

08/04/2003 JSM 2003 1

Algebraic Conditions for Binary Spuriousity

MILO SCHIELD
 Augsburg College Department of Business Administration
 Director, W. M. Keck Statistical Literacy Project

THOMAS V.V. BURNHAM
 Cognitive Consulting

August 4, 2003

www.augsburg.edu/ppages/~schield schield@augsborg.edu

08/04/2003 JSM 2003 2

Associations Confounded No test for Confounding

In observational studies,
 associations are often *confounded (tangled up)*.

E is Effect of interest
 A is associated predictor
 B is confounder

08/04/2003 JSM 2003 3

Categorical Cube: Three Binary Variables

B: confounder.

08/04/2003 JSM 2003 4

Quantitative Rate Cube Non-Planar Data

$$AQ = P(E|A=0, B=XQ) = Rb \cdot XQ + Ra \cdot (1-XQ)$$

$$XQ = P(B|A=0)$$

E: effect

B: confounder.

08/04/2003 JSM 2003 5

Criteria for Spuriousity: A has "no effect" on E

Cornfield & Gastwirth used a cross-A rate equality model:

- $P(E|A \text{ and } B) = P(E|B) = P(E|non-A \text{ and } B)$
- $P(E|A \text{ and } non-B) = P(E|non-B) = P(E|non-A \text{ and } non-B)$

We used two regression models:

- A non-interactive model: $E = b_0 + \underline{b_1} \cdot A + b_2 \cdot B$
- An interactive model: $E = b_0 + (\underline{b_1 + b_3 \cdot B}) \cdot A + b_2 \cdot B$

A-E association is spurious if underlined factor is zero.

As viewed from confounder perspective: B-E

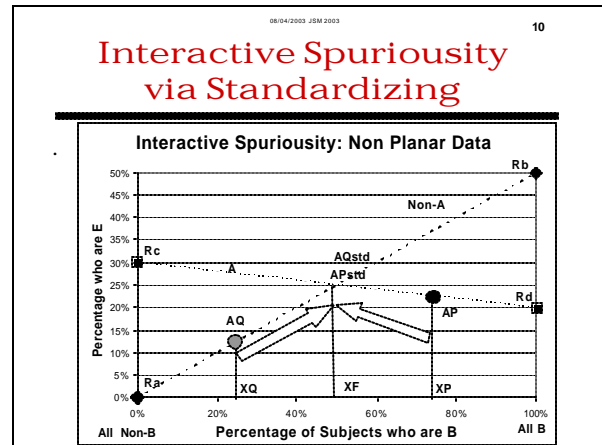
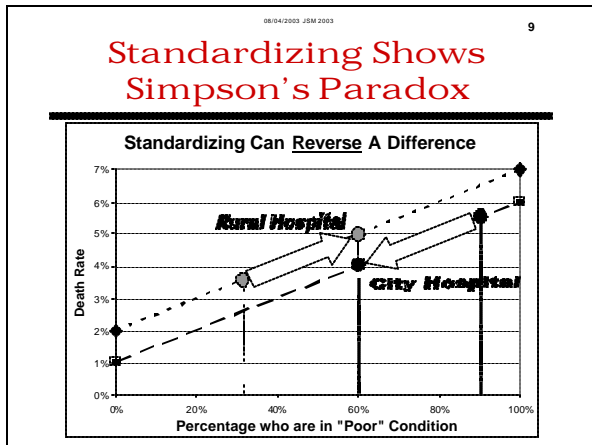
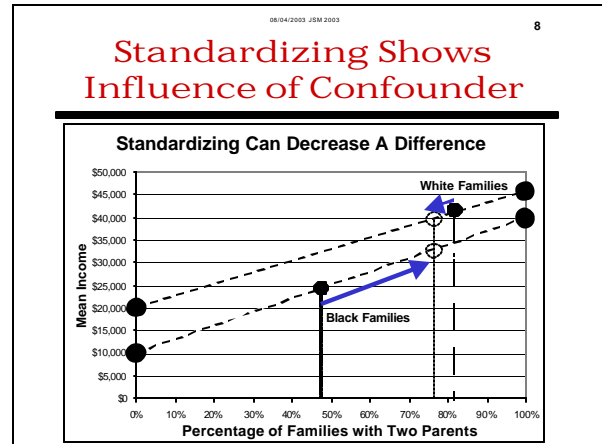
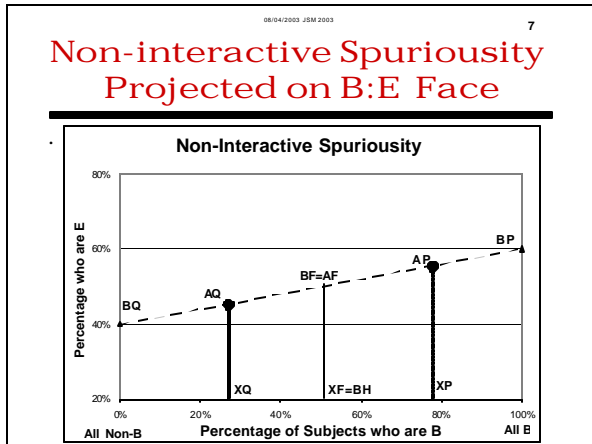
- Non-interactive model: B line \parallel A line
- Interactive model: Rate lines intersect at prevalence of B.

08/04/2003 JSM 2003 6

Non-Interactive Model: AP:AQ line and BP:BQ line

E: effect

B: confounder.



08/04/2003 JSM 2003 11

Spuriousity Results: New Necessary Condition

Gastwirth-Cornfield: $RR(E:B) > RR(E:A)$
New: $RR(E:B) - 1 > [RR(E:A) - 1][P(A)/P(B)]$

What cancer-gene effect size is necessary to make association between smoking and cancer spurious?

$RR(E:A)=9$ for cancer among smokers vs. non.
 $P(B) = 10\%$. 10% of adults have a cancer gene
 $P(A) = 40\%$. 40% of adults smoke, then

- Gastwirth-Cornfield: $RR(E:B) > 9$.
- New: $RR(E:B) > 33$

08/04/2003 JSM 2003 12

Conclusions

Spuriousity depends on model.
 Cornfield conditions more-generally valid.
 Standardizing illustrates interactive model.
 Spuriousity conditions for non-interactive and interactive models overlap.
 New equations for non-interactive spuriousity.
 New inequality for non-interactive model:
 $RR(E:B)-1 > [RR(E:A)-1] \cdot P(A) / P(B)$