Coincidence in Runs and Clusters

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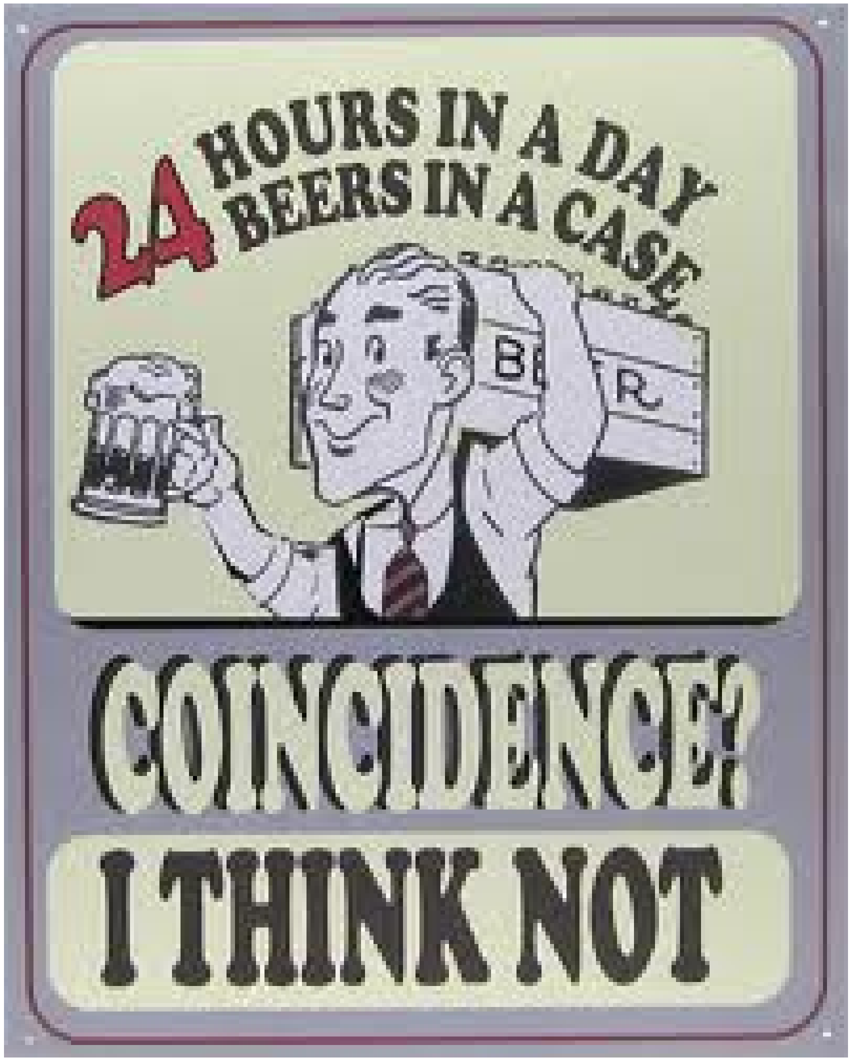
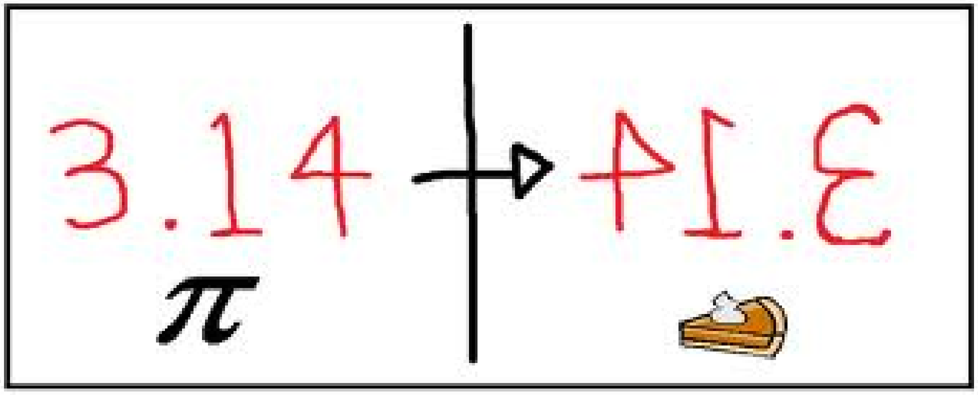
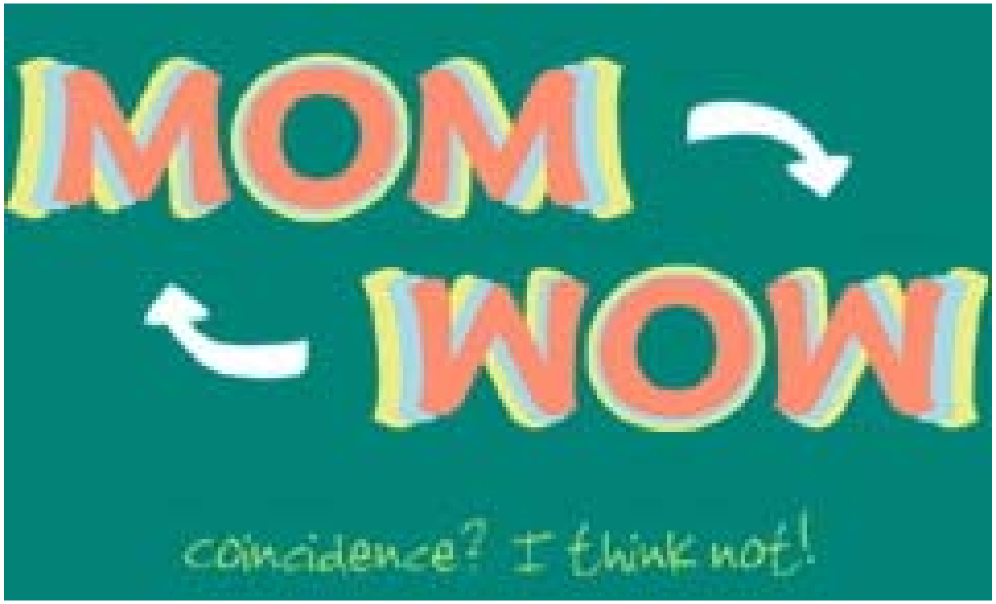
Paper at www.StatLit.org/pdf/2012Schield-MAA.pdf
Slides at www.StatLit.org/pdf/2012Schield-StatChat6up.pdf

Coincidence?



Coincidence?





March 2012

Chance: run of 10 heads?
One chance in 2^10 = 1 in 1,024

Three Heads in a Row

* has one chance in eight. P = 1/8

* is *expected* in 8 sets of three. N = 1/(1/8) = 8.

Binomial distribution: N*P = <Expected> = Mean.

If N = 1/P, then <Exp> = 1.

1	0	1		1	0	0
0	1	1		0	0	1
1	1	1		0	1	1
1	1	0		1	1	0

Map 8 sets of 3 each onto 10 tries

Run of 3 heads is generally found in (1/p)^k+(k-1) flips of a fair coin.										Schield (2012) V1	
Coin	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	
RUN											
3	1	0	1	1	1	0	0	1	1	0	
1	1	0	1								Set #1
2		0	1	1							Set #2
3			1	1	1						Set #3
2				1	1	0					Set #4
1					1	0	0				Set #5
1						0	0	1			Set #6
2							0	1	1		Set #7
2								1	1	0	Set #8

Distribution of longest run of heads in a set of 3

Longest Run	0	1	2	3
Expect #	1	4	2	1
Pctg of 8	12.5%	50.0%	25.0%	12.5%
	TTT	HTT, THT, TTH, HTH.	HHT, THH	HHH

Distribution of Longest Run of Successes

N=2: Counts: 1, 2, 1: TT; HT, TH; HH

N = 3: Counts: 1, 4, 2, 1.

0H: TTT. 1H: HTT, THT, TTH, HTH.

2H: HHT, THH. 3H: HHH.

N = 10: 1, 143, 360, 269, 139, 64, 28, 12, 5, 2, 1.

Mode=2, Median=3, Mean = 2.80.

Mean <> 3, but close enough as a rule-of-thumb.

Summary Statistics: Distribution of Longest Runs

K	N	Mode	Median	Mean
3	8	2	2	2.51
4	16	3	3	3.43
5	32	4	4	4.38
6	64	5	5	5.35
7	128	6	6	6.34
8	256	7	7	7.32
9	512	8	8	8.3
10	1,023	9	9	9.26

Runs: Adjacent Events

Law of Very-Large Numbers (Qualitative):
The very unlikely is almost certain given enough tries

RUNS RULE-OF-THUMB:

A run of events with 1 chance in N
is *generally found* in N tries.

90 tries/row, 15 rows: 1450 tries
Run of 10 (or more) is generally found

Conclusion

Students need to "see" that coincidences

1. **are more common than expected**
2. **depend on the context**
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4. **may still be signs of causation (Cholera)**

That runs with 1 chance in N are generally found in N tries.

Patterns in Rice

With rice scattered in two dimensions, people can often see shapes that are very unlikely.

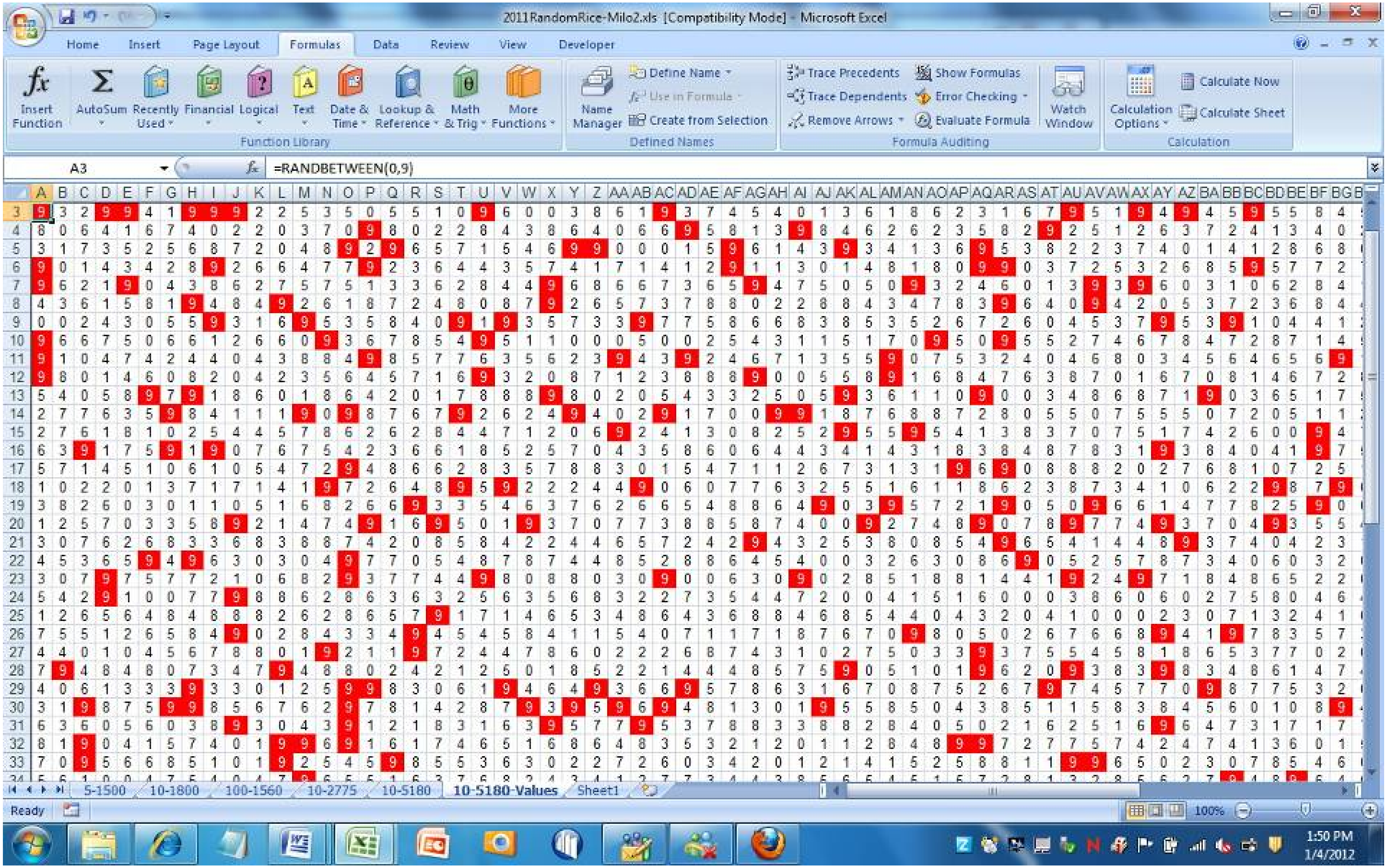
Let’s simulate rice in Excel where each cell has 1 chance in 10.

Patterns in Rice: # Touching
2:1/100; 4:1/10,000; 6: 1/1,000,000

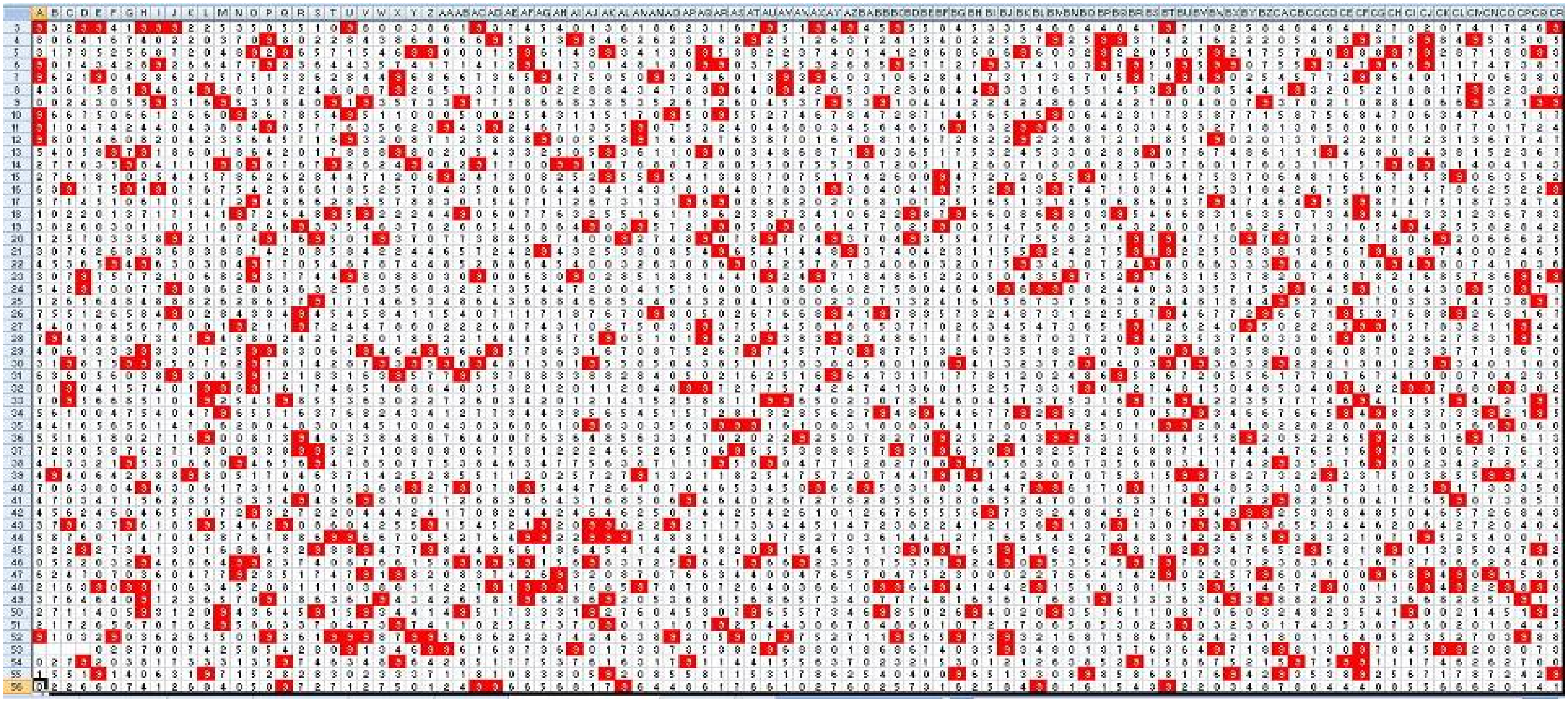
A3 ▼ *fx* =RANDBETWEEN(0,9)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
3	9	3	2	9	9	4	1	9	9	9	2	2	5	3	5	0	5	5
4	8	0	6	4	1	6	7	4	0	2	2	0	3	7	0	9	8	0
5	3	1	7	3	5	2	5	6	8	7	2	0	4	8	9	2	9	6
6	9	0	1	4	3	4	2	8	9	2	6	6	4	7	7	9	2	3
7	9	6	2	1	9	0	4	3	8	6	2	7	5	7	5	1	3	3
8	4	3	6	1	5	8	1	9	4	8	4	9	2	6	1	8	7	2
9	0	0	2	4	3	0	5	5	9	3	1	6	9	5	3	5	8	4
10	9	6	6	7	5	0	6	6	1	2	6	6	0	9	3	6	7	8
11	9	1	0	4	7	4	2	4	4	0	4	3	8	8	4	9	8	5
12	9	8	0	1	4	6	0	8	2	0	4	2	3	5	6	4	5	7

Patterns in Rice



3 touching: 1in 1,000
6 touching: 1 in a million



Patterns in Rice

In 2D, there are more ways for cells to connect:

2 horizontally (left side or right side)

2 vertically (above and below)

4 vertices (NE, SE, SW and NW corners)

8 TOTAL ways two random cells can connect.

Chance that 6 cells with rice will touch:

a.1 in 10^6: 1 in a million

b. (8-1)^6 = 262,144

Conclusion

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That runs with 1 chance in N are generally found in N tries.

The "Birthday" Problem

Q. What is the chance that two people in a group will have the same birth-date: month and day?

A. One chance in 365?

The “Birthday” Problem

Richard von Mises (1883-1953)

In a group of 28 people, a birthday match is “expected”.

The "Birthday" Problem
Math Answer

If the chance of an rare event is *p* and *p* =1/*k*, then this event is "expected" in k trials.

In a group of size N, there are (N-1)(N/2) pairs.

Solve for N(k). k = (N-1)(N/2) = (N^2-N)/2

Quadratic: N^2 – N - 2k = 0

Estimate: ~ N^2/2 = 1/p.

Trial and error: [27^2]/2 = 364 = 1/p = k

Q. Are students convinced? No!!!

49 Connections: Quadrant 1

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM								28 People	
		Month	10	11	11	9	4	7	6			
		Day	16	18	8	9	13	25	24			
Month	Day										Month	Day
8	20							1			7	25
10	29										8	16
4	11										11	6
3	3										11	29
1	3										8	3
3	30										3	24
10	28										1	15
		Month	5	2	6	2	1	7	5			
		Day	28	8	6	12	14	1	25			

49 Connections: Quadrant 2

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM								28 People
		Month	8	12	7	11	6	4	2		
		Day	28	2	15	15	5	24	2		
Month	Day									Month	Day
10	8									2	5
5	17									2	17
9	13									12	26
11	18									3	6
12	21							2		4	20
2	28									10	2
10	11									3	23
		Month	10	7	4	12	8	4	8		
		Day	22	22	10	6	4	20	21		

49 Connections: Quadrant 3

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM							28 People	
		Month	3	8	7	5	6	8	11		
		Day	4	5	25	27	19	4	26		
Month	Day									Month	Day
7	15									12	13
4	31									7	30
11	3									2	1
8	15									4	14
3	28									10	25
3	18									1	18
2	26		3							12	23
		Month	2	3	2	4	6	11	9		
		Day	26	26	23	6	30	11	8		

49 Connections: Quadrant 4

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM								28 People	
		Month	11	11	3	5	1	5	2			
		Day	5	27	17	3	5	19	4			
Month	Day										Month	Day
11	5		4								11	12
11	17										8	24
8	2										5	1
4	26										3	28
4	22										10	13
10	8										4	4
12	22										8	11
		Month	1	7	5	5	12	10	5			
		Day	2	1	23	7	20	14	14			

Connections: Quadrants 1 & 2

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM								28 People	
		Month	5	1	3	2	9	10	9			
		Day	24	3	11	22	10	19	31			
Month	Day										Month	Day
5	29										7	2
12	13										2	20
2	17							2			10	9
6	8										5	11
5	28										4	9
7	24						1				9	10
3	28										6	28
		Month	11	6	4	9	3	10	4			
		Day	3	1	12	4	17	9	8			

49 Connections: Top-to-Bottom

Schield (2011) **RICHARD VON MISES' BIRTHDAY PROBLEM** **28 People**

		Month	*11*	*8*	*10*	*10*	*8*	*10*	*3*		
		Day	*19*	*3*	*28*	*17*	*27*	*29*	*5*		
Month	Day					S				Month	Day
5	*23*									*1*	*12*
1	*1*									*11*	*17*
9	*6*									*12*	*3*
10	*13*									*7*	*29*
7	*14*									*2*	*17*
8	*30*									*4*	*2*
1	*8*									*8*	*17*
					N						
		Month	*12*	*3*	*10*	*9*	*12*	*9*	*5*		
		Day	*24*	*6*	*17*	*19*	*1*	*20*	*29*		

49 Connections: Side-To-Side

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM									28 People
		Month	2	3	10	6	6	9	6			
		Day	14	3	13	27	13	7	24			
Month	Day										Month	Day
1	24										1	31
9	8	E									6	28
12	6										12	24
12	28										10	1
10	27										11	19
9	18									W	9	8
4	12										4	16
		Month	8	8	6	5	7	4	7			
		Day	13	3	19	3	30	9	18			

21 Connections: Same-Side

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM								28 People	
		Month	3	2	2	3	9	3	5			
		Day	4	5	9	29	20	5	20			
Month	Day										Month	Day
6	22									E	4	1
10	8										7	10
5	5										3	26
11	23										3	10
3	27									E	4	1
10	2										9	8
2	21										5	7
		Month	8	1	10	12	9	5	5			
		Day	18	6	11	9	3	26	19			

Multiple Connections

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM								28 People	
		Month	11	7	5	7	12	6	9			
		Day	9	11	20	22	15	5	28			
Month	Day			S							Month	Day
1	17										12	17
10	9										7	6
8	29			1					2		7	11
3	24										4	21
2	9										5	24
6	29										1	14
11	12										3	12
									N			
		Month	11	12	9	12	4	4	7			
		Day	27	29	21	14	16	10	11			

Multiple Connections

Schield (2011)			RICHARD VON MISES' BIRTHDAY PROBLEM								28 People	
		Month	2	10	5	10	10	7	7			
		Day	19	12	12	20	17	20	5			
Month	Day			N		N					Month	Day
10	4			2							9	21
4	3					1					10	20
9	31										6	15
6	1									E	11	7
5	23									E	11	7
5	23										4	2
9	2										6	12
		Month	4	9	7	1	9	7	4			
		Day	12	21	12	19	16	4	27			

Connections and Chance

Pairs	GROUP	Details
196	Quadrants 1-4	49 pairs each
49	Side-to-Side	
49	Top-to-Bottom	
84	Within each side	21 pairs each
378	TOTAL	

A “birthday” match has one chance in 365.

In a group of 28, we have 378 pairs: (N-1)(N/2).

A match is expected: Match is more likely than not.

Conclusion

Students need to “see” that coincidences

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That runs with 1 chance in N are generally found in N tries.

ASA Chapter & StatChat
May 9 Wednesday 6 PM Augsburg

Wed May 9. Augsburg College. 6-9 PM. Supper

Chapter website: **www.amstat.org/chapters/twincities/**

SPEAKERS:

Marc Isaacson: **Teaching Activities**

Robert Raymond: **Untangling a Conundrum.**

Milo Schield: **Introducing the Matrixx Case**

Danny Kaplan: **Comments on US Supreme Court Matrixx Case: Is Significance Significant?**