Colloquium
University of Northern Iowa
December 14, 1998

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Probability: Classical and Bayesian

United on Probability Axioms
1. \( P(a) \geq 0 \) for all \( a \) in domain of \( P \)
2. \( P(t) = 1 \) if \( t \) is a tautology
3. \( P(a \lor b) = P(a) + P(b) \)
   if \( a, b \) and \( a \lor b \) are all in domain of \( P \)
   and if \( a \) and \( b \) are mutually exclusive
4. \( P(h|e) = P(h \& e)/P(e) \)

United on Bayes Theorems
Bayes version:
\[ P(h|e) = \frac{P(e|h)P(h)}{P(e)} \]
LaPlace version:
\[ P(h|e) = \frac{P(h)}{[P(h) + P(\neg h) LR]} \]
\[ LR = \text{Likelihood Ratio} = \frac{P(e|\neg h)}{P(e|h)} \]
\[ P(e) = P(e|h)P(h) + P(e|\neg h)P(\neg h) \]

Classical probability is objective:
- expresses fundamental laws regarding the assignment of objective physical probabilities to events in the outcome space of stochastic experiments
- independent of our feelings
- a property of the future: not of the past

Bayesian probability is epistemic — based on our context of knowledge
- expresses numeric degrees of uncertainty
- measures our strength of belief
- can be applied to the truth of propositions
### Probability: Classical versus Bayesian

- **Classical (Purely objective)**
  - Hypothesis testing with p-values
  - Confidence that fixed parameter is in a range

- **Bayesian strength of belief**
  - No hypothesis testing; no p-values
  - Probability fixed parameter is in fixed range

### Teaching Bayesian: Yes!

Realistic approach

“...differences of opinion are the norm in science and an approach [Bayesian] that explicitly recognizes such differences is realistic.” [Statistics: A Bayesian Perspective by Berry]

“The Bayesian approach is the only one capable of representing faithfully the basic principles of scientific reasoning.”

[Scientific Reasoning by Howson and Urbach]

### Teaching Bayesian: No!

“at best, premature”

“Surveys of the statistical methods actually in use suggest that Bayesian techniques are little used. Bayesians have not yet agreed on standard approaches to standard problems settings. Bayesian reasoning requires a grasp of conditional probability, a concept confusing to beginners. Finally, an emphasis on Bayesian inference might well impede the trend toward experience with real data...”

David Moore, 1997

### Bayesian Interpretation of Classical Hypothesis Tests

- Combines classical hypothesis test with Bayesian strength of belief.
- If prior belief about truth of null is 50%, \( P(\text{alternate is false|reject null}) = \text{p-value} \)
- Objectively determines prior strength of belief necessary to achieve a 95% probability that the alternate is true.

Milo Schield, 1995 ASA JSM

### “Bayesian Interpretation of Classical Confidence”

Interprets classical confidence as a Bayesian strength of belief.

One should be indifferent in betting on
- whether next ball is red (given 95% chance)
- whether a particular 95% confidence interval contains the population parameter

Milo Schield, 1996ASA JSM

### Conclusion

- Students take statistics to help them make better decisions.
- Decision making is Bayesian -- based on a strength of belief.
- Elementary statistics should include a Bayesian interpretation of classical statistical inference.
Focus on observational studies
Focus on confounding factors
Emphasize conditional probability
Clearly identify role of chance:
  * Highly unlikely if due to chance”
  * Highly unlikely to be due to chance”

Milo Schield, 1998 ASA JSM

Simpson’s Paradox: a reversal of an association due to a confounding factor.
Objectively determines the minimum effect size for a reversal in the three variable case.

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Elementary Statistics should be split:
  * Technical statistics for majors that use hypothesis tests (psychology, sociology, education, etc.)
  * Basic statistics for majors that don’t (humanities) and students that don’t (two-year schools)

Elementary Statistics: Technical versus Basic

Goal is statistical literacy: critical thinking about statistics
Opportunity to Improve:
  * Statistical education
  * Reputation of statistics
  * Attract national attention
  * Demonstrate leadership

Need more research on
  * assessment of statistical literacy
  * student comprehension/retention
  * selection of topics
  * development of teaching materials
  * value added for other majors
  * difficulty of training faculty

Elementary Statistics: Benefits of Changes

Elementary Statistics: Technical versus Basic

(To be continued)
• **US & Canada:**
  - 0 - 10% Pure Bayesian**
  - 10 - 30% Mixed Bayesian**
• **UK, Australia, & New Zealand:**
  - 20 - 40% Pure Bayesian**
  - 40 - 60% Mixed Bayesian**
  ** Estimated

Enrollment in elementary statistics

- 11,000 in 1970
- 20,000 in 1980 -- 6.0% growth/year
- 47,000 in 1990 -- 8.5% growth/year
- 69,000 in 1995 -- 7.7% growth/year

77% of all enrollment in elementary statistics is at the 4-year level

Why are more students taking stats?

- **Desire:** Students have a greater interest in understanding mathematical concepts such as variable, function, slope and correlation.
- **Necessity:** More students are required to take statistics for their major or graduation.
PROBABILITY:
CLASSICAL AND BAYESIAN

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Statisticians are

• *united* on the axioms of statistics (mathematics)

• *divided* on the meaning of chance (philosophy)
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Bayes version:

\[ P(h|e) = \frac{P(e|h) \cdot P(h)}{P(e)} \]

LaPlace version:

\[ P(h|e) = \frac{P(h)}{[P(h) + P(\sim h)] \cdot LR} \]

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\[ P(e) = P(e|h)P(h) + P(e|\sim h)P(\sim h) \]
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- expresses fundamental laws regarding the assignment of **objective** physical probabilities to events in the outcome space of stochastic experiments  
- independent of our feelings  
- a property of the future: not of the past
Probability: Classical versus Bayesian

Bayesian probability is **epistemic** --
*based on our context of knowledge*

• expresses numeric degrees of uncertainty

• measures our strength of belief

• can be applied to the truth of propositions
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P(h|e)  P(e|h)  P(e|~h)

“Statistical Literacy and Evidential Statistics”

- Focus on observational studies
- Focus on confounding factors
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Elementary Statistics: Technical versus Basic

- **Technical Statistics:**
  Statistical inference: sampling distributions, confidence intervals and hypothesis tests

- **Basic Statistics:**
  Reading tables, reading and interpreting graphs, and evaluating the results of observational studies.
Elementary Statistics: Benefits of Changes

- **Goal is statistical literacy:** critical thinking about statistics

- **Opportunity to Improve:**
  - Statistical education
  - Reputation of statistics

- **Attract national attention**
  - Demonstrate leadership
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Statistics Faculty
Bayesian: US and UK

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** Estimated
Math program enrollments
Two-year colleges

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- 69,000 in 1995 -- 7.7% growth/year
Enrollment in elementary statistics**

- **117,000 in 1990**
- **164,000 in 1995: 6.8% growth/year**

** taught just in math programs

77% of all enrollment in elementary statistics is at the 4-year level
Math program enrollments:
Four-year colleges

Enrollment: 1995 versus 1990

• 25% increase in elementary stats
• 10% decrease in math courses
• 20% decrease in upper-level math
• 26% decrease in upper-level stats
Why are more students taking stats?

• **Desire:** Students have a greater interest in understanding mathematical concepts such as variable, function, slope and correlation.

• **Necessity:** More students are required to take statistics for their major or graduation.