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Twenty Questions about Precalculus

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Editor's note: This paper is the text of the keynote address given by Lynn Steen in October 2001 at the conference Rethinking the Preparation for Calculus. (For an overview of the conference, please see the preceding paper in this volume by Jack Narayan and Darren Narayan.)

Introduction

Approximately fifteen years ago a workshop similar to this one took place at Tulane University where a merry band of reformers sought to make calculus lean and lively. I had the opportunity to address that workshop with a list of twenty questions for calculus reformers. Thus I thought it appropriate to take a similar approach to this current workshop, to help launch your work by asking twenty questions about precalculus. (For comparison, I reproduce in Appendix A the questions that I put before the calculus reformers at Tulane. There you will find not 20 but 28 questions, the extra eight being added to the manuscript as a result of issues raised during the workshop. The full text with elaborations on each question can be found in [3].)

At the time of the Tulane workshop I was President of the Mathematical Association of America, and in that capacity had some degree of oversight responsibility for MAA's many committees. Even as the Tulane rebels were training their sights on calculus, I was well aware that then, as now, more college students study precalculus than calculus. On several occasions I asked the CUPM subcommittee on the First Two Years (later to be renamed CRAFTY—Calculus Reform and the First Two Years) whether in order to fulfill the mission implied by their title they might be interested in looking at the mathematics course that is the most common of all taken during students' first two years in college, namely precalculus. Their answer was consistently negative: precalculus, in their judgement, was an unfortunate leftover from high school mathematics. Despite enrollment evidence, they said, college mathematics begins with calculus.

With this fifteen-year-old experience as backdrop, I checked current data to see what enrollments look like now. Figure 1 offers a sobering portrait of undergraduate mathematics prepared by combining recent data from two sources—the (forthcoming) quinquennial CBMS 2000 survey [2] and the annual AMS survey [1]. (Enrollments included in this figure are predominantly in departments of mathematical and statistical science. They do not count the many statistics, computer science, and applied mathematics courses found outside departments of mathematics or statistics.) Clearly precalculus (and its alter ego college algebra) is the single most common mathematics course in undergraduate education. Data aside, it also appears to be the rock on which college students' mathematics education most often founders. That dark secret is why we are all here.

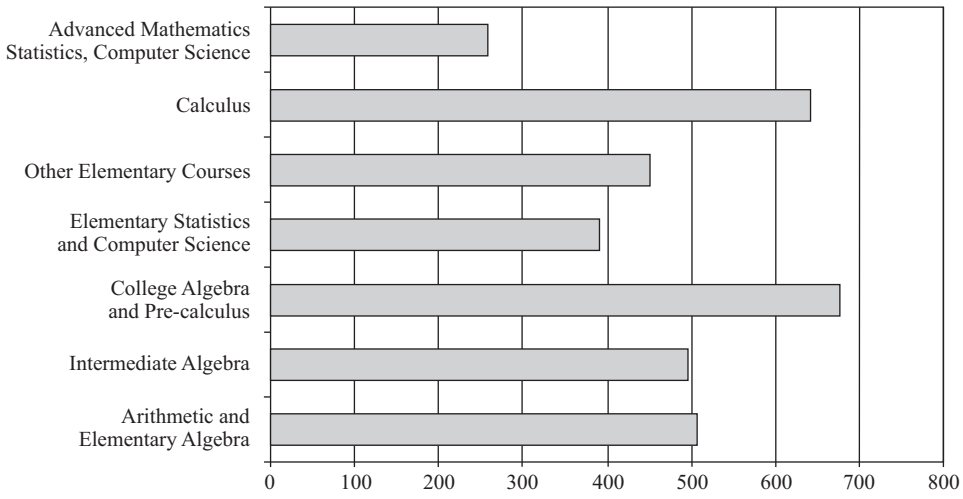

Figure 1.

One can approach the challenges of precalculus from several perspectives. For example, a managerial perspective would suggest a cycle of setting goals, developing strategies, implementing changes, assessing outcomes, reflecting on results, and making adaptations. A journalist's paradigm, in contrast, seeks insight by asking questions: what, who, why, when, where, and how? To actually make changes that improve student learning, the managerial paradigm is really the only effective option: set a goal, make some changes, look at the results, and then regroup. But to reflect on the issues, to "rethink precalculus" as this workshop intends, nothing can beat the journalist's simple questions.

What?

What exactly is precalculus? Is it the same as college algebra? (In this analysis, I ignore whatever differences there may be between them.) Does precalculus have an intellectual core like geometry or calculus? Does it have a center or a town square? Or is it more like a mathematical strip mall that just fills space between high school and college?

What is the real goal of precalculus? Is it really to prepare students for calculus, or does it have other purposes, either benign or sinister? Isn't it also, *de facto*, a ubiquitous prerequisite for a wide range of quantitatively-oriented college courses, a steady source of tuition revenue that reliably exceeds marginal costs, and an accepted means of screening students for access to the economic rewards of higher education?

What effect does calculus have on the nature of the precalculus course? What differences are there in preparation for reformed calculus, for traditional calculus, for mainstream calculus, or for non-mainstream (business) calculus? Can a single course provide suitable preparation for all flavors of calculus? Can precalculus possibly be made lean and lively?

Who?

Who takes precalculus? Is its clientele students who are reviewing (or relearning) what they once learned, students who did not learn what they once studied, students who never had the opportunity to learn precalculus topics, or students who declined the opportunity? In most courses, the answer is "all of the above." Can a single course really serve all these different students?

Who should take precalculus? Does precalculus serve well the quantitative needs of students preparing for fields that do not require calculus? Does it offer any lasting benefit for students who never take any

further mathematical or quantitative course? For that matter, does precalculus really benefit the students it was created to serve—those who need calculus but are not ready for it?

Who should teach precalculus? University mathematicians? Teaching assistants or adjuncts? Experienced secondary school teachers (who perhaps teach the very same course during the day to high school students)? What about on-line tutorials? Is a Ph.D. in mathematics an appropriate credential for teaching precalculus? Might mathematicians' uncommon facility with elementary mathematics make them peculiarly inappropriate as empathetic teachers of precalculus?

Who benefits from precalculus? Who loses? Does precalculus have disparate impact on at-risk populations? For whom, if anyone, does precalculus serve as a pump? For whom is it a filter? Some believe its primary beneficiary is the budgets of mathematics departments for whom it serves as a cash cow. Maybe it is just a means of shifting tuition income from a required large enrollment course to low enrollment advanced electives—that is, from the mathematically weak to the mathematically strong.

Why?

Why is calculus so important for under-prepared undergraduates? Is preparing for calculus really a wise use of college students' time and energy? Might the siren call of calculus replace more important goals for students who enter college unprepared for calculus? Shouldn't more under-prepared undergraduates be steered in other quantitative directions?

Why do students take precalculus? Is it to prepare for calculus, to meet the prerequisite of a particular course or program of study, to fulfill a general education option, or to fulfill a graduation requirement? Are any of these reasons defensible, or are they simply traditional?

Why is precalculus so often part of general education? Does precalculus advance students' mathematical or quantitative literacy? Does anyone believe that precalculus is the right mathematics course to prepare students well for lives in the 21st century? Does it reveal important insights into the nature, power, and beauty of mathematics? Can precalculus possibly serve two masters—calculus and culture?

Why should students take precalculus? Does precalculus have value for the majority of students who take the course? Are its concepts and skills independently useful apart from their role in calculus? How many ever use the skills they learn in precalculus? Is precalculus a sensible choice for the last mathematics course a student ever takes?

Why do so many prospective elementary school teachers take precalculus? In the majority of departments, precalculus (or college algebra) is the second most common course taken by students preparing for K–3 certification [2]. Does this make any sense? Does precalculus really provide teachers with deep understanding of the mathematics they will be teaching?

When?

When should students take precalculus? Is there an optimal window for learning precalculus? Isn't precalculus taught and learned better in high school? Currently only about 25% of high school graduates take precalculus in high school, even though over 60% enroll in some form of postsecondary education. Shouldn't higher education tell students and schools that it is more important for more students to finish precalculus in high school than for more students to finish calculus?

Where?

Where do precalculus students come from? What have been their mathematical backgrounds? What are their major programs of study or career interests? How many are returning after having interrupted their study

of mathematics? How do students' prior mathematical experiences influence their views of mathematics, their confidence in their own abilities, and their likelihood of success with precalculus?

Where do precalculus students go? How many precalculus students eventually take calculus? (*Answer*: Relatively few.) How many take other courses that utilize ideas from precalculus? (*Answer*: A few more.) How many complete a year of calculus with good grades and incentive to continue their study of mathematics? (*Answer*: Embarrassingly few.) For how many is precalculus the end of their study of and interest in mathematics? (*Answer*: Far too many.)

How?

How should the changing role of mathematics influence the nature of precalculus? In the last two decades mathematical practice has become increasingly algorithmic and digital. New applications range from genomics to cinema, from manufacturing to Wall Street. How, if at all, should the content of precalculus reflect this expanded interface of mathematics with the rest of the world?

How do articulation agreements constrain precalculus? Are inter-institutional agreements on syllabi and standards essential instruments of quality control? Or do tight curriculum specifications lead to curricular sclerosis? Are the transparency benefits of articulation agreements worth the cost of inflexibility and stifled innovation? On balance, do students gain or lose from these protocols?

How well aligned is precalculus with common placement tests? Do commercial or homegrown placement tests reflect the same level and type of performance expectations as a precalculus course? Do they accurately place students into or out of precalculus? Are they fair to students?

How should technology influence precalculus? Is technology a means or an end? Is its role to help students learn traditional mathematics, or is technology now so much part of the way mathematics is practiced that it has itself become an important goal of instruction? Is the use of numerical, graphing, and CAS systems a prerequisite to learning calculus?

How do you measure success? This may be the toughest question of all. Fewer than one in four students, perhaps as few as one in ten, achieve the *prima facie* goal of precalculus: to succeed in calculus. Without clarity about goals, it is impossible to gauge success. Without data on students' future academic careers, success is unknowable. And without external validation, precalculus may never improve.

Conclusion

These questions suggest an overwhelming agenda for a course of enormous importance, but a course that is all but invisible to the mathematical community. I wonder how much has really changed in the last fifteen years since CRAFTY's predecessor declined to take up the challenge? Neither enrollment patterns, course prerequisites, nor general education requirements have changed very much. Nor, I suspect, have mathematicians' attitudes about what constitutes appropriate college mathematics. Does the mathematical profession now consider precalculus a challenge worth working on, or do they still see it as a peripheral problem best ignored? Can any mathematician earn tenure by teaching or improving precalculus? (That's a rhetorical question.)

In addition to seeking answers to the twenty questions I have suggested, the merry band of reformers assembled for this conference will need to think hard about where precalculus fits into the agenda of mathematics, of science, and of our nation. Rethinking precalculus may lead to some surprising conclusions.

References

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2. Lutzer, David, *et al. CBMS 2000: Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States.* Providence, RI: American Mathematical Society, 2002.

3. Steen, Lynn A. "Twenty Questions for Calculus Reformers." In *Toward a Lean and Lively Calculus: Report of the Tulane Calculus Conference.* Ronald G. Douglas, Editor. Washington, DC: Mathematical Association of America, 1986, pp. 157–165.

Appendix A

Twenty Questions for Calculus Reformers
Lynn Arthur Steen, January, 1986 (From [3])

1. Should fewer students study calculus?
2. Is calculus an appropriate filter for the professions?
3. Will computer science dethrone calculus?
4. Do students really learn the major ideas of calculus?
5. Has calculus become a cookbook course?
6. Does calculus focus excessively on closed-form formulas?
7. Should calculus students learn to use or to imitate computers?
8. What new topics are essential for calculus in a computer age?
9. Which topics in calculus are no longer essential?
10. Do engineers still need the traditional calculus?
11. Should calculus be a laboratory course?
12. Is there any reason to teach high school calculus?
13. Why do U.S. students perform so poorly on international tests?
14. Is there any value to precalculus remedial programs?
15. Why do calculus books weigh so much?
16. Can one design a good calculus course from a survey?
17. Is calculus a good course to train the mind?
18. Can calculus courses convey cultural literacy?
19. Does calculus contribute to scientific literacy?
20. What will calculus be like in the year 2000?

Added after workshop discussion:

21. Do students ever read their calculus books?
22. Should precalculus be a prerequisite for calculus?
23. Is teaching calculus most like teaching a foreign language?
24. Should the student-faculty ratio for calculus be limited?
25. Do student evaluations favor calculation-based courses?
26. Are there enough qualified calculus teachers?
27. Who will be the calculus teachers in the year 2000?
28. Should calculus be taught only by experienced teachers?