| | A | B | C | D | E | F | G | H | I | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Assume Households (#) have a Log-Normal Distribution by Income. | | | | | | | | | 2 |
| | | **Households Log-Normal Distribution** | | | | | **Households Normal Distribution** | | | 3 |
| | **Income** | Median | **50** | | | | mu | 3.912 | =LN(C4) | 4 |
| | | Mean | **80** | | | | mu+S^2/2 | 4.382 | =LN(C5) | 5 |
| | | | | | | | Sigma^2 | 0.940 | =2*(H5-H4) | 6 |
| | | Mode | 19.5 | =EXP(H4-H6) | | | Sigma | 0.970 | =SQRT(H6) | 7 |
| | | PDF# (Mode) | 1.32E-02 | =LOGNORM.DIST(C7,H4,H7,0) | | | | | | 8 |
| | | Std.Dev | 99.9 | =SQRT((EXP(H6)-1)*EXP(2*H4+H6)) | | | | | | 9 |
| | %HH by HH$ < Ave$ | | 0.686 | =NORM.S.DIST(SQRT(LN(C5/C4)/2), 1) | | | | | | 10 |
| | Gini Coefficient | | 0.507 | =2*NORM.S.DIST(H7/SQRT(2),1)-1 | | | | | | 11 |

| | A | B | C | D | E | F | G | H | I | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Since households have a Log-Normal distribution by Income with mu# and sigma#, it follows that | | | | | | | | | 13 |
| | total Income has a Log-Normal Distribution by HH Income [Aitchinson & Brown (1963)] | | | | | | | | | 14 |
| | with parameters mu$ = (mu# + sigma#^2) and sigma$ = sigma# | | | | | | | | | 15 |
| | | **Total Income Log-Normal Distribution** | | | | | **Total Income Normal Distribution** | | | 16 |
| | Median$ | | 128 | =EXP(H17) | | | mu$ | *4.852* | =H4+H6 | 17 |
| | Mean$ | | 205 | =EXP(H20) | | | Sigma$ | 0.970 | =H7 | 18 |
| | | | | | | | Sigma$^2 | 0.940 | =H18^2 | 19 |
| | Mode | | 50.0 | =EXP(H17-H19) | | | mu$+S$^2/2 | 5.322 | =H17+H19/2 | 20 |
| | PDF (Mode) | | 5.14E-03 | =LOGNORM.DIST(C20,H17,H18,0) | | | | | | 21 |
| | StdDev=Mean*CV | | 255.8 | =C18*SQRT(((C18/C17)^2)-1) | | | | | | 22 |
| | %Tot$ by HH$ > Ave$ | | 0.686 | =1-NORM.DIST(LN(C5),H17,H18,1) | | | | | | 23 |

| | A | B | C | D | E | F | G | H | I | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **CELL** | **FormulaText()** | | | | **CELL** | **FormulaText()** | | | 25 |
| | Add, B33. C33: Manual entry (Already entered) | | | | | F33 | =1-C33 | | | 26 |
| | B34 | =LOGNORM.INV(A34,H$4,H$7) | | | | G33 | =F33/E33 | | | 27 |
| | C34 | =LOGNORM.DIST(B34,H$17,H$18,1) | | | | H33 | =C$5*F33/E33 | | | 28 |
| | E33 | =1-A33 | | | | I34 | =H34/B34 | | | 29 |

| | A | B | C | D | E | F | G | H | I | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ----------BOTTOM-UP---------- | | | **Table 1** | ---TOP_DOWN--- | | Times=Share | Above | AboveAve$ | 31 |
| | **Pct#** | #Cutoff$ | %$cdf | | %#down | %$down | %$/%# | Ave$ | /Cutoff$ | 32 |
| | 0% | 0.0 | 0.00% | | 100% | 100.0% | 1.0 | 80 | | 33 |
| | 10% | 14.4 | 1.22% | | 90% | 98.8% | 1.1 | 88 | 6.1 | 34 |
| | 20% | 22.1 | 3.51% | | 80% | 96.5% | 1.2 | 96 | 4.4 | 35 |
| | 30% | 30.1 | 6.76% | | 70% | 93.2% | 1.3 | 107 | 3.5 | 36 |
| | 40% | 39.1 | 11.07% | | 60% | 88.9% | 1.5 | 119 | 3.0 | 37 |
| | 50% | 50.0 | 16.61% | | 50% | 83.4% | 1.7 | 133 | 2.7 | 38 |
| | 60% | 63.9 | 23.69% | | 40% | 76.3% | 1.9 | 153 | 2.4 | 39 |
| | 70% | 83.1 | 32.81% | | 30% | 67.2% | 2.2 | 179 | 2.2 | 40 |
| | 75% | 96.2 | 38.40% | | 25% | 61.6% | 2.5 | 197 | 2.1 | 41 |
| | 80% | 113.1 | 44.91% | | 20% | 55.1% | 2.8 | 220 | 1.9 | 42 |
| | 85% | 136.6 | 52.67% | | 15% | 47.3% | 3.2 | 252 | 1.8 | 43 |
| | 90% | 173.2 | 62.25% | | 10% | 37.8% | 3.8 | 302 | 1.7 | 44 |
| | 95% | 246.4 | 75.03% | | 5% | 25.0% | 5.0 | 400 | 1.6 | 45 |
| | 98% | 366.2 | 86.09% | | 2% | 13.9% | 7.0 | 557 | 1.5 | 46 |
| | 99% | 477.0 | 91.26% | | 1% | 8.7% | 8.7 | 699 | 1.5 | 47 |
| | 99.5% | 607.5 | 94.59% | | 0.5% | 5.4% | 10.8 | 866 | 1.4 | 48 |
| | 99.9% | 1000.4 | 98.30% | | 0.1% | 1.7% | 17.0 | 1,358 | 1.4 | 49 |
| | 99.95% | 1214.8 | 98.99% | | 0.05% | 1.0% | 20.3 | 1,623 | 1.3 | 50 |
| | 99.99% | 1840.4 | 99.70% | | 0.01% | 0.3% | 29.8 | 2,388 | 1.3 | 51 |
| | http://www.statlit.org/Excel/2014-Schield-Explore-LogNormal-Incomes-Excel2013.xlsx | | | | | | | | | 52 |

| CELL | FormulaText() | | | CELL | FormulaText() | | | |
|------|---------------|---|---|------|---------------|---|---|---|
| A62 | Manual entries (Already enetered) | | | F62 | =LOGNORM.DIST(A62,H$17,H$18,0) | | | 54 |
| B62 | =LOGNORM.DIST(A62,$H$4,$H$7,0) | | | G62 | =F62/C$21 | | | 55 |
| C62 | =B62/C$8 | | | H62 | =F62/C$8 | | | 56 |
| D62 | =LOGNORM.DIST(A62,H$4,H$7,1) | | | I62 | =LOGNORM.DIST(A62,H$17,H$18,1) | | | 57 |
| | To make reading Table 2 easier, hide columns B-C and F-H.    Unhide to read Table 1 | | | | | | | 58 |

(53 appears at top right of CELL/FormulaText header block)

| A | B | C | D | E | F | G | H | I | |
|---|---|---|---|---|---|---|---|---|---|
| Table 2 | Distribution of Subjects by Income | | | | Distribution of Total Income by Amount | | | | 60 |
| Income | PDF# | % of mode | CDF# | | PDF$ | % of $mode | %of#mode | CDF$ | 61 |
| 1 | 1.20E-04 | 0.91% | 0.00% | | 1.50E-06 | 0.03% | 0.01% | 0.00% | 62 |
| 2 | 8.31E-04 | 6.31% | 0.05% | | 2.08E-05 | 0.40% | 0.16% | 0.00% | 63 |
| 3 | 2.04E-03 | 15.46% | 0.19% | | 7.63E-05 | 1.48% | 0.58% | 0.01% | 64 |
| 4 | 3.46E-03 | 26.25% | 0.46% | | 1.73E-04 | 3.36% | 1.31% | 0.02% | 65 |
| 5 | 4.90E-03 | 37.25% | 0.88% | | 3.07E-04 | 5.96% | 2.33% | 0.04% | 66 |
| 7 | 7.52E-03 | 57.12% | 2.13% | | 6.58E-04 | 12.79% | 5.00% | 0.14% | 67 |
| 8 | 8.62E-03 | 65.46% | 2.94% | | 8.62E-04 | 16.76% | 6.55% | 0.21% | 68 |
| 9 | 9.57E-03 | 72.67% | 3.85% | | 1.08E-03 | 20.93% | 8.17% | 0.31% | 69 |
| 10 | 1.04E-02 | 78.79% | 4.85% | | 1.30E-03 | 25.21% | 9.85% | 0.43% | 70 |
| 12 | 1.16E-02 | 88.14% | 7.05% | | 1.74E-03 | 33.85% | 13.22% | 0.73% | 71 |
| 14 | 1.24E-02 | 94.27% | 9.46% | | 2.17E-03 | 42.23% | 16.50% | 1.12% | 72 |
| 16 | 1.29E-02 | 97.91% | 12.00% | | 2.58E-03 | 50.13% | 19.58% | 1.60% | 73 |
| 18 | 1.31E-02 | 99.65% | 14.60% | | 2.95E-03 | 57.40% | 22.42% | 2.15% | 74 |
| 20 | 1.32E-02 | 99.97% | 17.23% | | 3.29E-03 | 63.98% | 24.99% | 2.78% | 75 |
| 22 | 1.31E-02 | 99.25% | 19.86% | | 3.59E-03 | 69.87% | 27.29% | 3.47% | 76 |
| 24 | 1.29E-02 | 97.77% | 22.45% | | 3.86E-03 | 75.09% | 29.33% | 4.21% | 77 |
| 26 | 1.26E-02 | 95.74% | 25.00% | | 4.10E-03 | 79.66% | 31.12% | 5.01% | 78 |
| 28 | 1.23E-02 | 93.33% | 27.49% | | 4.30E-03 | 83.63% | 32.67% | 5.85% | 79 |
| 30 | 1.19E-02 | 90.67% | 29.91% | | 4.48E-03 | 87.04% | 34.00% | 6.73% | 80 |
| 33 | 1.14E-02 | 86.39% | 33.41% | | 4.69E-03 | 91.23% | 35.63% | 8.10% | 81 |
| 36 | 1.08E-02 | 81.96% | 36.74% | | 4.86E-03 | 94.42% | 36.88% | 9.54% | 82 |
| 40 | 1.00E-02 | 76.08% | 40.90% | | 5.01E-03 | 97.39% | 38.04% | 11.51% | 83 |
| 43 | 9.45E-03 | 71.80% | 43.82% | | 5.08E-03 | 98.80% | 38.59% | 13.03% | 84 |
| 46 | 8.91E-03 | 67.68% | 46.57% | | 5.12E-03 | 99.63% | 38.92% | 14.56% | 85 |
| 50 | 8.23E-03 | 62.50% | 50.00% | | 5.14E-03 | 100.00% | 39.06% | 16.61% | 86 |
| 55 | 7.45E-03 | 56.54% | 53.92% | | 5.12E-03 | 99.52% | 38.87% | 19.18% | 87 |
| 60 | 6.74E-03 | 51.17% | 57.46% | | 5.05E-03 | 98.25% | 38.38% | 21.73% | 88 |
| 65 | 6.10E-03 | 46.35% | 60.67% | | 4.96E-03 | 96.40% | 37.66% | 24.23% | 89 |
| 70 | 5.53E-03 | 42.03% | 63.57% | | 4.84E-03 | 94.16% | 36.78% | 26.68% | 90 |
| 75 | 5.03E-03 | 38.18% | 66.21% | | 4.71E-03 | 91.63% | 35.79% | 29.07% | 91 |
| 80 | 4.57E-03 | 34.73% | 68.61% | | 4.57E-03 | 88.91% | **34.73%** | 31.39% | 92 |
| 90 | 3.80E-03 | 28.89% | 72.78% | | 4.28E-03 | 83.21% | 32.50% | 35.82% | 93 |
| 100 | 3.19E-03 | 24.20% | 76.27% | | 3.98E-03 | 77.45% | 30.25% | 39.95% | 94 |
| 110 | 2.69E-03 | 20.41% | 79.20% | | 3.70E-03 | 71.84% | 28.06% | 43.79% | 95 |
| 120 | 2.28E-03 | 17.32% | 81.67% | | 3.42E-03 | 66.52% | 25.98% | 47.35% | 96 |
| 130 | 1.95E-03 | 14.79% | 83.78% | | 3.16E-03 | 61.53% | 24.04% | 50.64% | 97 |
| 140 | 1.67E-03 | 12.70% | 85.59% | | 2.93E-03 | 56.90% | 22.23% | 53.68% | 98 |
| 150 | 1.44E-03 | 10.96% | 87.14% | | 2.71E-03 | 52.62% | 20.56% | 56.50% | 99 |
| 160 | 1.25E-03 | 9.51% | 88.49% | | 2.50E-03 | 48.69% | 19.02% | 59.10% | 100 |
| 170 | 1.09E-03 | 8.29% | 89.66% | | 2.32E-03 | 45.09% | 17.61% | 61.51% | 101 |
| 180 | 9.55E-04 | 7.25% | 90.68% | | 2.15E-03 | 41.78% | 16.32% | 63.74% | 102 |
| 190 | 8.39E-04 | 6.37% | 91.57% | | 1.99E-03 | 38.75% | 15.14% | 65.81% | 103 |
| 200 | 7.40E-04 | 5.62% | 92.36% | | 1.85E-03 | 35.98% | 14.05% | 67.74% | 104 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **210** | 6.55E-04 | 4.98% | 93.06% | | 1.72E-03 | 33.44% | 13.06% | 69.52% | 105 |
| **220** | 5.82E-04 | 4.42% | 93.68% | | 1.60E-03 | 31.11% | 12.15% | 71.18% | 106 |
| **230** | 5.18E-04 | 3.94% | 94.23% | | 1.49E-03 | 28.97% | 11.32% | 72.72% | 107 |
| **240** | 4.63E-04 | 3.52% | 94.72% | | 1.39E-03 | 27.01% | 10.55% | 74.16% | 108 |
| **250** | 4.15E-04 | 3.15% | 95.15% | | 1.30E-03 | 25.21% | 9.85% | 75.50% | 109 |
| **260** | 3.73E-04 | 2.83% | 95.55% | | 1.212E-03 | 23.56% | 9.20% | 76.76% | 110 |
| **270** | 3.36E-04 | 2.55% | 95.90% | | 1.133E-03 | 22.03% | 8.61% | 77.93% | 111 |
| **280** | 3.03E-04 | 2.30% | 96.22% | | 1.061E-03 | 20.62% | 8.06% | 79.03% | 112 |
| **290** | 2.74E-04 | 2.08% | 96.51% | | 9.941E-04 | 19.33% | 7.55% | 80.05% | 113 |
| **300** | 2.49E-04 | 1.89% | 96.77% | | 9.325E-04 | 18.13% | 7.08% | 81.02% | 114 |
| **310** | 2.26E-04 | 1.72% | 97.01% | | 8.755E-04 | 17.02% | 6.65% | 81.92% | 115 |
| **320** | 2.06E-04 | 1.56% | 97.22% | | 8.23E-04 | 16.00% | 6.25% | 82.77% | 116 |
| **340** | 1.71E-04 | 1.30% | 97.60% | | 7.28E-04 | 14.16% | 5.53% | 84.32% | 117 |
| **360** | 1.44E-04 | 1.09% | 97.91% | | 6.47E-04 | 12.58% | 4.92% | 85.69% | 118 |
| **380** | 1.21E-04 | 0.92% | 98.18% | | 5.77E-04 | 11.21% | 4.38% | 86.91% | 119 |
| **400** | 1.03E-04 | 0.78% | 98.40% | | 5.16E-04 | 10.03% | 3.92% | 88.00% | 120 |
| **420** | 8.81E-05 | 0.67% | 98.59% | | 4.62E-04 | 8.99% | 3.51% | 88.98% | 121 |
| **440** | 7.56E-05 | 0.57% | 98.76% | | 4.16E-04 | 8.08% | 3.16% | 89.86% | 122 |
| **460** | 6.51E-05 | 0.49% | 98.90% | | 3.75E-04 | 7.28% | 2.85% | 90.65% | 123 |
| **480** | 5.64E-05 | 0.43% | 99.02% | | 3.38E-04 | 6.58% | 2.57% | 91.36% | 124 |
| **500** | 4.90E-05 | 0.37% | 99.12% | | 3.07E-04 | 5.96% | 2.33% | 92.00% | 125 |
| **550** | 3.51E-05 | 0.27% | 99.33% | | 2.42E-04 | 4.70% | 1.83% | 93.37% | 126 |
| **600** | 2.57E-05 | 0.20% | 99.48% | | 1.93E-04 | 3.75% | 1.46% | 94.45% | 127 |
| **650** | 1.91E-05 | 0.15% | 99.59% | | 1.55E-04 | 3.02% | 1.18% | 95.31% | 128 |
| **720** | 1.30E-05 | 0.10% | 99.70% | | 1.17E-04 | 2.27% | 0.89% | 96.26% | 129 |
| **750** | 1.11E-05 | 0.08% | 99.74% | | 1.04E-04 | 2.02% | 0.79% | 96.59% | 130 |
| **800** | 8.62E-06 | 0.07% | 99.79% | | 8.62E-05 | 1.68% | 0.65% | 97.06% | 131 |
| **850** | 6.77E-06 | 0.05% | 99.83% | | 7.19E-05 | 1.40% | 0.55% | 97.46% | 132 |
| **900** | 5.37E-06 | 0.04% | 99.86% | | 6.04E-05 | 1.18% | 0.46% | 97.79% | 133 |
| **950** | 4.30E-06 | 0.03% | 99.88% | | 5.11E-05 | 0.99% | 0.39% | 98.07% | 134 |
| **1,000** | 3.48E-06 | 0.03% | 99.90% | | 4.35E-05 | 0.85% | 0.33% | 98.30% | 135 |
| **1,100** | 2.32E-06 | 0.02% | 99.93% | | 3.19E-05 | 0.62% | 0.24% | 98.67% | 136 |
| **1,200** | 1.59E-06 | 0.01% | 99.95% | | 2.39E-05 | 0.46% | 0.18% | 98.95% | 137 |
| **1,500** | 5.83E-07 | 0.00% | 99.98% | | 1.09E-05 | 0.21% | 0.08% | 99.44% | 138 |
| **2,000** | 1.48E-07 | 0.00% | 99.99% | | 3.70E-06 | 0.07% | 0.03% | 99.77% | 139 |
| **2,500** | 4.80E-08 | 0.00% | 100.00% | | 1.50E-06 | 0.03% | 0.01% | 99.89% | 140 |
| **3,000** | 1.84E-08 | 0.00% | 100.00% | | 6.90E-07 | 0.01% | 0.01% | 99.94% | 141 |
| **4,000** | 3.77E-09 | 0.00% | 100.00% | | 1.89E-07 | 0.00% | 0.00% | 99.98% | 142 |
| **5,000** | 1.04E-09 | 0.00% | 100.00% | | 6.49E-08 | 0.00% | 0.00% | 99.99% | 143 |
| **7,000** | 1.34E-10 | 0.00% | 100.00% | | 1.18E-08 | 0.00% | 0.00% | 100.00% | 144 |
| **10,000** | 1.35E-11 | 0.00% | 100.00% | | 1.68E-09 | 0.00% | 0.00% | 100.00% | 145 |

**DEFINITIONS**        **Study these closely before answering any questions.**

**Table 1 on Page 1: Sorted by Percentile: Percentage of subjects below stated income.**

| Cell | ID | Definition |
|------|-----|------------|
| A33 | Pct# | X: The percentile of subjects by income |
| B33 | #Cutoff$ | The income needed for a subject to be at the Xth percentile by income |
| C33 | %$cdf | The percentage of total income earned by subjects having an income **below** the cutoff |
| E33 | %#down | Y: The percentage of subjects having incomes **above** the cutoff. |
| F33 | %$down | Percentage of total income earned by the **top** Y percentage of subjects by income |
| G33 | %$/%# | TimesEqualShare: **Top-down** ratio of cumulative pctg of income to cumul. pctg. of subjects |
| H33 | Ave$ | The average income of subjects in the **top Y** percentage of subjects by income |
| I34 | /Cutoff$ | Tatio of average income above the Xth percentile to the cutoff income at the Xth percentile. |

**Table 2 on Page 2: Sorted by Income**

| | | |
|------|-----|------------|
| A62 | Income | X: The income (in $1,000) |
| D62 | CDF# | The percentage of **subjects** that have incomes **BELOW** X |
| I62 | CDF$ | The percentage of **total income** that is earned by subjects having incomes **BELOW** X |

**PRACTICE QUESTIONS:**

**Table 1 (Pg 1) is sorted by percentages (bottom up & top down); Table 2 (Pg 2) is sorted by Incomes**

| Q | Answer | Question: If Median = 50k and Mean = 80k, |
|---|--------|---------------------------------------------|
| 1 | 96.2K | what is the minimum income ($) needed to be in the top 25% of income earners (#)? |
|   | Cell B41 | % of subjects =>Table 1. Top 25% = Bottom 75% => Row 41.  Min Income Col B. |
| 2 | 16.6% | what percentage of income ($) is earned by the bottom 50% of income earners (#)? |
|   | Cell C38 | % of subjects =>Table 1. Bottom 50% => Row 38.  Pctg of total income =Col C. |
| 3 | 25.0% | what percentage of income ($) is earned by the top 5% of income earners? |
|   | Cell F45. | % of subjects =>Table 1.  Top 5% => Top Down Row 45.  Pctg of total income = Col F. |
| 4 | 5.0 | The top 5% of subjects have ____ times their equal share of total income |
|   | Cell G45 | % of subjects => Table 1.  Top 5% => Top Down Row 45.  Times their equal share=> Col G |
| 5 | 68.6% | what percentage of subjects (#) have income less than 80K |
|   | Cell D92 | Less than $ => Table 2. 80K => Row 92. Pctg of subjects below % => Col D. |
| 6 | 31.4% | what percentage of income ($) is earned by subjects (#)  making **less** than 80k |
|   | Cell I92 | Less than $ => Table 2. 80K => Row 92. Pctg of Income below => Col I |
| 7 | 68.6% | what percentage of income ($) is earned by subjects (#)  making **more** than 80k |
|   |  | arithmetic  100% - 31.4% |

## Distribution of Households by Income;
## Distribution of Total Income by Amount

**Households by Income**
**Median: $50K,**
**Mean=$80K,**
**Mode=$19.5K**

**Distribution of Total Income by**
**Amount of Income**
**Mode: $50K**
**Median: $128K**
Ave $205K

Percentage of Maximum

Income ($1,000)

Log Normal Distribution of Households by Income           Income/House: Mean=80K; Median=50K

## Distribution of Total Income
## by Income per Household

Mode: 50K

Median:
128K

Cumulative Distribution Function (CDF):
Percentage of Total Income below amount

Probability Distribution Function (PDF):
as a percentage of the Modal PDF

Unit Incomes ($1,000)

LogNormal Dist of Units by Income           Median=50K; Mean=80K

## Pctg of Income vs. Pctg. of Households

**Top 50% (above $50k): 83% of total Income**
**Top 10% (above $175k: 38% of total Income**
**Top 1% (above $475k): 8.7% of total Income**
**Top 0.1% (above $1M): 1.7% of total Income**

Perfect Equality

**Gini Coefficient: 0.507**
**Bigger means more unequal**

Percentage of Income

Percentage of Households

Log Normal Distribution of Households by Income

Income/House: Mean=80K; Median=50K

## Cumulative Distribution of Total Income and of Households by Income

Mean Income per household: 80K

Percentage of HH with Incomes below this Amount

68.61%

31.39%

Percentage of Total Income held by HH having Incomes below this Amount

Unit Incomes ($1,000)

LogNormal Dist of Units by Income

Median=50K; Mean=80K