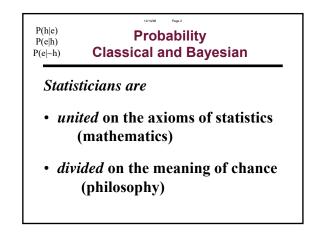
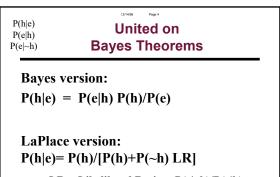
$\begin{array}{c} P(h e) \\ P(e h) \\ P(e \sim h) \end{array}$	PROBABILITY: CLASSICAL AND BAYESIAN
	Colloquium University of Northern Iowa December 14, 1998
	MILO SCHIELD
	Augsburg College www.augsburg.edu/ppages/schield schield@augsburg.edu



$\begin{array}{c} P(h e) \\ P(e h) \\ P(e \sim h) \end{array}$	United on Probability Axioms
	$0 \ge 0$ for all a in domain of P
3. P(a	= 1 if t is a tautology (b) = P(a) + P(b)
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LR = Likelihood Ratio = P(e|~h)/P(e|h) P(e) = P(e|h)P(h) + P(e|~h)P(~h)

P(h|e) Probability: P(e|h) Probability: P(e|~h) Classical versus Bayesian

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P(h|e) **Probability:** P(e|h) **Classical versus Bayesian** P(e|~h)

12/14/98 Page 7

- Classical (Purely objective) Hypothesis testing with p-values Confidence that fixed parameter is in a range
- Bayesian strength of belief No hypothesis testing; no p-values Probability fixed parameter is in fixed range

P(h|e) **Teaching Bayesian: Yes!** P(e|h) **Realistic approach** P(e|~h)

12/14/98 Page 8

"...differences of opinion are the norm in science and an approach [Bayesian] that explicitly recognizes such differences is *realistic.*" [Statistics: A Bayesian Perspective by Berry]

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Finally, an emphasis on Bayesian inference might well impede the trend toward experience with real data ... " David Moore, 1997

Page 10 P(h|e) "Bayesian Interpretation of P(e|h) **Classical Hypothesis Tests**" P(e|~h)

- Combines classical hypothesis test with Bayesian strength of belief.
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Milo Schield, 1995 ASA JSM

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Interprets classical confidence as a Bayesian strength of belief.

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P(e|~h)

P(h|e)

P(e|h)

Page 12 Conclusion

• Students take statistics to help them make better decisions.

12/14/98

- Decision making is Bayesian -- based on a strength of belief.
- Elementary statistics should include a **Bayesian interpretation of classical** statistical inference.

12/14/98 P(h|e) "Statistical Literacy and P(e|h) **Evidential Statistics**" P(e|~h)

Page 13

- Focus on observational studies
- Focus on confounding factors
- Emphasize conditional probability
- Clearly identify role of chance:
 - Highly unlikely if due to chance"
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Page 15

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P(h|e) **"Statistical Literacy** P(e|h) and Simpson's Paradox" P(e|~h)

12/14/98 Page 14

- Simpson's Paradox: a reversal of an association due to a confounding factor.
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P(h|e) **Elementary Statistics:** P(e|h) **Technical versus Basic** P(e|~h)

Elementary Statistics should be split:

- Technical statistics for majors that use hypothesis tests (psychology, sociology, education, etc.)
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Page 16 P(h|e) **Elementary Statistics:** P(e|h) **Technical versus Basic** P(e|~h) **Technical Statistics:**

- Statistical inference: sampling distributions, confidence intervals and hypothesis tests
- **Basic Statistics:** Reading tables, reading and interpreting graphs, and evaluating the results of observational studies.

Elementary Statistics: Benefits of Changes

12/14/08 Page 17

- Goal is statistical literacy: critical thinking about statistics
- Opportunity to Improve: Statistical education **Reputation of statistics**

P(h|e)

P(e|h)

P(e|~h)

Attract national attention ٠ **Demonstrate leadership**

12/14/98 Page 18 P(h|e) **Elementary Statistics:** (To be continued) P(e|~h)

- Need more research on
- assessment of statistical literacy •
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- selection of topics ٠

P(e|h)

- development of teaching materials •
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12/14/98 P(h|e) Statistics Faculty P(e|h) **Bayesian: US and UK** P(e|~h)

Page 19

- US & Canada: 0 - 10% Pure Bayesian** 10 - 30% Mixed Bayesian**
- UK, Australia, & New Zealand: 20 - 40% Pure Bayesian** 40 - 60% Mixed Bayesian** ** Estimated

12/14/98 P(h|e) Math program enrollments P(e|h) **Two-year colleges** P(e|~h)

Page 20

Enrollment in elementary statistics

- 11,000 in 1970
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Page 21 P(h|e) Math program enrollments P(e|h) Four-year colleges P(e|~h)

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Page 22 P(h|e) Math program enrollments: P(e|h) Four-year colleges $P(e|\sim h)$

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P(h|e) Math program enrollments: P(e|h) **Statistics** P(e|~h)

12/14/08 Page 23

Why are more students taking stats?

- Desire: Students have a greater interest in understanding mathematical concepts such as variable, function, slope and correlation.
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Page 1

P(e|h) $P(e|\sim h)$

P(h|e)

PROBABILITY: CLASSICAL AND BAYESIAN

Colloquium **University of Northern Iowa**

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Probability Classical and Bayesian

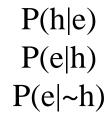
Statisticians are

- *united* on the axioms of statistics (mathematics)
- *divided* on the meaning of chance (philosophy)

P(h|e)P(e|h) $P(e|\sim h)$

United on Probability Axioms

- **1.** $P(a) \ge 0$ for all a in domain of P
- **2.** P(t) = 1 if t is a tautology
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United on Bayes Theorems

Bayes version: P(h|e) = P(e|h) P(h)/P(e)

LaPlace version: P(h|e)= P(h)/[P(h)+P(~h) LR]

 $LR = Likelihood Ratio = P(e|\sim h)/P(e|h)$ $P(e) = P(e|h)P(h) + P(e|\sim h)P(\sim h)$

 \bigcirc



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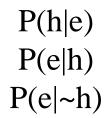
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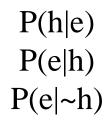
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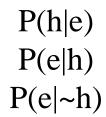
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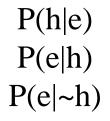
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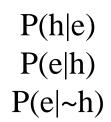
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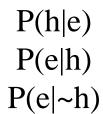
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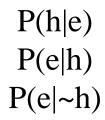
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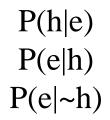
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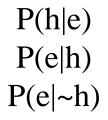


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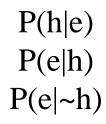
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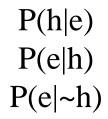
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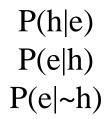


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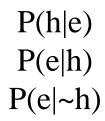
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