Math of Association in Quantitative Literacy

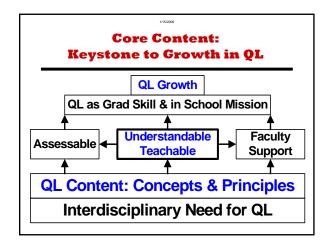
MILO SCHIELD

Augsburg College Dept of Business Administration Director, W. M. Keck Statistical Literacy Project

MAA Quantitative Literacy

15 January 2006

Slides 2006SchieldMAA6up.pdf www.StatLit.org Schield@Augsburg.edu



QL Numbers in Context

"The essence of QL is to use mathematical and logical thinking in context." Lynn Steen 2004

QL must have defining core concepts that are

- based on the role of context in arguments
- mathematically sound
- understandable by students and faculty
- useful to students in their everyday lives
- teachable by non-math faculty.

QL: Four Core Concepts

Whether QL is a separate course or is infused in other courses, it must have core concepts.

Here are some good candidates:

Four key math tools that control for context:

- 1. Arithmetic comparisons (% more than)
- 2. Ratios (percentages, rates, probability)
- 3. Comparisons of ratios (likely, prevalent)
- 4. Standardizing (compare apples w. apples)

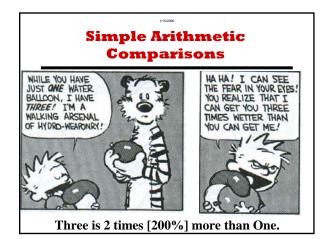
#1: Numeric Comparisons Control For Context

Qualitative vs. quantitative

- Napoleon was shorter than many French soldiers
- Napoleon 4" shorter than average French soldier
- Women live longer than men
- Women can expect to live 7 years longer than men

If interest rates increase from 1% to 2%.

- Double (two times as much as)
- 100% increase (100% more; 1 times more than)
- 1 percentage point increase Not a 1% increase!



#2: Ratios Control For Context

Part-whole ratios are conditional probabilities. \bullet P(B|A)

Algebra is clean and unambiguous. Ordinary English is messy and ambiguous But students speak English – not Algebra

- Q. Can these both be true for the same group?
- 1. Unemployment is up Number is up
- 2. Unemployment is down Rate is down

#2 Ratios Control For Context

Q1. Are these percentages the same?

- 1. The percentage of men WHO ARE runners
- 2. The percentage of men **AMONG** runners

Q2. Are these rates the same?

- 3. The women's death rate
- 4. The death rate of women
- 5. The rate of death among women
- 6. The women's rate of death

Q/L: Interpreting Medical Tests 99.9% accurate! Greater Than 99.9% Accurate Reliable to lines Used by Booters and Magnitak Confidential and Anonymous Results 24 Hours a Day One Spot** Technology HIV-1 TEST SYSTEM for the Detection of Antibodies to HIV-1

"99.9% Accurate" Statistical Prevarication:

Q. Is this accuracy in prediction?

99.9% of those testing positive have HIV?
 NO!

"99.9%" involves confirmation, not prediction Confirmation:

• 99.9% of those with HIV test positive

Prediction is typically a different number: Suppose that 0.1% of a population have HIV. 50% of those testing positive, will have HIV

#3: Comparisons of Ratios Control For Context Two Ways

Is marijuana a gateway drug to heroin?

- 1. 90% of heroin addicts first used marijuana
- 2. 99% of heroin addicts first used milk

Are men psychologically stronger than women?

- 3. Widows are more likely **AMONG** suicides than widowers [are].
- 4. Widows are *less* likely **TO** commit suicide than widowers [are].

#3: Common Named Comparisons

- DP: Differential Prevalence/Risk
- RP: Relative Prevalence/Risk
- OR: Odds Ratio
- Fraction of cases attributable to an exposure in the exposure group: AFG* in the population: AFP*
- * Used to estimate number of cases **due to** an exposure (deaths due to second-hand smoke).

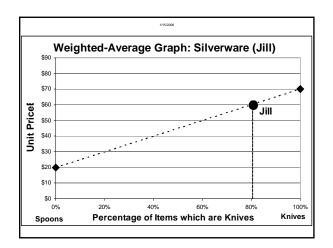
#4: Standardizing Ratios Controls For Context

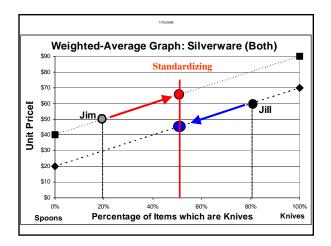
Once you have ratios (percentages, rates or averages) or comparisons of ratios, many students mistakenly think no more can be done.

Standardizing takes into account the influence of confounders on ratios.

Standardizing links mathematics, confounding and context in ways that everyone should know.

Standardizing involves multivariate thinking.





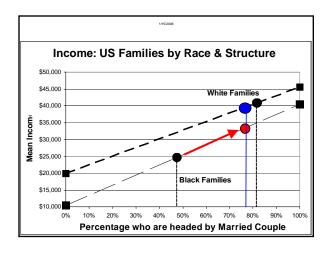
#4: Numbers in Context: Multivariate Thinking

Let's try an example in Public Affairs:

Average family income:

- \$41,000 for US white families
- \$25,000 for US black families
- \$16,000 is the black-white income gap

Is this evidence of structural racism in America?



#4: Numbers in Context: Seeing Confounding

Mexico has better medical care than the US.

- Death rate in Mexico: 5 per 1,000 population
- Death rate in US: 8.7 per 1,000 population

Utah schools (227) better than Oklahoma (225)

NAEP score: 4th grade Math in 2000n.

OK higher than UT for low-income kids & for high-income kids. OK had more low-income kids

#4: Math of Confounding: Not Elementary

Some say that QL skills involve "sophisticated reasoning with elementary mathematics rather than elementary reasoning with sophisticated mathematics."

I disagree.

I believe that quantitative/statistical literacy involves "sophisticated reasoning with both elementary and sophisticated mathematics."

#4: Confounding involves Differential Calculus

Confounding involves the distinction between a total derivative and a partial derivative.

$$\frac{dz(x,y)}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial x}$$

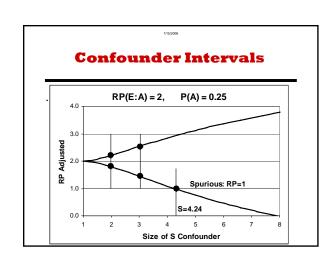
#4: Math of Confounding QL may Involve New Math

In mathematics, a course of study is identified and distinguished by the type and level of math.

So long as QR/QL is distinguished by school math, it is hard to justify as a college-level course.

Burnham and Schield (2006) have introduced some new math involving confounder influence, confounder resistance and confounder intervals.

If valid and practical, this new math could give QR/QL unique math credentials.



Recommendations

Review/critique Schield & Burnham (2006) MAA paper: *Confounders as Mathematical Objects*.

This paper is dense: 150 equations with new concepts and new ratio-comparison notation.

Those completing an in-depth review will be acknowledged in the paper submitted for formal publication.

References

- "Statistical Literacy and the Liberal Arts at Augsburg College" in Peer Review. Copy at www.StatLit.org/pdf/2004SchieldAACU.pdf
- "Confounders as Mathematical Objects" by Schield and Burnham. 150 equations. Copy at www.StatLit.org/pdf/2006SchieldBurnhamMAA.pdf
- "Statistical Literacy Online at Capella University" by Marc Isaacson. Copy at www.StatLit.org/pdf/2005IsaacsonASA.pdf.