Rash Task [from Kahan, Peters, Dawson, and Slovic, 2013, http://dx.doi.org/10.2139/ssrn.2319992]
Medical researchers have developed a new cream for treating skin rashes. New treatments often work but sometimes make rashes worse. Even when treatments don't work, skin rashes sometimes get better and sometimes get worse on their own. As a result, it is necessary to test any new treatment in an experiment to see whether it makes the skin condition of those who use it better or worse than if they had not used it.

Researchers have conducted an experiment on patients with skin rashes. In this experiment, one group of patients used the new cream for two weeks, and a second group did not use the new cream. The total number of patients in the two groups was not the same, but this does not prevent assessment of the results.

	Rash Got Better	Rash Got Worse
Patients who DID use the new skin cream	223	75
Patients who DID NOT use the new skin cream	107	21

Is using the new cream better, the same, or worse than not using the new skin cream?

Lucky Dip Task [© 2012, MARS, Shell Center, University of Nottingham, http://map.mathshell.org/materials/lessons.php?taskid=409]
Dominic has devised a simple game. Inside a bag he places 3 black and 3 white balls.
He then shakes the bag.
He asks Amy to take two balls from the bag without looking.

If the two balls are the same color, then Amy wins.
If they are different colors then Dominic wins.

Is the game fair, meaning Dominic and Amy have equal probability of winning?
If not, then who is most likely to win?

Survey Task [© 2010 The College Board. AP Statistics 2010, Form B, Question 5]
An advertising agency in a large city is conducting a survey of adults to investigate whether there is an association between highest level of educational achievement and primary source for news. The company takes a random sample of 2,500 adults in the city. The results are shown in the table below.

Primary Source for News	Not High School Graduate	High School Graduate but Not College Graduate	College Graduate	Total
Newspapers	49	205	188	442
Local television	90	170	75	335
Cable television	113	496	147	756
Internet	41	401	245	687
None	77	165	38	280
Total	370	1,437	693	2,500

(a) If an adult is to be selected at random from this sample, what is the probability that the selected adult is a college graduate or obtains news primarily from the internet?

(b) If an adult who is a college graduate is to be selected at random from this sample, what is the probability that the selected adult obtains news primarily from the internet?

(c) When selecting an adult at random from the sample of 2,500 adults, are the events
"is a college graduate" and "obtains news primarily from internet" independent?

Taxicab Task [see Kahneman & Tversky 1972; wording from Bar-Hillel 1980, *Acta Psychologica 44*, 211–233]
A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data.

(a) 85% of the cabs in the city are Green and 15% are Blue.
(b) A witness identified the cab as Blue.
(c) The court tested the reliability of the witness under the same circumstances that existed on the night of the accident, and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time.

What is the probability that the cab was actually Blue? Explain your reasoning.

ELISA Task [© 2009 The College Board. AP Statistics 2009, Form B, Question 2]
The ELISA tests whether a patient has contracted HIV. The ELISA is said to be positive if it indicates that HIV is present in a blood sample, and the ELISA is said to be negative if it does not indicate that HIV is present in a blood sample. Instead of directly measuring the presence of HIV, the ELISA measures levels of antibodies in the blood that should be elevated if HIV is present. Because of variability in antibody levels among human patients, the ELISA does not always indicate the correct result.
As part of a training program, staff at a testing lab applied the ELISA to 500 blood samples known to contain HIV. The ELISA was positive for 489 of those blood samples and negative for the other 11 samples. As part of the same training program, the staff also applied the ELISA to 500 other blood samples known to not contain HIV. The ELISA was positive for 37 of those blood samples and negative for the other 463 samples.

(a) When a new blood sample arrives at the lab, it will be tested to determine whether HIV is present. Using the data from the training program, estimate the probability that the ELISA would be positive when it is applied to a blood sample that does not contain HIV.

(b) Among the blood samples examined in the training program that provided positive ELISA results for HIV, what proportion actually contained HIV?

(c) When a blood sample yields a positive ELISA result, two more ELISAs are performed on the same blood sample. If at least one of the two additional ELISAs is positive, the blood sample is subjected to a more expensive and more accurate test to make a definitive determination of whether HIV is present in the sample. Repeated ELISAs on the same sample are generally assumed to be independent. Under the assumption of independence, what is the probability that a new blood sample that comes into the lab will be subjected to the more expensive test if that sample does not contain HIV?

Answers
Rash: The new skin cream is worse; $\frac{75}{298} = 25\%$ got worse with the new cream, more than $\frac{21}{128} = 16\%$ who got worse without it. (Two-sided Chi-square p ≈ 0.05; a statistician might say not significantly worse.)

Lucky Dip: Dominic is more likely to win. Given any first ball, Amy has 2/5 chance of matching that ball's color on the second draw, while the other color has 3/5 chance. Other solution paths include enumerating outcomes, trees, and combinations. Saying Amy had 20% probability and Dominic 30%, considering just one color, was marked incorrect.

Survey: (a) $\frac{693}{2500} + \frac{687}{2500} - \frac{245}{2500} = \frac{1135}{2500} = 0.454$. (b) $\frac{245}{693} = 0.354$, reading the table. (c) No, they are not independent. P(college) * P(internet) = $\frac{693}{2500} \times \frac{687}{2500} = 0.076$, different from joint probability P(college and internet) = $\frac{245}{2500} = 0.098$.

Taxicab: P(actual blue | identified blue) $= \frac{12}{29} = 0.41$. Use Bayes' formula, or consider when the witness would say blue: Actually blue, correctly seen (0.15)×(0.80) = 0.12; and actually green, incorrectly seen (0.85)×(0.20) = 0.17.

ELISA: (a) $\frac{37}{500} = 0.074$. (b) Of $489 + 37 = 526$ positive results, 489 had HIV: $\frac{489}{526} = 0.930$.
(c) P(false positive) = $\frac{37}{500}$ from (a). For the more expensive test, the sample must yield results of positive, positive; or positive, negative, positive. Computing, $\frac{37}{500} \times \frac{37}{500} + \frac{37}{500} \times \frac{463}{500} \times \frac{37}{500} = 0.0054 + 0.0051 = 0.0105$.