A First-Year Interdisciplinary Quantitative Reasoning Course: A Pre-Statistics Course

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Abstract: In this session, we will describe a new Quantitative Reasoning course developed at DePaul University by an interdisciplinary group of faculty. The purpose of the course is to help students become confident and critical users of quantitative information of all kinds. The course addresses the growing need for quantitative and computer literacy in response to the enormous expansion in the use of quantitative methods and information in the social and physical sciences as well as in civic life and ideally precedes formal statistics courses and data analysis courses. Students work in small groups on data sets from many different disciplines, such as psychology, environmental science, economics, finance, sociology, and history. They learn to critique quantitative arguments, make simple analyses of data, and present their results both in writing and orally. Students are introduced to spreadsheets, word processors, email, presentation software, and the Internet. In the session, we will give some history of the development of the course, describe its key characteristics and the specific skills addressed, and provide some examples of class activities.

Introduction

In this paper, we describe a new Quantitative Reasoning course developed at DePaul University by an interdisciplinary group of faculty. This course, which addresses important changes in society, assists students on their way to becoming confident and critical users of quantitative information of all kinds and functions as an excellent Pre-Statistics course. We describe the need for this new course, the history of its development, and its content.

The quantification of society

Although some have traced the development to the late middle ages [6], western societies since the 17th century have embraced a program of increasing quantification and "objectification" [7, 15]. The hallmarks of this program are (1) the keeping of permanent quantitative records, (2) the development of techniques to display and interpret data, and (3) the acceptance of inference from recorded data as an objective (or at least less biased) approach to drawing scientific, social, or economic conclusions. The development is closely tied to the rise of modern science and western democracy. In science, the process began with the path-breaking work of Kepler and Galileo in the early 17th century. Applied to society, quantification first appeared in the work of John Graunt (1662) who analyzed parochial records of baptism, marriages, and deaths. In 1669, Huygens created the first mortality tables and calculated life expectancies. William Petty in his *Political Arithmetic* (1690) argued that the specificity of number conferred objectivity and that true political knowledge would arise of a full quantitative accounting of social and economic facts. In 1694, the English Parliament passed a census law, and like today it was controversial.

We cannot do justice to this rich and important history here; we can only point the reader to some excellent recent accounts [7, 15]. However, we must note that this process accelerated greatly in the twentieth century, intensified by technology. Technological improvements impacted the process in two ways. First, calculating devices made laborious computations easier so that a much wider group of people could make complicated calculations formerly done by an elite of specialists in natural science, social science, economics, or public policy. Second, communication networks made for a far greater dissemination of data, whether internally in organizations or externally from public sources (governments, scientific agencies, and trade organizations). Today data informs decision making at every level in the private and the public sphere, and it is questionable whether one can understand (much less actively participate in) critical public policy debates without acquaintance with the data that underlies them (e.g., in the year 2000, global warming, social security reform, international monetary policy, free trade, welfare reform). Considerations such as these have led to a number of reports and initiatives on enhancing the quantitative reasoning abilities of college graduates [1, 8, 9, 10, 11, 12, 18, 19]. John Allen Paulos' *Innumeracy* [13] and Edward Tufte's books [21, 22, 23] brought the topics of quantitative and graphical literacy to a broad public audience.

The University's Role

Universities have played and continue to play a key role in the process of quantification. Researchers at universities have developed most of the techniques of statistics and data analysis, and universities have trained the specialists who collected, analyzed, and disseminated quantitative information. The creation of statistics departments and the requirement of statistics courses in many academic programs are examples of the leadership displayed by many universities. But the technologically-induced expansion of quantification in the twentieth century has led to a critical moment for universities especially with regard to general education.

General education mathematics requirements at universities are, in most cases, not preparing students for the quantitative work they need to do within academic disciplines and in the workplace. Why? First, general education mathematics requirements often emphasize algebra and abstract symbol manipulation. The handful of word problems students do are hopelessly artificial and are more often than not the butt of jokes. Second, in their mathematics courses students do not see mathematics in complex social and scientific contexts they encounter in other disciplines and in the workplace. Students do not develop essential skills of critical examination of data without these contexts, and our colleagues from other disciplines often tell us that students can't even do the mathematics at all when it is presented in a real context. Third, at most universities, technology is not integrated into mathematics; most commonly, computer literacy is a separate requirement. This state of affairs does harm to both students' quantitative abilities and their ability to use technology productively. All professionals who work with data use technology extensively because of its power, ease of use, and enhancement of productivity; so should our students. Teaching our students to use technology does not detract from the teaching of ideas; it goes hand in hand with it.

DePaul's Response

In the mid-nineties, DePaul addressed the question of the appropriate mathematics requirement while in the process of developing a new general education program. At that time, the only quantitative requirement for students whose majors did not require them to take calculus was about the level of beginning high school algebra. Students could meet the requirement by passing an examination which they could re-take as many times as they wished. The skills necessary to pass the exam were neither sufficient nor appropriate to allow students to do well in higher-level college courses with a quantitative component. Students were graduating without the ability to understand or interpret the most basic quantitative data or information that they would be sure to encounter after they left the university.

In response to this situation, the faculty committee developing the new general education program decided to include a quantitative reasoning requirement for all students whose major did not require calculus. This requirement would be met by having students take a newly developed Quantitative Reasoning course designed to enable students who have completed it to:

(1) Make and analyze quantitative arguments expressed in numerical, graphical, verbal, or symbolic forms.
(2) Interpret and create graphs summarizing quantitative data.
(3) Understand and use reasoning involving percentage change and proportional relationships.
(4) Make reasoned estimations.
(5) Use computer tools to analyze data.
(6) Make simple mathematical models (especially linear and exponential) and understand the limitations of mathematical models.

The course would have intermediate algebra as a prerequisite and most students would be expected to take the course during their first year. Thus, in the spring of 1996, we were faced with the opportunity to design the course we wanted and the challenge of having it ready for implementation in the fall of 1997.

Our first decision was to include as many faculty members as possible in the process. We did this by inviting all faculty members from the College of Liberal Arts and Sciences, the College of Commerce, the School of Education, and the School of Computer Science, Telecommunications and Information Systems to a "town hall" meeting on quantitative reasoning in the curriculum. Discussion focused on the following questions:

- What should be in a foundation course in Quantitative Reasoning?
- How much can we expect from one course?
- What other components should there be in an effective Quantitative Reasoning program?
- What is the role of technology?

Approximately 40 faculty representing a diverse set of disciplines expressed interest in our questions, and 25 attended the meeting and came up with some preliminary answers. As a result, four faculty members (one computer scientist, one psychologist, and two mathematicians) agreed to develop a prototype for the new course to be piloted in the 1996-1997 academic year. Two of the important decisions at this point were to have students look at data from a variety of disciplines and to integrate the use of technology as a fundamental analytical tool.

During 1996-1997, four sections of the prototype were taught in a new computer classroom dedicated to the course. Although the instructors tended to use different curriculum materials, the sections shared some features that have become standard for the course: students used spreadsheets to analyze real data and presented their results in word documents; much of class was spent in the computer classroom with students working collaboratively on activities; and students were expected to do a group project. Meanwhile, the instructors teaching the course plus a group of interested faculty met several times to discuss full implementation in 1997-1998. At that time, we realized that we had a need for good curriculum materials that would allow students to experience the usefulness and power of quantitative reasoning in different contexts. At the same time, course materials should be flexible enough to allow faculty from different disciplines some degree of freedom in designing their individual sections.

It is important to note that institutional support has been critical to the success of the Quantitative Reasoning course. This includes funding the establishment and maintenance, including some staff and student support, of a state-of-the-art computer classroom (the Quantitative Reasoning Center), and allowing class size to be capped at 25. As we prepared for full implementation, two further forms of support were provided. First, we created the position of Director of the Quantitative Reasoning Center and did a nation-wide search to hire someone who would coordinate curriculum and faculty development efforts. Second, with the help of federal funding, we were able to provide support for faculty development.

Under the leadership of the new Director, we moved to full implementation of the course over the next three years, offering 18 sections (over three quarters) in 1997-1998 and 54 sections in 1999-2000. As we worked with a large and diverse group of faculty to develop curriculum materials of our own and associated instructional strategies, key characteristics of the course began to emerge:

(1) The emphasis is on reasoning
(2) Context is emphasized and is drawn from many disciplines
(3) Technology is integral
(4) Active learning is emphasized
(5) Faculty share across the disciplines

We realized that our overarching goal is to help our students to become confident and critical users of quantitative information of all kinds, verbal, tabular, and graphical. We particularly emphasize the use and misuse of quantitative information in public policy and scientific issues and teach students to recognize the limitation of quantitative methods as well as the insights they provide.

Students learn by working with actual data set in collaborative hands-on computer activities. The course is intended to be an ideal pre-statistics course providing students with the data skills, technology skills, and confidence that an instructor would like beginning statistics students to have before they begin formal study of statistics. We were influenced by the work of A. Rossman and K. Sommers at Dickinson College [16], the work of D. Pierce, D. Wright, and L. Roland at Western Oregon State [14], and the CHANCE project [17].

The Course Today

The most important feature of the course is the extensive use of the student computer activities. About two-thirds of the course is held in a computer classroom where students work collaboratively on activities that integrate the technology (primarily Word, Excel, and PowerPoint) into the reasoning material. The students work in groups of two or three, with the instructor facilitating, providing assistance and guidance. The computer classroom ends up being a busy place with students interacting with each other, with the computer, and with the instructor. The activities themselves, which are based on real data sets and frequently involve public policy issues, and the interactive format create a very engaged classroom. Students have universally praised the activities for enhancing their learning. The course makes heavy use of the Web. All the activities and Excel files are accessible through the Web.

To see how this works, let us look at two examples of these activities. In an early activity related to ratios, students examine data from the Center for Budget and Policy Priorities on income inequality in the US [2]. The data gives the average of each quintile of household income for each state. We focus on the average for the top fifth and the bottom fifth; the ratio of these two averages for a given state is called the income gap. Students start by examining this data, sorting by bottom fifth income and top fifth. They add a new column with the income gap for each state and are asked to describe carefully in words the meaning of the this quantity (*how many times* more the wealthiest fifth earn than the poorest fifth). Using the map feature of Excel, students make shaded maps based on the data and describe the geographic patterns and outliers. Data from each of three decades is available so that students can examine changes over time. Finally students write a short essay critiquing the use of quantitative information in an article from the *New York Times* on the subject; they are also asked to examine the public policy implications of the data.

An example from the middle of the course has students study the minimum wage in the US. They first graph the minimum wage in nominal dollars and carefully describe the graph. They are asked to make a connection between the graph and which political party controlled Congress or the Presidency. Students then convert the minimum wage figures into constant dollars and once again graph it. They are asked to describe this graph (it differs dramatically from the first one) and to notice certain patterns within it. They then critique a misleading and incorrect graph of the same data that was published by the Associated Press in 1997. Finally they are asked to read some recent newspaper articles that describe the current state of minimum wage legislation in Congress and to formulate a position, necessarily informed by the historical data (not necessarily exclusively so), on whether the minimum wage should be raised and if so by how much.

Throughout the course, students do activities of this type; their finished products are Word documents consisting of their analysis and embedded Excel tables, graphs, and maps. Some of the topics for which activities have been written include inflation, higher education costs, incarceration rates, carbon dioxide levels, global temperature, and sunspot numbers. In addition, instructors give demonstration and lectures or lead discussions for approximately one third of the class. Out of class assignments reinforce the activities and are similar in character. A supplementary text, a custom published version of Bennett and Briggs *Using and Understanding Mathematics: A Quantitative Reasoning Approach* [3] is used. Students individually take one or two midterm exams and a final exam at the computer.

An essential part of the course is the final project. This work usually also done collaboratively, demands a greater synthesis than the activities. In the most common final project, students are asked to choose a section of the *Statistical Abstract of the US* [23] and prepare a briefing to the President on their topic. They are to present important trends (e.g., Population, Vital Statistics, Health, Crime, Income, Energy, Federal Budget, Education) that have public policy importance and where leadership is critical. Some instructors give an environmental focus to the final project using data from WorldWatch [4] and the Commission on Environmental Quality [5]. In another form of the final project, students analyze raw survey data (similar in quality to the General Social Survey); they use the Pivot Table Report in Excel to calculate percentages but do not perform hypothesis tests. In all cases, students prepare a written report with analyses and charts and make an oral presentation using presentation software (PowerPoint).

What follows are some of the mathematical topics covered during the course.

Estimations. Students learn how make carefully reasoned estimations in the spirit of what physicists call “Fermi problems” [24] Students start from commonly accepted facts (e.g., population of the US, the average height and weight of a person, average household size), reason to an estimate of a not so obvious quantity, and finally express their estimate to reasonable degree of precision. Examples: Estimate the height of a ten-story building. Estimate the annual revenue generated by vehicle licensing in a state. Estimate the amount of carbon emitted into the atmosphere by the United States each year. We cultivate this skill in order to build students’ confidence, help students be more critical when number mentioned in median and elsewhere, and help students to judge whether a computed answer is reasonable.

Percentages and Rates. Through the examination of real data sets and articles from the media, students learn the distinction between absolute and relative quantities. The inappropriate use of absolute quantities when relative ones (rates) are called for is one of the most common student errors we have encountered. For example, the annual number of traffic deaths in a state is not a useful comparative measure of traffic safety. Percentages are probably the most common quantitative concept in the media; but college students and adults in general have great difficulty using and interpreting them correctly.

Graphing. Students learn to make and interpret a great variety of graphs and charts including shaded maps. Students learn when pie charts, bar charts, multiple bar charts, x-y graphs, and maps are appropriate. Students examine the effects of scaling axes. We also introduce some useful terminology from Precalculus: global and local maxima and minima, increasing, decreasing, concavity, and periodicity. Students critique incorrect or misleading or graphics from the media.

Proportional Reasoning. Most sections teaching students how to use the Consumer Price Index (CPI) to compare prices in different years and to calculate the inflation rate. Proportional reasoning also arises in geometric scaling problems.

Modeling. Students make linear and exponential models for a variety of phenomena. The make extrapolations and interpolations and contrast the long term behavior of linear and exponential models. Students learn to appreciate the limitations of their models and those of others. An important concept is that when modeling, the further one is from the data the less confidence one has in a prediction; one makes this precise in a first course in statistics.

Descriptive Statistics. Students make histograms to examine distributions and learn the properties of the standard measures of central tendency and measures of dispersion, especially resistance (or lack of resistance) to outliers.

Association. Students learn the basic technique of determining correlation and interpreting it when it exists. Emphasized here is the importance of looking for common underlying causes that might give rise to a correlation and avoiding the strong temptation to infer causality.

Cross tabs. Working with real survey data, students use the Pivot Table Report feature of Excel to create cross tabs and calculate the percentages for each case. Students do not use t-tests or chi-square tests. We have found that students have difficulty interpreting even simple cross tab tables, typically confusing rows and columns. This material is a very useful pre-statistics topic.

Faculty Development

A large interdisciplinary team of instructors teaches the course. Instructors from history, communications, psychology, geography, computer science, chemistry, physics, environmental science, statistics, political science, and sociology have taught or are about to teach the course. Each instructor receives training in the course. Each instructor receives his or her own website along with a standard set of all the activities, assignments, samples exams, and sample syllabi. They can freely adapt these materials and have read access to the other instructors' websites so that they can easily share course materials. A great deal of cross-fertilization of ideas has resulted through this open environment. In addition, the instructors meet weekly to share their experiences and help each other with pedagogy and the material itself. Particularly stimulating has been the interplay of ideas that occurs when the faculty from different disciplines interact and are stimulated by each others' diverse viewpoints and experiences.

Response to the Course

The response to the course has been overwhelmingly positive from students, faculty who have taught the course, and the faculty who have taught students in subsequent courses. Students consistently praise the hands on activities and report greater mathematical confidence and quantitative ability. Faculty who teach the course enjoy the format, learn a great deal about quantitative reasoning themselves, and report satisfaction that they are giving students needed reasoning skills. Faculty who teach students in subsequent courses, especially statistics, disciplinary quantitative methods, and general education science courses, report that students who have taken quantitative reasoning are significantly more capable in handling data and are far less afraid of working with numbers.

Possible Future Plans

We are currently discussing a variety of possible future directions. These include:

- Conducting research on student understanding (we have started a research project on college students learning of percentages).
- Adding more open-ended case studies to our collection of activities.
- Introducing the use of databases to the student work, allowing for the exploration of large data sets.
- Introducing material on probability through the use of simulations.

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