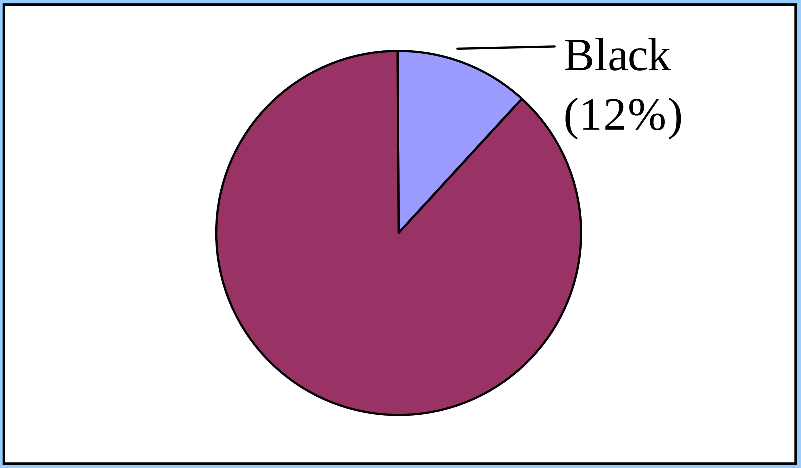
If **blacks** are *more likely* among murder victims than among the general population,

then blacks are *more likely* to be **murder victims** than are those in the general population.

Bayes' Comparison

Blacks: 48% of murder victims: 12% of population.





If **blacks** are *4 times as likely* among murder victims as among the general population,

then blacks are *4 times as likely* to be **murder victims** as are those in the general population.

Bayes' Theorem
Bayes' Comparison

Bayes' theorem can be easily proved using algebra:

1. $P(A|B) \equiv P(A \cap B) / P(B)$. $P(B|A) \equiv P(A \cap B) / P(A)$
2. $P(A|B)P(B) = P(A \cap B) = P(B|A)P(A)$
3. $P(A|B) = P(B|A)\,[P(A)/P(B)]$ if $P(B) > 0$

Bayes' comparison is a rearrangement of Bayes' theorem:

4. $P(A|B) / P(A) = P(B|A) / P(B)$

Over-involvement: If $P(A|B) > P(A)$ then $P(B|A) > P(B)$

Comparison: If $P(A|B)/P(A) = k$, then $P(B|A)/P(B) = k$.

Racially-Motivated Hate Crimes

In 2000-2003, 79% of the victims of racially-motivated hate crimes were black. [2006 USSA, Table 305] Only 12% of the US population are black.

Q. Are non-blacks more or less likely to commit a racially-motivated hate crime?

Under-Involvement: If non-blacks (88% of the population) commit 79% of the racially-motivated hate crimes, then non-blacks are **LESS** likely to commit such crimes than are those in the US population.

Low Birth-Weight Babies

45% of low birth-weight babies had moms who were in the bottom 20% of IQ. [The Bell Curve, p. 381]

Q. Are low-IQ moms are more likely to have a low birth-weight baby?

Bayes' compare: If 20% of the moms had 45% of the low birth-weight babies, then low-IQ moms are **2.25 times** as likely to have a low birth weight baby as are moms in the population.

Men in Prison

Males are 77% of those in prison. [2006 USSA, Table 298] Suppose men are 50% of the population.

Q. Are men more likely to go to prison?

Bayes' Compare: If men (50% of the population) are 77% of those in prison, then men are **1.54 times** as likely to go to prison as those in the population.

Conclusions

To be statistically literate, students need to understand conditional probability as presented in pie charts and in ordinary English.

To meet the GAISE challenge, statistical educators must focus on providing *guidelines and training* on how to read and interpret conditional probabilities in the everyday media.

If this cannot be done within a traditional statistical-inference course, then perhaps we need a separate course in statistical literacy.

References

Bayes, T. 1763. "An Essay Toward Solving a Problem in the Doctrine of Chances", Philosophical Transactions of the Royal Society of London 53, 370-418. [1]

Rossman, A. and T. Short (1995). Conditional Probability and Education Reform: Are They Compatible? *Journal of Statistical Education*, v.3, n.2.[2]

Stanford Encyclopedia of Philosophy: Bayes Theorem [3]

Schield, M. and T. Burnham (2002). Algebraic Relationships between Relative Risk, Phi and Measures of Necessity and Sufficiency in a 2x2 Table. *ASA Proceedings of the Section on Statistical Education*, pp. 3089-3094. [4]

[1] See www.stat.ucla.edu/history/essay.pdf

[2] See www.amstat.org/publications/jse/v3n2/rossman.html