(handout slides uploaded to conference app/website)

"Helping English Language Learners Navigate Probability Vocabulary and Concepts"

Amy Wagler & Larry Lesser
The University of Texas at El Paso

75-minute breakout based largely on our paper (with Berenice Salazar) in the November 2016 *Statistics Education Research Journal*

Definition of ELL

students who experience "enough limitations that he or she cannot fully participate in mainstream English instruction" (Goldenberg, 2008, p. 10), which includes those beginning to learn English who could benefit from language support and those who are proficient in English but may need additional assistance in social or academic situations (Hoffstetter, 2003).

our work on language/ELLs in statistics

http://www.math.utep.edu/Faculty/lesser/ELL.html

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- Nov. 2016 *SERJ*: case study of ELLs using bilingual probability applet

importance/rationale for topic

- Importance of language
- Spanish is the second-most spoken language in the world and is by far the most common language of ELLs in US
- ELL-friendly teaching practices can help *all* students

Question for group discussion

What are some examples of how language can be challenging for any student learning probability?

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versus

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Language in probability: specifying sets of discrete events

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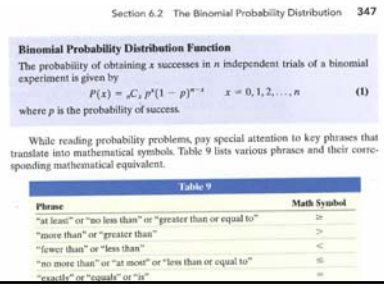

Section 6.2 The Binomial Probability Distribution 347

Binomial Probability Distribution Function

The probability of obtaining x successes in n independent trials of a binomial experiment is given by

$$P(x) = {}_nC_x\, p^x (1-p)^{n-x} \qquad x = 0, 1, 2, \ldots, n \qquad (1)$$

where p is the probability of success.

While reading probability problems, pay special attention to key phrases that translate into mathematical symbols. Table 9 lists various phrases and their corresponding mathematical equivalent.

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Language in probability: lexical ambiguity (e.g., the word *random*)

Kaplan, Rogness, Fisher (2014)

Figure 1a. Random Zebras (Colloquial).

Figure 1b. Random Hat (Statistical).

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college ELL interview excerpt (from Lesser & Winsor, 2009):

M: The second event is 'quarter lands on tails.'

S2: What is *tails* on the quarter?

[Mexican coins: seal (or sun) and eagle;
other Latin America: cara[face] y cruz[cross], shield, crown]

tossing (asymmetric) moon blocks

(can disrupt equiprobability bias)

- each crescent-shaped block has flat(*yang*) and curved(*yin*) sides
- used in China, Hong Kong, Taiwan, etc. to indicate - (2 yins) or + (1 of each) fortune
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Specific Context: Coin Flipping

- Real-world: decided some precinct delegates in Iowa political caucuses, opening NFL kickoffs, etc.
- Statistics: flips are Bernoulli trials, have the simplest equiprobable sample space, are a benchmark for randomness, etc.
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Coin-flipping can illustrate probability misconceptions

misconception	Example: A person believes….
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multilingual probability resources

- Terms in 29+ languages at http://isi.cbs.nl/glossary
- Multilingual collections of applets (e.g., NLVM or Shodor)

Google the words NLVM coin tossing applet

explore the NLVM coin tossing applet

in English, Spanish, French, or Chinese!

questions adapted from protocol

- What is the longest run in this sequence?

H T T H H T H H H T T H

- For a 100-flip sequence, how long do you think the longest run will be?

Research Questions

- What is the nature of how Spanish-speaking ELLs use a bilingual applet when learning probability?
- When does it appear that language plays a factor when Spanish-speaking ELLs explore probability with the applet?

timeline

STEP	DATES
Study design, IRB process	Oct. 2011 – Feb. 2012
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Mock Interviews

- In the next slide, a set of three questions from the protocol are provided
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  - Choose roles (interviewer, ELL interviewee, non-ELL interviewee, recorder)
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Mock Interviews

En tus propias palabras, ¿Que significa la "mayor racha" o el mayor número de caras sucesivas?

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En la secuencia A, ¿Cuál es la mayor racha o el mayor número de caras sucesivas?
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Para una secuencia de 100-lanzamientos, ¿Qué tan larga crees que sea la racha más larga de número de caras o escudos?

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Debrief

- Points of consensus:
- Questions that arose:
- Themes:

from Lesser, Wagler, & Salazar (2016)

B: in your own words…what does 'longest run' mean to you?

P1: …the more, the most, hmmmm, the fastest to flip the coin, like [short pause] many times but so fast [nervous laugh]

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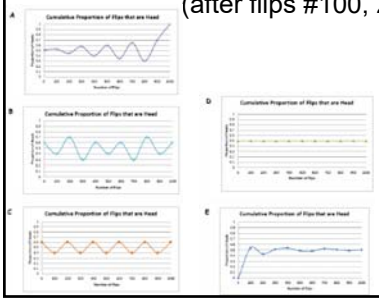
Sketch a plausible graph of the cumulative proportion of flips that are heads

(after flips #100, 200, 300,….,etc.)



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THIS MORNING'S VOCABULARY QUIZ

Multiple meanings of run in statistics

- Difference of *x*-coordinates (e.g., slope is "rise over run")
- A sequence of at least 2 consecutive identical outcomes (e.g., "what is the longest run of heads?")
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- Run the [experiment/simulation/program]

Multiple meanings of run in statistics

What is a run in a designed experiment?

A run is an experimental condition or factor level combination at which responses are measured. Usually, each run corresponds to a row in the worksheet and results in one or more response measurements, or observations. For example, you do a full factorial design with two factors, each with two levels. Your experiment has four runs:

Run	Factor 1	Factor 2	Response
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NOTE
When doing an experiment, the run order should be randomized.

Each run corresponds to a design point, and the entire set of runs is the design. Multiple executions of the same experimental conditions are considered separate runs and are called replicates.

Language recommendations (using Spanish as a resource!)

- replace "in the long run" by "in the long term" (*en el largo plaza*); Sullivan (2010) uses "long-term proportion"
- replace "longest run of heads" by "largest number of successive [consecutive] heads" (*el mayor número de caras sucesivas*)

Also,

- replace "face of the coin" by "side of the coin" (to avoid confusion with *cara*[heads])

Another reason to distinguish similar-sounding phrases

- "long run" and "longest run"
- Mean, median, mode: Lesser & Winsor (2009) & CLASS survey

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What are your questions …or suggestions?

- Our ELL work and URLs of the resources: www.math.utep.edu/Faculty/lesser/ELL.html
- Contact us:

Lesser@utep.edu or awagler2@utep.edu

Thank You
Mahalo
Kiitos
Tack
Toda
Grazie
Thanks
Obrigado
Takk
Merci
Gracias

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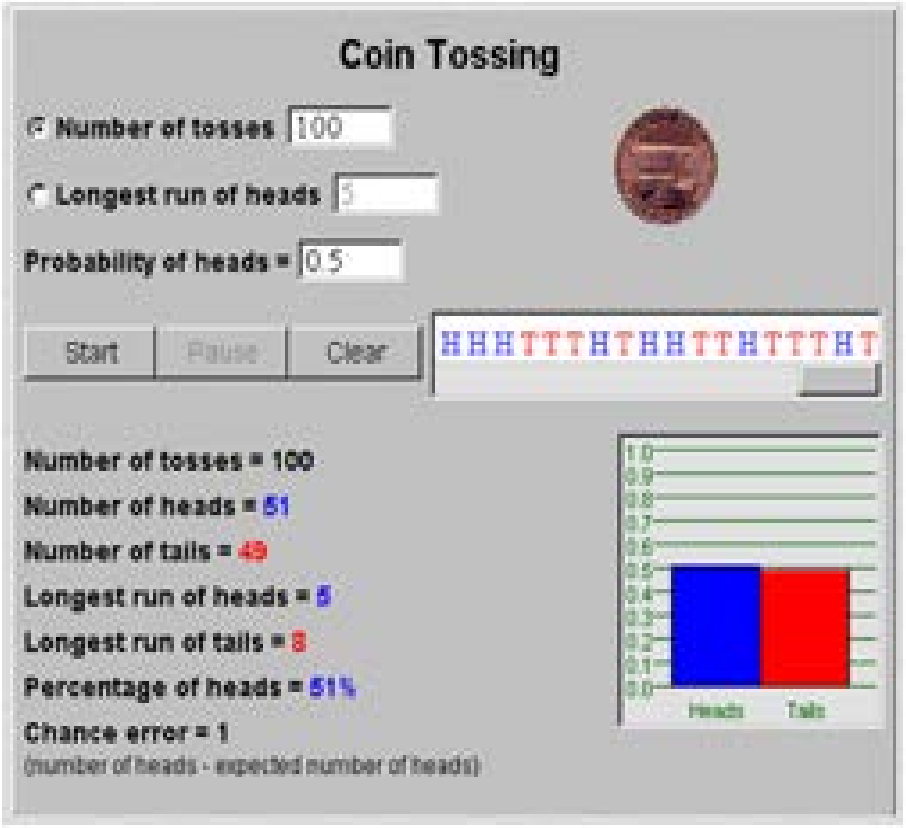

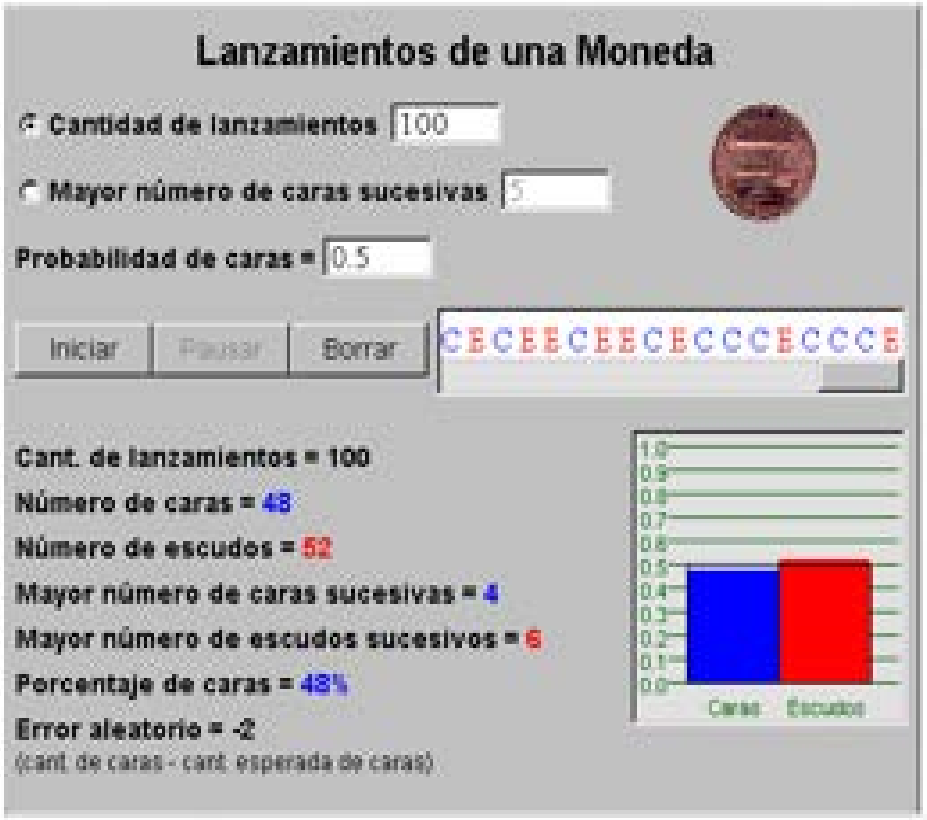

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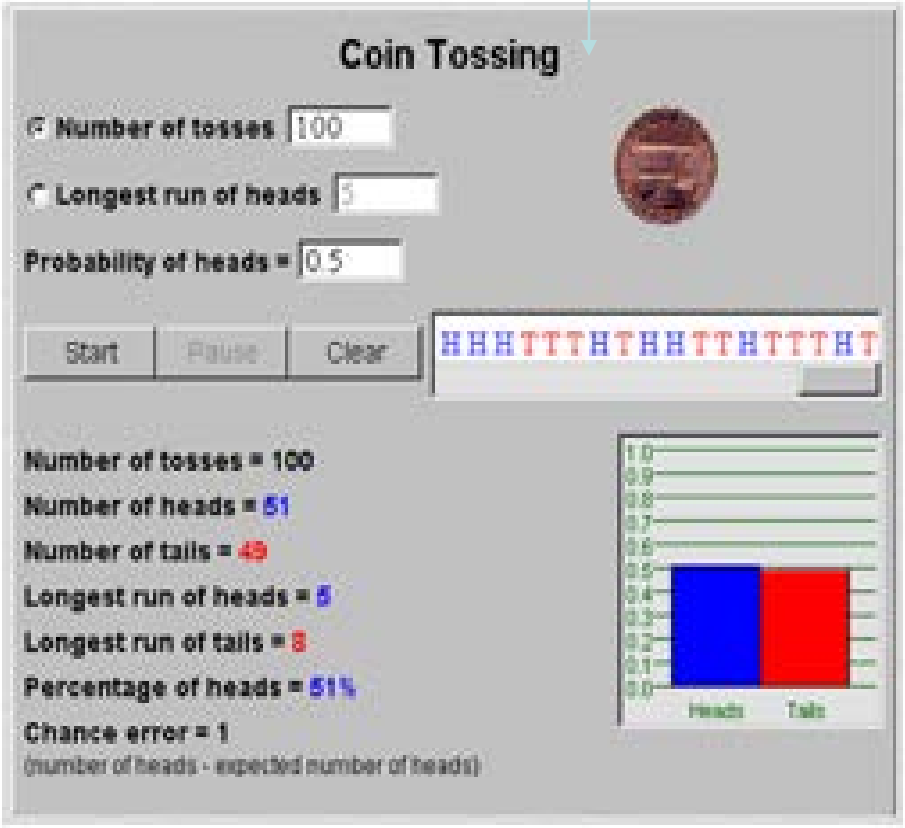

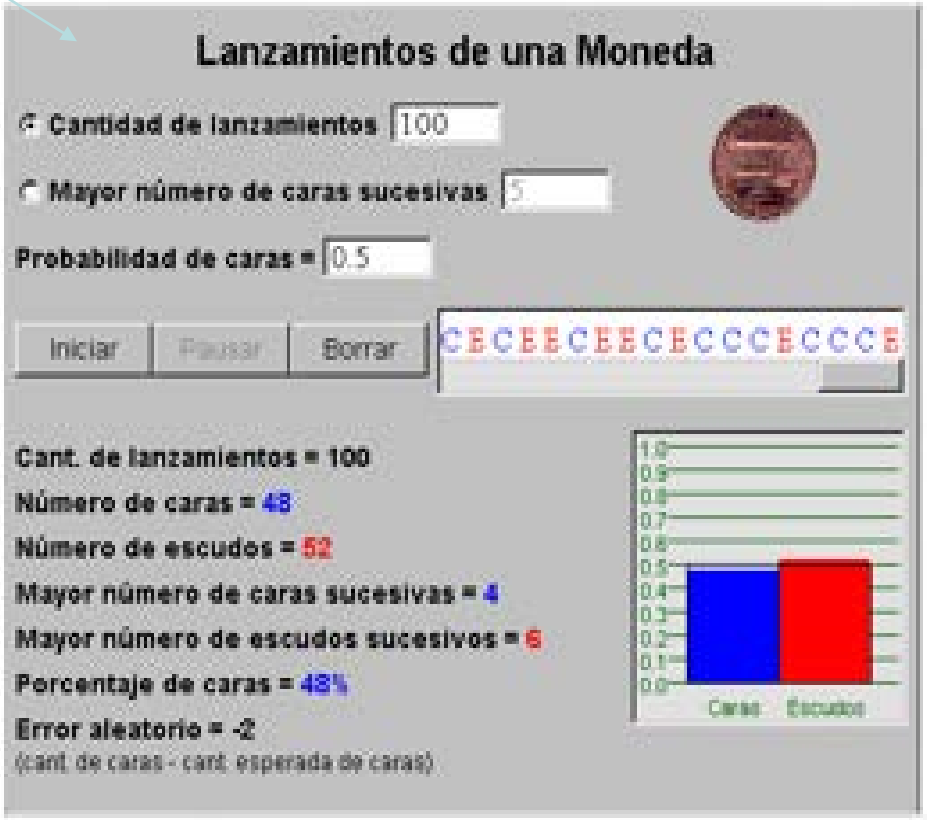

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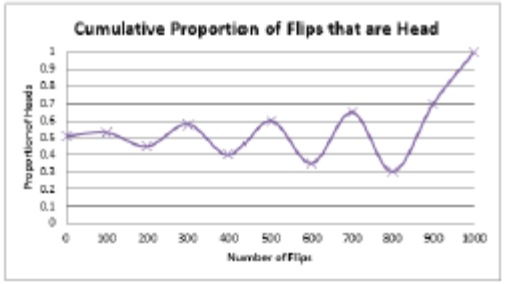

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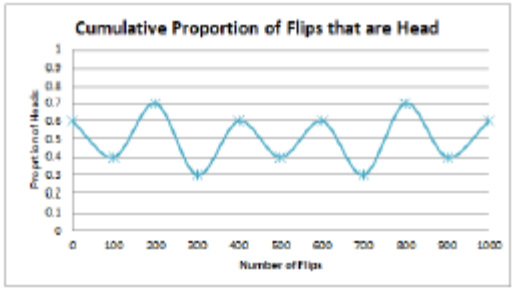

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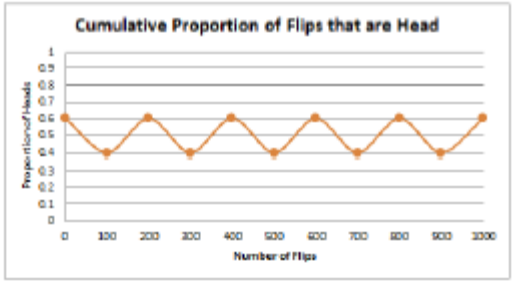

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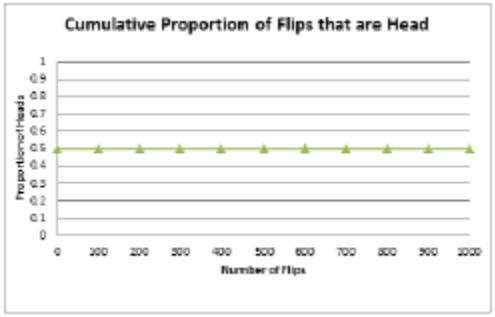

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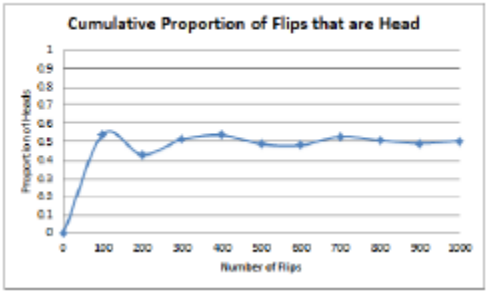

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www.nytimes.com/2011/05/29/opinion/29winchester.html

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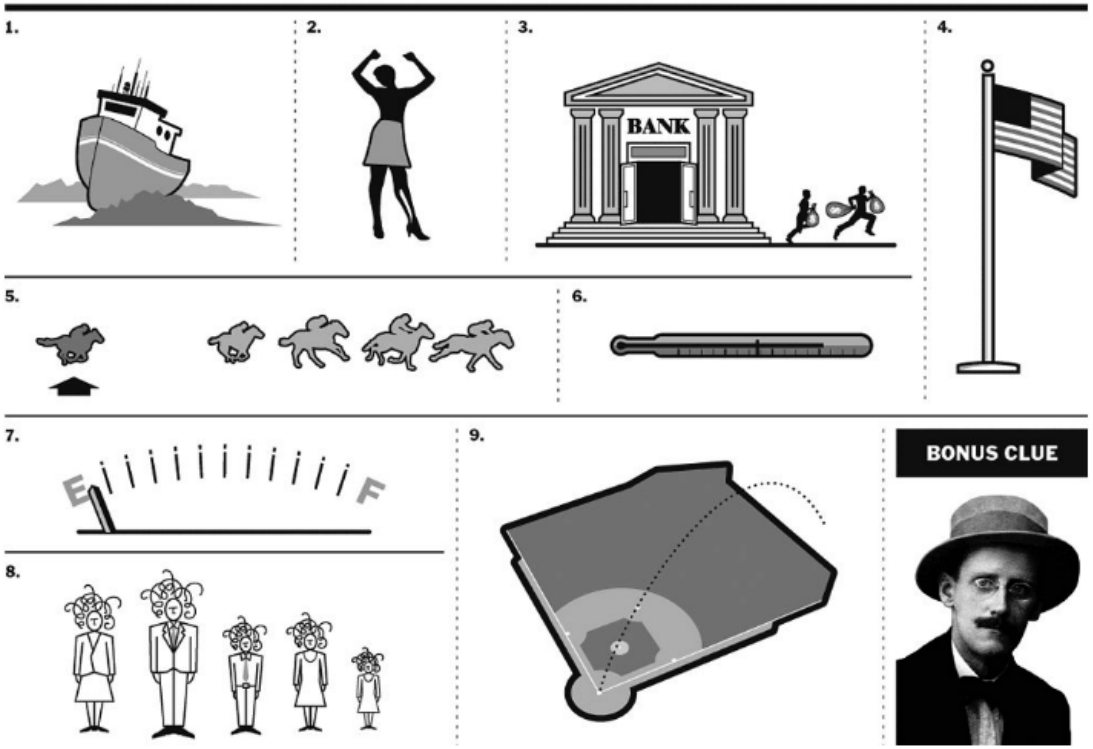

ANSWERS: 1. Run aground. **2.** A run in her stocking. **3.** Take the money and run. 4. Run it up the flagpole. 5. Also-ran. **6.** Run a fever. **7.** Running on empty. **8.** Runs in the family. **9.** Home run. **BONUS:** "Along the riverrun" from "Finnegans Wake" by James Joyce. Sam Potts

Multiple meanings of run in statistics

- Difference of *x*-coordinates

  (e.g., slope is “rise over run”)
- A sequence of at least 2 consecutive identical outcomes (e.g., “what is the longest run of heads?”)
- In the long run
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Learn more about Minitab 17

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> **NOTE**
> When doing an experiment, the run order should be randomized.

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Language recommendations (using Spanish as a resource!)

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- replace “longest run of heads” by “largest number of successive [consecutive] heads” (*el mayor número de caras sucesivas*)

Also,

- replace “face of the coin” by “side of the coin” (to avoid confusion with *cara*[heads])

Another reason to distinguish similar-sounding phrases

- “long run” and “longest run”
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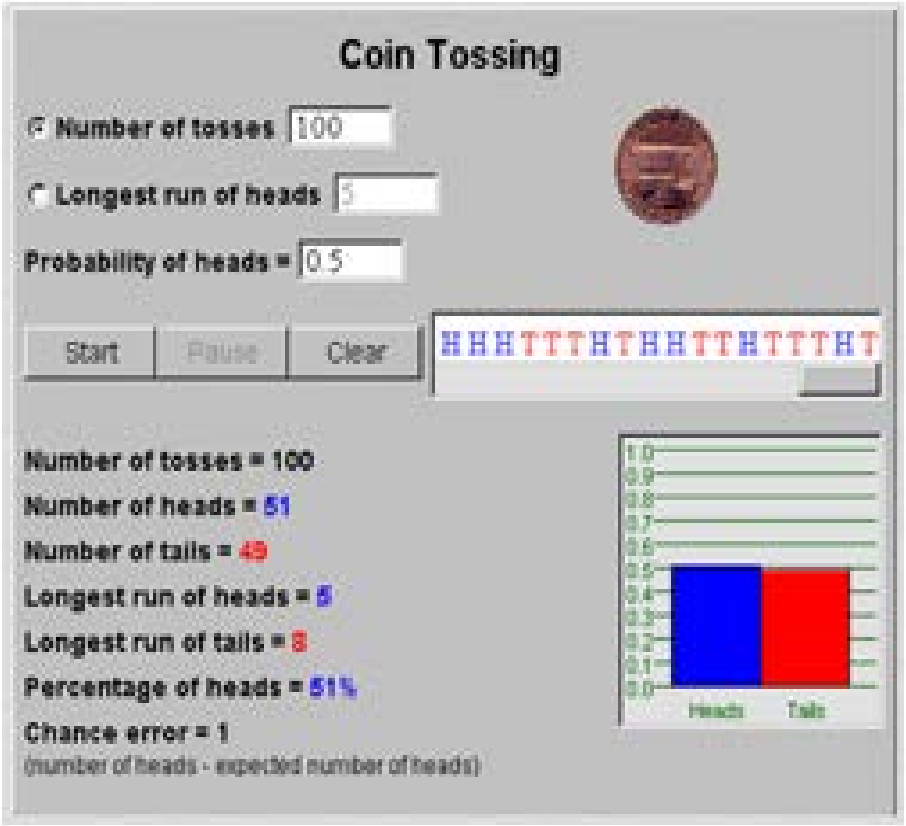
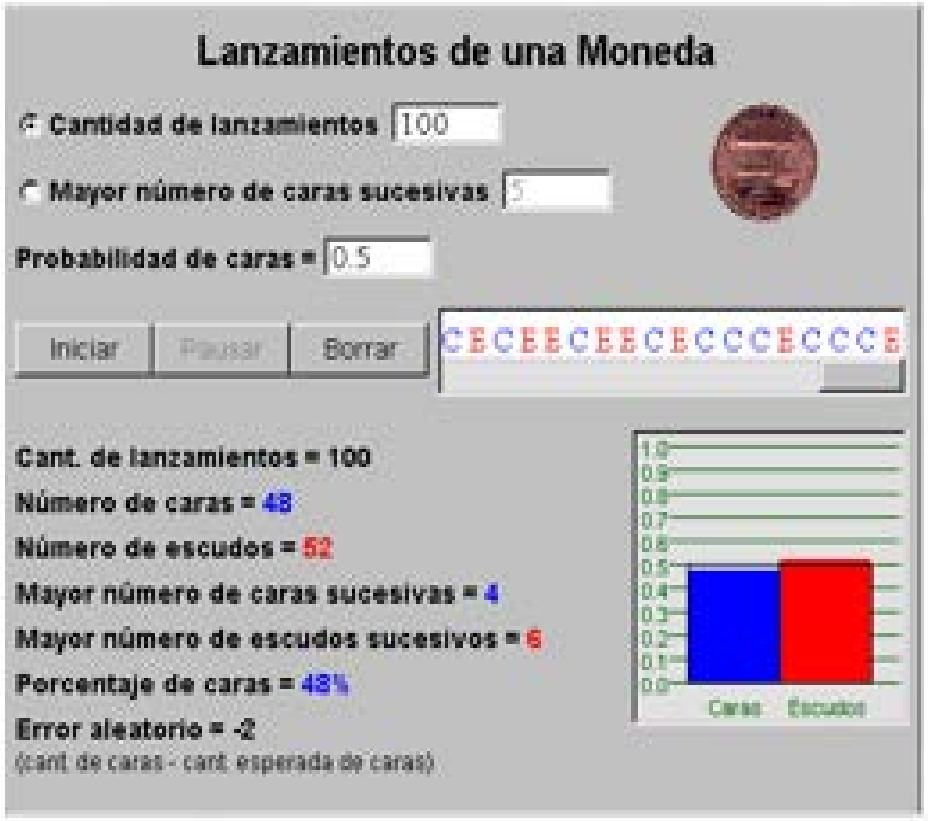
Pedagogical discussion

- Visuals (e.g., the sequence of flips, the bar chart of flips accumulated) help!





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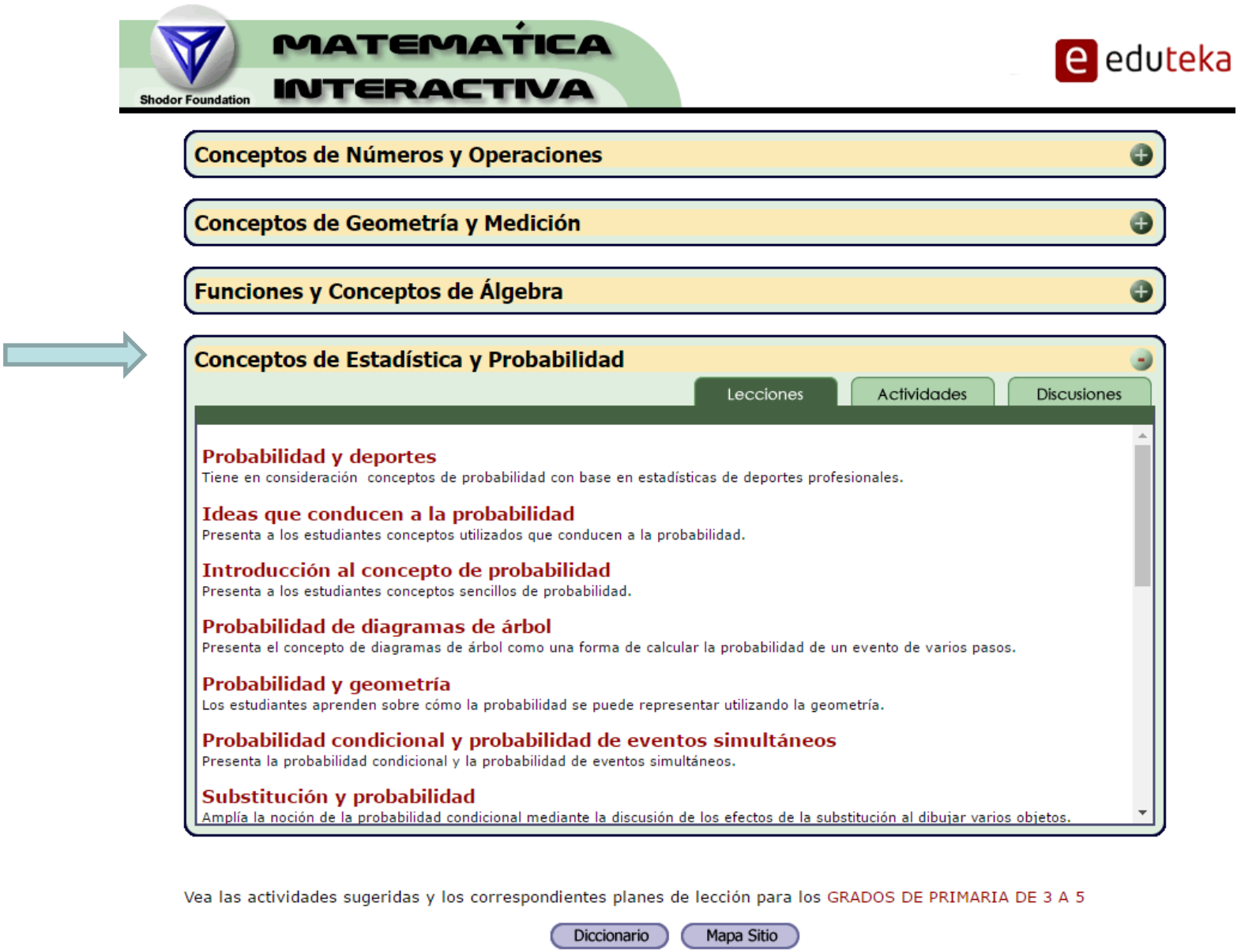

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Language	Description
English	arithmetic mean ; arithmetic average ; mean ; average
French	moyenne arithmétique ; moyenne ; valeur moyenne ; valor medio
German	arithmetisches Mittel ; arithmetischer Mittelwert ; Durchschnitt ; Mittelwert ; Mittel
Dutch	rekenkundig gemiddelde ; gemiddelde
Italian	media aritmetica ; media ; valore medio
Spanish	media aritmética ; promedio aritmético ; promedio ; media
Catalan	mitjana aritmètica, valor mitjà ; mitjana
Portuguese	média aritmética ; média
Romanian	medie aritmetică
Danish	aritmetisk gennemsnit ; middelværdi ; gennemsnit ; middeltal
Norwegian	aritmetisk gjennomsnitt ; forventning ; aritmetisk middel ; gjennomsnitt
Swedish	aritmetiskt medelvärde
Greek	αριθμητικός μέσος
Finnish	aritmeettinen keskiarvo ; keskiarvo ; keskiluku ; keskiluku (epämäär.)
Hungarian	számtani átlag ; átlag ; középérték
Turkish	aritmetik ortalama ; ortalama
Estonian	aritmeetiline keskmine ; keskväärtus ; keskmine
Lithuanian	aritmetinis vidurkis ; vidutinė reikšmė ; vidurkis
Slovenian	aritmetična sredina ; enostavna aritmetična sredina ; srednja vrednost ; povprečje ; povprečna vrednost ; sredina
Polish	średnia arytmetyczna ; średnia ; przeciętna ; obliczać przeciętną ; obliczać średnią ; stanowić średnią ; wartość przeciętna ; środek (jako zasób)
Russian	Средняя арифметическая
Ukrainian	арифметичне середнє
Serbian	аритметичка средина ; аритметички просек ; средина ; просек
Icelandic	meðaltal ; beint meðaltal ; einfalt meðaltal ; samlagningarmeðaltal ; hreint meðaltal ; reikningslegt meðaltal ; meðalgildi ; væntanlegt gildi ; væntigildi ; vongildi
Euskara	batezbesteko aritmetiko ; batezbesteko balio ; batezbesteko
Farsi	miyangine hesabi ; mot v set ; miyangin
Persian-Farsi	میانگین حسابی ; میانگین ; متوسط
Arabic	الوسط الحسابي والمعدل الحسابي ، المتوسط ، المعدل
Afrikaans	rekenkundige gemiddelde ; gemiddelde
Chinese	算术平均（值）；等差中项
Korean	산술평균 ; 평균

What are your questions …or suggestions?

- Our ELL work and URLs of the resources: www.math.utep.edu/Faculty/lesser/ELL.html

- Contact us:

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Thank You
Mahalo
Kiitos
Tack
Toda
Grazie
Obrigado
Thanks
Takk
Merci
Gracias