Comments on Burrill's 2020 IASE Paper:

STATISTICAL LITERACY AND QUANTITATIVE REASONING: RETHINKING THE CURRICULUM

By Milo Schield

Abstract: This paper reviews Burrill (2020) as presented at the International Statistical Association Round Table on July 10, 2020. Burrill's paper is a great statistical literacy paper. It is great in three ways. 1) One of her graphs illustrated some of the most basic aspects of statistical literacy: the difficulties in reading and interpreting percentages in graphs, tables and statements. 2) Her calculation of the inverse of one of the percentages in that graph illustrates a most important concepts in Statistical Literacy. 3) In her proposal of the essential concepts for consumers of databased information, Burrill includes confounding which aligns her paper with the ASA GAISE 2016 update: a major advance for statistical literacy.

Introduction:

This review is a selective – not comprehensive. There are three aspects that make it a great statistical literacy paper. These three aspects are presented in the following three sections.

1. Decoding percentages presented in a graph

"…those who analyze data should not simply be data crunchers but should interrogate the data to develop insight, make recommendations and predictions (GAISE 2, 2020). As an example, consider Figure 7, which appeared in an analysis of achievement gaps in the state of Michigan (Flores, 2014).

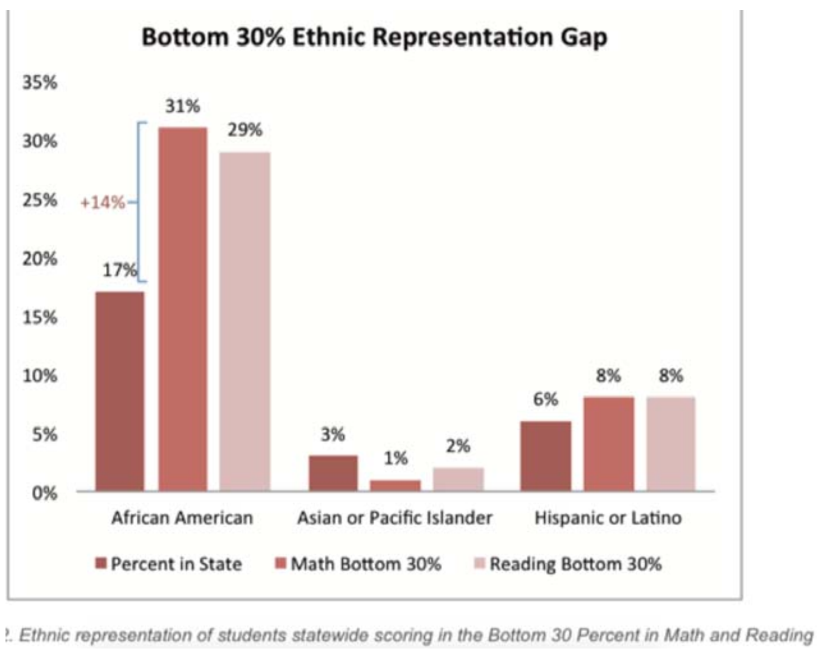


Figure 1 Achievement gaps in math and reading

Reading percentage graphs can be much more difficult than reading percentage tables. Tables usually have margin values; margin values provide insight as to which indexes are parts (numerator) and which are wholes (denominator). This graph is particularly difficult. I think my students would have great difficulty reading this graph. Normally the part is listed in the title or on or above the bars; the whole is listed below or to the left of the bars. The opposite is true here.

Suppose that students had been given these tables:

Percent in State

All	AA	Asian	Hispanic	Other
100%	17%	3%	6%	74%

Math Bottom 30%

All	AA	Asian	Hispanic	Other
100%	31%	1%	8%	60%

Table 1: Two-by-four Percentage Tables

The margin values in both tables are sums: 100%. From these margin value without any help from the titles or the context, student should be able to infer that AA is part in both tables. So 17% of the students taking this exam are AA and 31% of the students in the Math bottom 30% are AA. But students weren't given these tables. They were given Figure 1.

Graphs seldom, if ever, have margin values. Unfortunately, there is nothing in the wording of the titles or the bars that indicates whether something is the numerator (part) or denominator (whole) for these percentage ratios. Here are two plausible interpretations for each of the percentage bars on the left:

Among high school students:
1. "17% of African Americans (AA) live in Michigan" versus "17% of Michigan students are AA".
2. "31% of AA are in the Math bottom 30%" versus "31% of the Math bottom 30% are AA".
3. "29% of AA are in the Reading bottom 30%" versus "29% of the Reading bottom 30% are AA".

Deciding on #1 is easiest. The percentage of AA living in Michigan is probably less than 17%. Furthermore even if 17% of AA live in Michigan, that information seems irrelevant in interpreting the other statistics which are based entirely on statewide tests in Michigan. Thus, the first bar says that "17% of Michigan students are African-American."

Deciding on #2 and #3 is harder. Both are plausible for bars involving AA. If there were no ethnic disparities, 30% of each group would be in the bottom 30% of any test. There are two clues that this interpretation is wrong. First, the columns for Asians and Hispanics are much closer to their prevalence in the population (3% or 6%) than to the 30% in the bottom 30% of the test. Second, the bars in a bar chart of percentages typically involving a common part with distinct wholes. Since AA is the part for the 17% bar, it is probably the part for the Math and Reading bars.

The result is an ethnic disparity: 17% of Michigan students are AA, but 31% of those in the Math-test bottom 30% are AA: a difference 14 percentage points – a ratio that is nearly a factor of two.

So what percentage of AA students are in the bottom 30% of a given test? This is a much more useful percentage. Isn't that just the 31%? No. "31% of those in the Math bottom 30% are AA". That's not the same as "31% of AA are in the Math bottom 30%".

Confusing these two percentages is the confusion of the inverse – a major problem in reading and interpreting percentages. See Utts (2003).

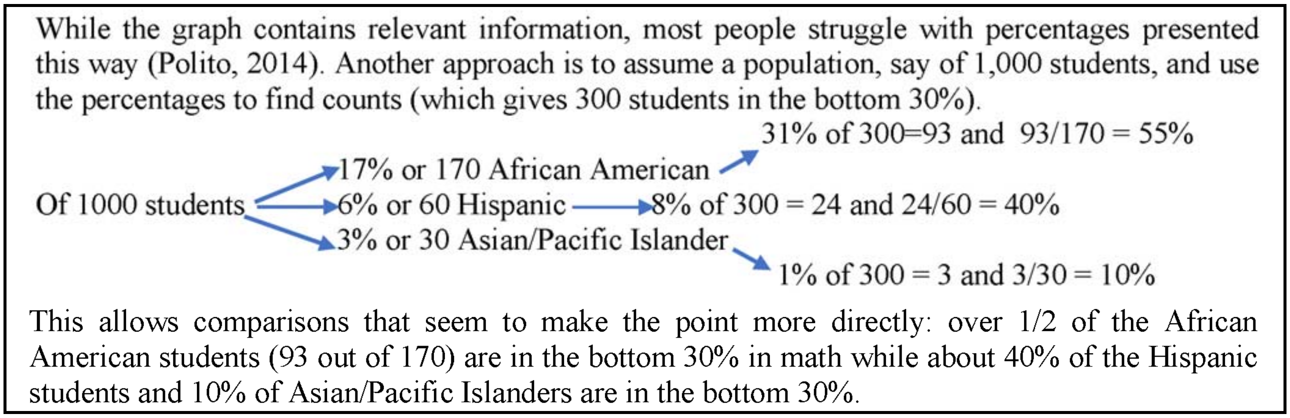
Note: When talking about percentages in this article, they are all part-whole percentages. They are never percentage-change percentages.

2. Calculating the Inverse using Counts and Bayes Theorem

The graph gives three percentages for Michigan students:

1. 17% of those students taking these tests are African American
2. 30% of those students taking these tests are in the bottom 30%.
3. 31% of those in the bottom 30% of the Math test are African-American.

The goal is to calculate the inverse: What percentage of African-American students are in the bottom 30% in the Michigan Math test? Distinguishing a percentage from its inverse, and calculating the size of the inverse when given a percentage are two essential topics in statistical literacy.


While the graph contains relevant information, most people struggle with percentages presented this way (Polito, 2014). Another approach is to assume a population, say of 1,000 students, and use the percentages to find counts (which gives 300 students in the bottom 30%).

This allows comparisons that seem to make the point more directly: over 1/2 of the African American students (93 out of 170) are in the bottom 30% in math while about 40% of the Hispanic students and 10% of Asian/Pacific Islanders are in the bottom 30%.

Figure 2: Burrill's arithmetic solution

Anyone can follow the arithmetic: 17% of 1,000 is 170. 31% of 300 = 93. 93/170 = 55%. But, where did the 300 come from: the sum of 170, 60 and 30 is 260. There is no explanation of why these numbers were chosen and why some were multiplied while others were divided. There is no statement of the intermediate ratios in ordinary English. One may wonder how the 55% became equal to "the fraction of AA students who are in the bottom 30% in Math.

To establish context, create a two-by-two table. Let the rows be the groups and the columns be the cases and the non-cases. Later this 2x2 table can be expanded to handle more racial groups.

Counts	Non	Case	ALL
Group1			
Group2		**93**	**170**
ALL		**300**	**1000**

Table 2: Two-by-two Count Table with four Numbers

N1: Start with a large total number: say 1,000. The number doesn't matter as long as the questions involve ratios. A large number avoids fractional people. Put it in the lower-right All-All cell.

N2: Let Group2 be African-Americans (AA). Since 17% of those taking the test are AA, 170 of the 1,000 are AA. Put the 170 in the 'All' cell associated with Group2 row.

N3: Let a "Case" mean "being in the bottom 30% in the math test." Since 30% of those taking the test will (of necessity) be in the bottom 30%, 300 of the 1,000 students will be in the bottom 30%. Put the 300 in the 'All' cell for the Cases column.

N4: Of the students in the bottom 30% of the Math test, 31% are AA. 31% of 300 is 93: those students in the math bottom 30% and AA. Enter 93 in the cell in the Cases column in the Group2 row.

Once these four numbers are entered, the numbers in all the other cells can be determined. Once the counts are completed, one can generate row percentages and column percentages.

Counts	Non	Case	ALL
Group1	623	207	830
Group2	77	**93**	**170**
ALL	700	**300**	**1000**

Col%	Non	Case	ALL
Group1	89%	69%	83%
Group2	11%	**31%**	**17%**
ALL	100%	100%	100%

Row%	Non	Case	ALL
Group1	75%	25%	100%
Group2	45%	**55%**	100%
ALL	70%	**30%**	100%

Counts	Non	Case	ALL
Group1	a	b	g
Group2	c	**d**	**h**
ALL	e	f	n

Table 3: Two-by-two Table with Counts, and Row and Column Percentages

Q. What percentage of Group2 (African-Americans) are cases (in the bottom 30% of the Math test)?
A. 55%: (93/170) as shown in the lower-left table of row percentages.

Bayes Rule:

Could this result have been obtain algebraically? Yes. Use Bayes Rule. Bayes rule holds that:

4. P(Case|Group2)/P(Case) = P(Group2|Case)/P(Group2)

Bayes rule can be verified using the cell labels in the lower-right table. Note that

5. d/n = (d/h)(h/n) = (d/f)(f/n). Let S = d/h, H = h/n, N = d/f, F = f/n. So S*H = N*F or S/F = N/H.

We can solve equation 4 for P(Case|Group2).

6. P(Case|Group2) = P(Group2|Case)* [P(Case)/P(Group2)] = 31% * (30%/17%) = 55%

For more on Bayes comparisons, see Schield (2008).

3. Essential Concepts includes Confounding

Burrill asked "what topics in mathematics, statistics and data are important to be “data literate”? This is a most important question. An excerpt of her reply is in Figure 3 (taken from her Table 3).

Her title, *Essential concepts for informed consumers of data-based information*, is beautiful. It clearly identifies her audience. All too many statistical educators posit a list of topics but omit their choice of audience. The choice of audience is what sets the standard for which topics are essential. Empirically validating these topics from the needs of this audience is a major task.

Consider the statistics regularly encountered by "informed consumers of data-based information." Most are social statistics based on observational studies. Confounding is a dominant influence.

Figure 3 is an excerpt of Burrill's list and classification of some essential concepts for informed consumers of data-based information.

Table 3 Essential concepts for informed consumers of data-based information

Concepts related to numeracy: students should	**Concepts related to statistics**: students should	**Concepts related to data**: students should
Distinguish between frequency and relative frequency	Connect significance and chance knowing unusual events can occur by chance	Find and interpret a model to describe a relationship between two variables (regression)
Interpret risk/ relative risk	Interpret margin of error in a context	Understand what correlation does and does not tell you
Distinguish between counts and percentages	Identify possible confounding factors	

Figure 3: Burrill's Table 3 (Headings and the bottom three lines)

Burrill takes a broad approach. That is shown in two interesting features of this table:

First, consider the column headings. Numeracy involves basic descriptive statistics. Statistics involves randomness and confounding. Data involve modeling and presentation. There are benefits to these three categories. Data is included without getting into Data Science. Numeracy is included without treating it as basic arithmetic. Statistics is included without an inordinate focus on hypothesis tests.

Second, consider the entry in the bottom row, center (Statistics) column: "Identify possible confounding factors." This inclusion is extremely important. Confounding is not listed in the index of many – if not most – introductory statistics textbooks. See Appendix A for other examples. Burrill maintains the focus on confounding that was recommended in the GAISE (2016) update.

RECOMMENDATIONS:

1. In section 1, write out the percentages associated with Figure 1 in ordinary English.
2. In section 2, write out the percentages, write out the percentages in ordinary English.
3. In section 2, consider using a box approach in calculating the inverse in section 2.
4. In section 2, consider introducing Bayes law, rule or theorem.
5. In section 3, in Figure 3, in the empty box on the bottom row, data column, insert "control for confounding using standardization or multivariate regression". See Schield (2004, 2020)

CONCLUSION

Burrill's 2020 paper, *Rethinking the Curriculum*, is a major contribution to statistical literacy: It features three activities that are essential to statistical literacy: (1) Translating bars in graphs into percentage grammar descriptions, (2), Converting a part-whole percentage into its inverse form, and (3) Identifying confounding as an essential item for informed consumers in interpreting data-based information.

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Appendix A.

Utts (2003) noted that “it is important for students of statistics to understand the distinction between randomized experiments and observational studies, and to understand how the potential for confounding variables limits the conclusions that can be made from observational studies” (p. 75)

Tintle et al., (2013) designated confounding as a major obstacle in learning from data. Confounding was

Gaise (2016) highlighted confounding in Appendix B (Multivariable Thinking). "… consider how a third variable can change our understanding of the relationship between two variables." (p. 34) "Statistical thinking with an appreciation of Simpson’s paradox would alert a student to look for the hidden confounding variables." (p. 38)

Schield (2019) noted the absence, presence and absence of confounding over the past 20 years.