A. INPUT

Generate sum of K identical log-normal distributions

K=# losses	**1,600**	Manual entry
Median/loss	**50**	Manual entry
Mean/loss	**100**	Manual entry
Percentile	**97.5%**	Manual entry

Std. Dev.P	173	=SQRT((EXP(C19)-1)*(EXP(2*C17+C19)))
Skew.P	0.87	=3*(C6-C5)/C9
Coef Variation	1.73	=C9/C6
Mode.P	12.5	=EXP(C17-C19)

B. COMPUTATIONS AND CELL FORMULA

Underlying Normal for the individual log-normal

mu	3.912	=LN(C5)
mu+S^2/2	4.605	=LN(C6)
Sigma^2	1.386	=2*(C18-C17)
Sigma	1.177	=SQRT(C19)

Generate the sum of K independent log-normal distributions:
Use the Fenton-Wilkinson approximation
Invalid if Mean > 2.7 * Median

Distribution of Total Losses (#) is approximately Log-Normal

Underlying Normal for Distribution of Total Losses

SIGMA^2	0.002	=LN(1+(EXP(C19)-1)/C4)
SIGMA	0.043	=SQRT(C29)
MU	11.982	=LN(C4*EXP(C17))+(C19-C29)/2
MU+S^2 / 2	11.983	=C31+C29/2

Note
MEAN = K*Mean
As K increases, MEDIAN approaches the MEAN
For K=1: SIGMA = sigma, MEDIAN=median, MEAN=mean

Cell Formula for Output

H4 =EXP(C31)/C4
H5 =EXP(C32)/C4
H6 =EXP(C31-C29)/C4
H7 =SQRT((EXP(C29)-1)*(EXP(2*C31+C29)))/C4

H12 =LOGNORM.INV(C7,C$31,C$30)/C4

H17 =H$5*G17
I17 =LOGNORM.DIST(H17*C$4,C$31,C$30,1)
Pull these two down

I2 =IF(C19<2, "Valid", "*INVALID*")

C. OUTPUT Valid
Dist of Ave Losses for Log-Normal Sum

MEDIAN	**99.9**	
MEAN	**100.0**	
MODE	**99.72**	
STDEV	**4.33**	
SKEW3	**0.065**	=3*(H5-H4)/H7
C. VAR	**0.043**	=H7/H5

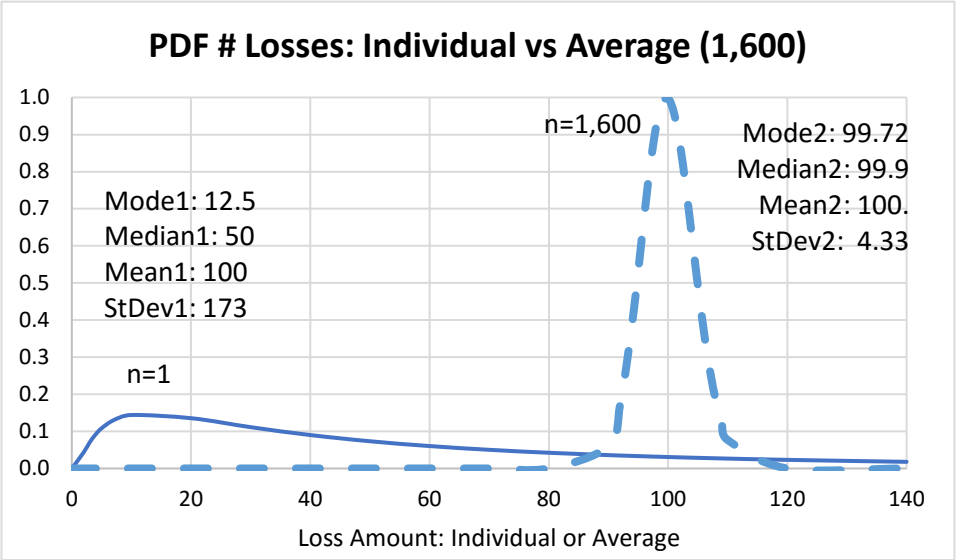
Cumulative Losses at: 97.5%

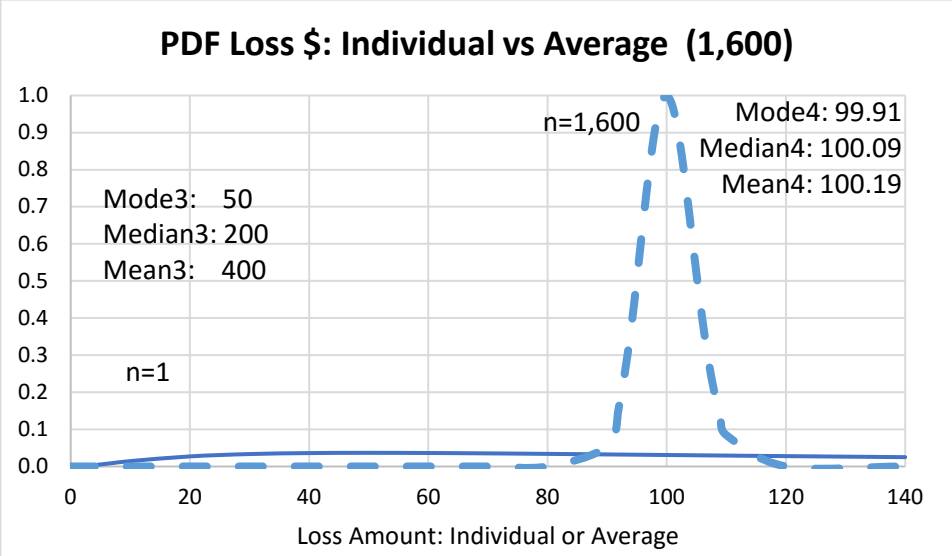
Amount	108.75	
Ratio	1.09	=H12/H5
Z	2.021	=(H12-H5)/H7

Distribution of Average Losses

Scale	Loss$	CDF
0.0001	0.0	0.000
0.02	2.0	0.000
0.03	3.0	0.000
0.04	4.0	0.000
0.05	5.0	0.000
0.07	7.0	0.000
0.1	10.0	0.000
0.2	20.0	0.000
0.3	30.0	0.000
0.4	40.0	0.000
0.5	50.0	0.000
0.6	60.0	0.000
0.7	70.0	0.000
0.8	80.0	0.000
0.9	90.0	0.008
0.92	92.0	0.028
0.94	94.0	0.080
0.96	96.0	0.178
0.97	97.0	0.248
0.98	98.0	0.328
0.99	99.0	0.417
1	100.0	0.509
1.01	101.0	0.599
1.02	102.0	0.684
1.03	103.0	0.759
1.04	104.0	0.823
1.05	105.0	0.875
1.06	106.0	0.914
1.07	107.0	0.944
1.08	108.0	0.964
1.09	109.0	0.978
1.1	110.0	0.987
1.2	120.0	1.000
1.4	140.0	1.000

Source: http://leo.ugr.es/pgm2012/submissions/pgm2012_submission_6.pdf


PDF # Losses: Individual vs Average (1,600)
n=1,600
Mode2: 99.72
Median2: 99.9
Mean2: 100.
StDev2: 4.33
Mode1: 12.5
Median1: 50
Mean1: 100
StDev1: 173
n=1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
20
40
60
80
100
120
140
Loss Amount: Individual or Average


PDF Loss $: Individual vs Average (1,600)
n=1,600
Mode4: 99.91
Median4: 100.09
Mean4: 100.19
Mode3: 50
Median3: 200
Mean3: 400
n=1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
20
40
60
80
100
120
140
Loss Amount: Individual or Average

NOTE: Distribution of average losses (#) is different from distribution of total loss ($)

			Distribution of Total Loss ($)	
PDF of Mode1 for the distribution of individual lo	Mu3	5.298	=C17+C19	
0.0136 =LOGNORM.DIST(C12,C17,C20,0)	Sigma3	1.177	=C20	
	Mode3	50.00	=EXP(R4-R5^2)	
PDF of Mode2 for the distribution of average los	PdfMode3	0.003	=LOGNORM.DIST(R6,R4,R5,0)	
0.0923 =LOGNORM.DIST(H6*C$4,C$31,C$3	Mu4	11.984	=C31+C29	
	Sigma4	0.043	=C30	
	Mode4	100	=EXP(R8-R9^2)/C4	
	PDFMode4	0.092	=LOGNORM.DIST(R10*C4,R8,R9,0)*C4	

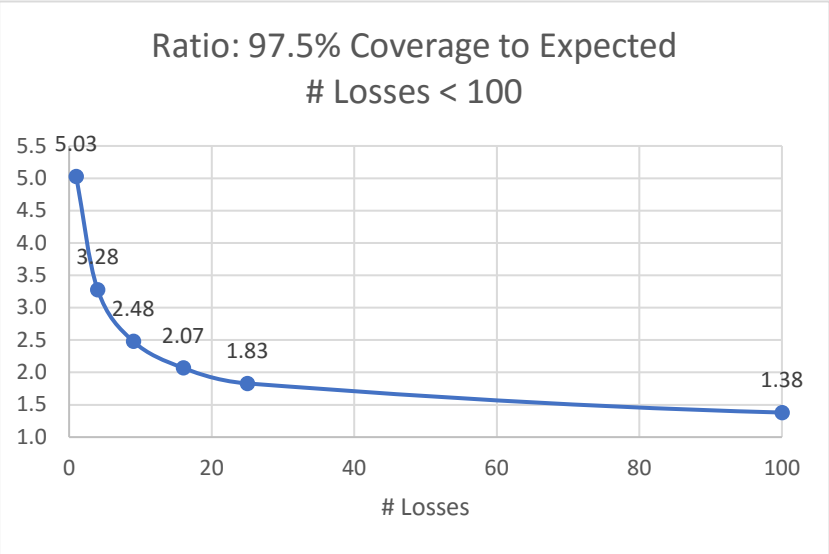
Cell	Formula for Output
L17	=LOGNORM.DIST(H17,C$17,C$20,0)/MAX(L$8,L$5)
M17	=LOGNORM.DIST(H17*C$4,C$31,C$30,0)*C$4/MAX(L$8, L$5)
N17	=LOGNORM.DIST(H17,R$4,R$5,0)/MAX(R$7,R$11)

pdf1/max	pdf2/max	pdf3/max	pdf4/max
0.000	0.00	0.000	0.000
0.044	0.00	0.001	0.000
0.070	0.00	0.002	0.000
0.092	0.00	0.004	0.000
0.108	0.00	0.005	0.000
0.130	0.00	0.009	0.000
0.144	0.00	0.014	0.000
0.136	0.00	0.027	0.000
0.111	0.00	0.033	0.000
0.090	0.00	0.036	0.000
0.073	0.00	0.037	0.000
0.060	0.00	0.036	0.000
0.050	0.00	0.035	0.000
0.042	0.00	0.034	0.000
0.036	0.06	0.032	0.054
0.035	0.18	0.032	0.163
0.034	0.39	0.032	0.371
0.033	0.68	0.032	0.654
0.032	0.82	0.031	0.792
0.032	0.92	0.031	0.906
0.031	0.99	0.031	0.978
0.031	1.00	0.031	1.000
0.030	0.96	0.031	0.969
0.030	0.87	0.031	0.892
0.030	0.76	0.030	0.780
0.029	0.62	0.030	0.650
0.029	0.49	0.030	0.517
0.028	0.37	0.030	0.392
0.028	0.27	0.030	0.285
0.027	0.18	0.030	0.198
0.027	0.12	0.030	0.132
0.027	0.08	0.029	0.084
0.023	0.00	0.028	0.000
0.018	0.00	0.025	0.000

	1	
Mode3	50.0	=R6
Median3	200	=EXP(R4)
Mean3	400	=EXP(R4+R5^2/2)
%Vm3	72.20%	=LOGNORM.DIST(Q19,R$4,R$5,1)
%Vm1	27.80%	=LOGNORM.DIST(C6,R$4,R$5,1)
%Cover	97.5%	=I11
Cover$	2,010	=LOGNORM.INV(Q22,R4,R5)/1
Xsloss$	$1,910	=Q23-C6
XS/Exp	19.10	=Q24/C6

N	1,600	100	25	9	4
Mode4	99.91	98.53	94.49	86.60	75.59
Median4	100.09	101.49	105.83	115.47	132.29
Mean4	100.19	103.00	112.00	133.33	175.00
%Vm4	50.86%	53.43%	56.68%	60.57%	64.58%
StDev4					
%Vm1	49.14%	46.57%	43.32%	39.43%	35.42%
%Cover	97.5%	97.5%	97.5%	97.5%	97.5%
Cover$	108.96	142.16	204.72	330.38	573.17
Xsloss$	$8.96	$42.16	$104.72	$230.38	$473.17
XS/Exp	0.09	0.42	1.05	2.30	4.73

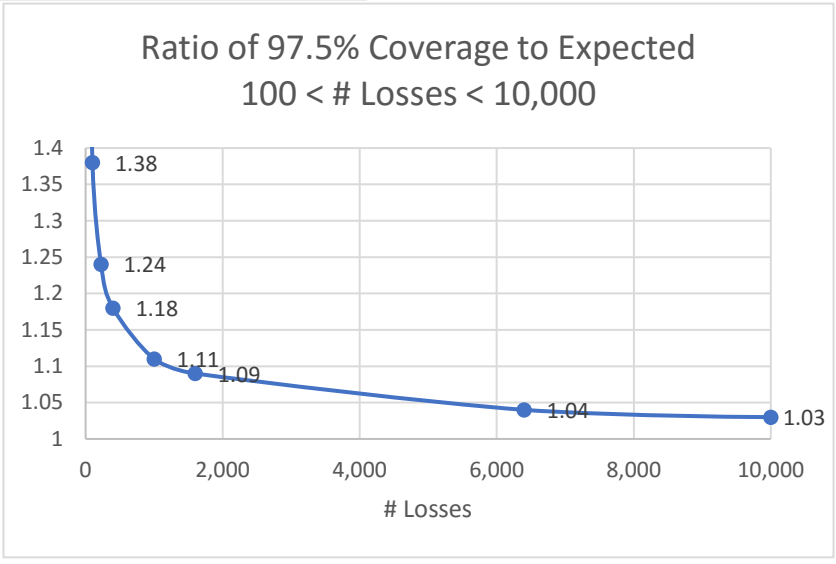
U	V	W	X	Y	Z	AA	AB	AC	
Distribution of Average Losses			97.5%	Percentile Coverage			On Average		2
K=# Losses	Skew3	Coef.Var	Ratio	Z(cover/Expect)		SDev/sq(k)	SE	Ratio/SE	3
1	0.866	1.732	5.03	2.33		173	173	2.9	4
4	0.845	0.866	3.28	**2.63**		87	87	3.8	5
9	0.696	0.577	2.48	2.56		58	58	4.3	6
16	0.570	0.433	2.07	2.47		43	43	4.8	7
25	0.477	0.346	1.83	2.39		35	35	5.3	8
100	0.254	0.173	1.38	2.20		17	17	8.0	9
225	0.171	0.115	1.24	2.12		12	12	10.8	10
400	0.129	0.087	1.18	2.08		8.7	8.7	13.6	11
1,000	0.082	0.055	1.11	2.04		5.5	5.5	20.2	12
1,600	0.065	0.043	1.09	2.02		4.3	4	25.3	13
6,400	0.032	0.022	1.04	1.99		2.2	2.2	47.3	14
10,000	0.026	0.017	1.03	1.985		1.7	1.7	60.6	15
40,000	0.013	0.009	1.02	1.972		0.9	0.9	113.3	16
100,000	0.008	0.005	1.01	1.968		0.5	0.5	202.0	17
=C4	=H8	=H9	=H13	=H14					



Cell	Formula
AB4	=C$9/SQRT(V17)
AC4	=X17*C$6
AD4	=Y17*C$6/AC17

Pull all three down

Skew=Pearson2 skew



AE	AF	AG	AH	AI	AJ	AK	AL

APPENDIX: 34

If SIGMA = sigma, then
Exp(sigma^2) = 1 + [Exp(sigma^2) -1]/L
L = [Exp(sigma^2) - 1] / [Exp(sigma^2) - 1]
L = 1

Table 1 **Standard Error (for n losses) given the Standard Deviation for a single loss**

		Number of Identical Distributions					
	Std. Error	1	10	100	1,000	10,000	100,000
SIGMA	0.1	0.100	0.032	0.010	0.003	0.001	0.000
	1	1.000	0.398	0.131	0.041	0.013	0.004
	2	2.000	1.360	0.655	0.228	0.073	0.023
	3	3.000	2.588	2.099	1.486	0.770	0.279
	4	4.000	3.701	3.376	3.015	2.606	2.121
	5	5.000	4.764	4.516	4.253	3.974	3.672
	10	10.000	9.884	9.767	9.648	9.528	9.407

AH13 =SQRT(LN(1+(EXP($AG13^2)-1)/AH$12))

Table 2 **Mu for the sum of n losses given mu for a single loss**

Let MUx = Ln(L * exp(mu))

		Number of Identical Distributions					
	MUx	1	10	100	1,000	10,000	100,000
Mu	0.1	0.1	2.4	4.7	7.0	9.3	11.6
	1	1.0	3.3	5.6	7.9	10.2	12.5
	2	2.0	4.3	6.6	8.9	11.2	13.5
	3	3.0	5.3	7.6	9.9	12.2	14.5
	4	4.0	6.3	8.6	10.9	13.2	15.5
	5	5.0	7.3	9.6	11.9	14.2	16.5
	10	10.0	12.3	14.6	16.9	19.2	21.5

AH26 =LN(AH$25*EXP($AG26))

Background: Finding the best distribution to fit the claims data
www.actuaries.org/EVENTS/Congresses/Cape_Town/Papers/Non-Life%20Insurance%20%28ASTIN%29/22_final%20paper_Oyugi.pdf

Theoretically justified choice of distribution
https://www.emis.de/journals/HOA/JAM/Volume2012/838397.pdf
Low skew normal
https://arxiv.org/ftp/arxiv/papers/1502/1502.03619.pdf