Promoting Understanding of Numbers that Matter: Why Mathematicians Can’t Do It Alone

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Numbers that matter affect:

• The decisions students make in their personal and professional lives
• Students’ understanding of the political and social issues that are part of citizenry
• The way people vote, and the policies they support.

Chair of a pharmacology department in a medical school:

“The idea of using a computer for something other than email or downloading documents to print is one that only a small fraction of our class is comfortable with. I would say the same thing about things numerical...........The number of incidents due to miscalculated drug doses becomes more understandable.”

These are numbers that matter!

Journalist:

“In journalism we tend to use statistics frequently........ I believe it is important to learn the implications of statistics, what you can and cannot learn from it........ I think an important tool is the ability to guestimate or have a rough idea of the outcomes of arithmetic and algebraic problems: the ability to roughly convert measurements, to understand how interest is calculated, to understand relationship of time, distance and speed, and so on........ I notice that many people, some quite intelligent, do not have these tools........

What is Quantitative Literacy?

• The ability to identify, understand, and use elementary mathematics in everyday contexts.
• Fluency in adapting this ability to new contexts.
• A habit of mind: not a particular piece of knowledge or a particular skill.
• Looking for quantitative patterns everywhere

Is Quantitative Literacy Different from Mathematical Literacy?

Mathematical Literacy: Requires
• Wide knowledge of mathematics
• Ability to abstract away from context

Quantitative Literacy: Requires
• Fluency in elementary mathematics
• Thinking in context
Is It Possible to be Mathematically Literate Without Being Quantitatively Literate?

- A certain level of mathematics is needed for quantitative literacy.
- However, it is possible to have learned the mathematics without being able to apply it in context.

UK Gas Prices: £8 out of £10 spent on gas was tax

A group of US students who heard this story knew what 8 percent was of 10, and what percent 8 was of 2, but not where the caller got 400% and 80%.

- Were these students mathematically literate?
- Were these students quantitatively literate?

Are Calculus Students Necessarily Quantitatively Literate?

If \( f(t) \) is the population of the US in millions at time \( t \) in years, what is the meaning of the statements \( f(2006) = 300 \) and \( f'(2006) = 2.87? \)

How do these statements relate the BBC’s observation\(^1\) that a new person is added to the US population every 11 seconds?

- Some calculus students could answer; some could not.

What Mathematics is Needed for Quantitative Literacy?

People can reasonably differ. A possible list:

- Arithmetic, including percentages, graphs.
- Estimation.
- Elementary probability and statistics.
- Basic geometry of measurement (volume, area, perimeter).
- Elementary growth patterns: linear (constant quantity per unit time) and exponential (constant percentage per unit time).

Watching your Blood Pressure: Pre-Hypertension

Risk of Heart Disease and Stroke doubles for every 20 point increase in systolic blood pressure and every 10 point increase in diastolic blood pressure over 115/75

Gina Shaw, WebMD, September 11, 2003

(Systolic blood pressure is the first number, diastolic is the second)
What Does this Mean in Practice?

If your blood pressure is
- 135/85, you have TWICE the risk
- 155/95, you have FOUR TIMES the risk
- 175/105, you have EIGHT TIMES the risk

Mathematically, this is exponential growth

What Does Your Credit Card Cost?

- Suppose your credit card charges 18% interest per year, compounded monthly, and requires you to pay at least 2.5% of your balance each month.
- If you charge $2000 to your credit card for a new computer, and pay it off at the minimum rate, how long will it take? How much will you eventually pay?

• Over 30 years
• $4925.72; nearly $3000 more than original loan

If you add $1 to monthly minimum payment
- Time reduced to 22 years
- Payments reduced to $4531.43, a savings of $394.29

If you pay $50 per month
- Time reduced to just over 5 years
- Payments reduced to $3076.84, a savings of $1848.88

On January 28, 1986, Space Shuttle Challenger Exploded

- What was cause of disaster?
- In early 1987, Richard Feynman showed the following data to a congressional hearing
- This data was available to NASA in January 1986.

A Graph Shows the Effect of Temperature on O-Rings

- O-rings more likely to break at low temperatures
- Temperature on January 28, 1986, was 31°F

Deaths in Two Tragedies

- More than 10 times as many men died
- Original population 2/3 men
- Sinking of Titanic, April 14, 1912
- Almost 4 times as many women died
- Original population half women
- Tsunami, village in Indonesia, December 26, 2004

What could this data have told NASA?

• Temperature (F) 0 70 80 90 100
• Number of damaged O-rings 0 1 2 3 4

Mathematics: Exponential growth and Geometric series, or Excel

Mathematics: Ratios and Approximation
Compressing 12-15 Billion History of Universe into One Year
(De Karen Kolehmainen, physics, CSUSB)

- January 1: Big bang
- Early February: Milky Way and Other Galaxies
- Mid August: Our Solar System Forms
- Early September: Life Starts on Earth
- Dec 31, evening: Early Man
- Dec 31, half a minute before midnight: Human Civilization begins

Mathematics: Scaling, Ratios, and Approximation

The Human Cost of the War in Iraq
Mortality Study 2002-2006

- Estimate of 654,965 excess deaths since 2003
- Cluster sample to estimate death rate (deaths/1000/year)
- 95% confidence interval for violent deaths: (426,369, 793,663)

Mathematics: Statistical Inference

Is There Racial Profiling in LA?

LAPD motor vehicle stops resulting in search (01/01/04-06/30/04)

<table>
<thead>
<tr>
<th>Race of driver</th>
<th>Total</th>
<th>Search</th>
<th>No search</th>
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<tbody>
<tr>
<td>White</td>
<td>5849</td>
<td>106104</td>
<td></td>
</tr>
<tr>
<td>African-American</td>
<td>9882</td>
<td>49439</td>
<td>155543</td>
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</tbody>
</table>

Approximating the Data

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</tr>
</thead>
<tbody>
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<td>White</td>
<td>6000</td>
<td>110000</td>
<td></td>
</tr>
<tr>
<td>African-American</td>
<td>10000</td>
<td>50000</td>
<td>160000</td>
</tr>
</tbody>
</table>

Numbers in Thousands

- About 5% of white drivers were searched when stopped
- About 20% of African-American drivers were searched when stopped

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</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>6</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>African-American</td>
<td>10</td>
<td>50</td>
<td>160</td>
</tr>
</tbody>
</table>

Some Further Questions about LAPD data

- Could the differences in percentages have happened by accident if there has been no racial profiling?
- The data was for January 1- June 30, 2004. Might a different period have shown substantially different percentages?
- Is the difference between 5% and 20% large enough to provide evidence that the LAPD uses racial profiling?
How Do We Teach Quantitative Literacy?

- Basic mathematics needed.
- How do we get students to use mathematics in diverse contexts?
- Your ideas?

Teaching Quantitative Literacy

- Students only see mathematics as useful when they see it used in other fields.
- Many applications should involve making a decision (not a calculation).
- Faculty should model quantitative literacy themselves.
- Coordination between faculty teaching quantitative literacy is important.

How Do We Teach Quantitative Literacy?

- It takes a “conspiracy”
- How does one foster such a conspiracy?
- Your ideas?