Chapter 3.4.2

DEVELOPING MATHEMATICAL LITERACY

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Abstract: *Mathematical literacy* has received increasing attention in many countries over the last few years. This is partly driven by concerns of employers that too many students leave school unable to function mathematically at the level needed in the modern world of work. Further, it is increasingly recognised that people can only tackle many of the challenges of modern life effectively if they are mathematically literate in key areas. Planning in personal finance, assessment of risk, design in the home or on the computer screen, and critical appraisal of the flood of statistical information from advertising, politicians and the press – these are just a few of the domains where mathematics is an essential tool in sensible decision making, not just an exotic luxury. Mathematical literacy, like literacy in language, is empowering.

1. WHAT DO WE MEAN BY MATHEMATICAL LITERACY?

The term and its variants are used in a variety of ways[1] alongside other terms with overlapping meanings – *quantitative literacy, numeracy, functional mathematics, quantitative reasoning* and more, are all used. Here we shall mainly call it *mathematical literacy* (ML), focussing on the core idea:

> *Mathematical literacy is the capacity to make effective use of mathematical knowledge and understanding in meeting challenges in everyday life.*

In contrast, mathematics in school largely continues the legacy of Euclid, Newton, and Euler – a school-based, "scholastic" discipline of major importance conveying the basic ideas of geometry, algebra, and calculus. It also equips a minority of students well for their chosen specialised professions –

in mathematics, physics, and traditional engineering. Valuable as this may be, and it is of immense and irreplaceable value, a *central* goal of schooling in the modern age must be to prepare *all* students for life in an increasingly technological society. That is what ML is all about: *mathematics acting in the daily lives of citizens.*

In this respect, school mathematics does not look so good. Most adults use little of the mathematics they first learned in secondary school in their every day lives. Much of that mathematics could be useful for mathematical literacy but the additional modelling skills that would make it so are not covered in most curricula.

Here we shall summarise key developments in recent years, including recent thinking in the US, described in *Mathematics and Democracy: the case for quantitative literacy* (Steen, 2002), and the suggestions in the UK Government Tomlinson report (2004) that *functional mathematics* should be at the core of learning for all, with additional *specialist mathematics* for students who want more. Of particular note is the OECD's *Program for International Student Assessment* (PISA, n.d.). This is important because it represents an international consensus on mathematical literacy. The PISA test instrument, while its exclusively short items are not "cutting edge", is a big step forward. It complements the narrower view of mathematics embodied in the Third International Mathematics and Science Study (TIMSS, n.d.), which devotes little serious attention to applications, and none to modelling or other non-routine problem solving. In PISA's definition (OECD, 2003, p. 24):

> *Mathematics literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen*

This broader conception looks at the life circumstances, contexts and needs of an individual, and considers the importance of their capacity to engage with and use mathematics in those life contexts. It involves recognising mathematical features of phenomena in the world around us, making judgments about those phenomena informed by mathematical understanding, and generally using mathematics as a tool for dealing with the phenomena.

The above definitions, and other variations on them, all convey three important ideas. First, ML is much more than arithmetic or basic skills. Second, ML requires something quite different from traditional school mathematics. Third, ML is inseparable from its contexts. In this respect ML is more like writing than like algebra, more like speaking than like history. ML has no special math content of its own, but finds appropriate content for the context. Moreover, like writing and speaking, the standard of excellence increases with the sophistication and importance of the issue being analyzed. Mathematics plays a parallel role in mathematical literacy to that of language in literacy[2].

Unlike teachers of language, most mathematics teachers rarely try to link mathematics lessons to the everyday lives of their students who, consequently, don't expect it. Making ML a reality in most classrooms will need a revised '*classroom contract*'[3], supported by new and well-engineered classroom materials and professional development support – a major challenge for design and development.

In the rest of this section, we shall:

- outline the characteristics of a 'good ML problem';
- discuss what mathematical literacy looks like as it develops;
- touch on some areas of controversy.

Then, in the next section, we shall discuss how teachers and curricula may develop ML in the classroom as part, along with other modelling and applications, of the learning of other mathematical competencies.

2. WHAT KINDS OF PROBLEM?

Exemplification clarifies meaning so, after this general discussion of mathematical literacy and its curriculum implications, we offer a few examples of the kinds of task that one would expect students to tackle in curriculum and assessment that is focussed on ML. We begin with some assessment tasks – always good way to communicate learning goals, since they are brief and specific. For practical reasons, PISA uses a mixture of multiple choice and short answer items (OECD, 2003, pp. 57-92), illustrated by these first two tasks:

Example 3.4.2-1: Rock Concert

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert?

2000, 5000, 20000*, 50000, 100000

Example 3.4.2-2: Robberies

A TV reporter showed the graph below and said: "The graph shows that there is a huge increase in the number of robberies from 1998 to 1999."

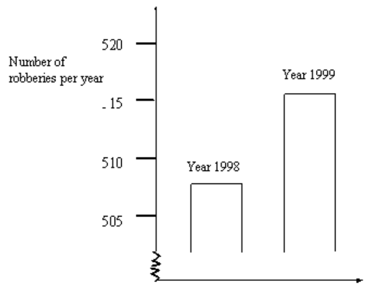

Figure 3.4.2-1. Graph of the number of robberies per year

Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

Of course, ML assessment or curriculum can and should have much more substantial and extended tasks than these. The *Numeracy through Problem Solving* (NTPS, 1987-89) modules provide examples of areas that motivate students to remarkably good, extended reasoning – for many students, work of much higher standard than their usual formal mathematics. The flavour is given by examples in the previous chapter and by *Design a Board Game.* Students begin a design process by critiquing and improving a number of badly-designed games. Both mathematical and non-mathematical faults are considered; the clarity of the rules, the fairness and interest of the game, the geometry of the board design and so on.

Example 3.4.2-3: Snakes and Ladders

Read the description of a game given in Fig. 3.4.2-2, then answer the questions below.

This is a game for two players. You will need a coin and two counters.

Rules

- Take it in turns to toss the coin.
  If it is heads, move your counter 2 places forward.
  If it is tails, move your counter 1 place forward.
- If you reach the foot of a ladder, you must go up it.
  If you reach the head of a snake, you must go down it.
- The winner is the first player to reach 'FINISH'.

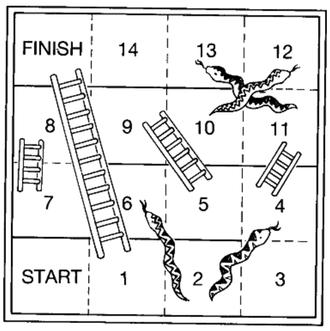

Figure 3.4.2-2. Description of a game

Questions:
1. Suppose you start by tossing a head, then a tail, then a head. Where is your counter now?

List and describe all the faults you notice with the board.

The goal here was to check that the student understands the basic principles of board game design, notably that the *board* and the *rules* must work together, and can apply a careful logical analysis. As in many well-engineered complex tasks, there is a 'ramp' of difficulty – some of the faults are harder to find than others.

At the end of each NTPS module there were written examinations at two levels, *Standard* and *Extension*, to assess how far the student can transfer the skills and insights they have developed in doing the three-week module to less- or more-distant contexts.

What are the general principles for task types for ML? Typical ML challenges involve real data, non-routine procedures, and complex reasoning, yet often require only relatively elementary mathematics. In contrast, school mathematics problems feature increasingly abstract concepts using simplified numbers, straightforward procedures and stylised applications.

> *Whereas school mathematics stresses elementary uses of sophisticated mathematics, mathematical literacy focuses on sophisticated uses of (often) elementary mathematics.*

Steen and Forman (2001) have summarised the characteristics of good ML problems as part of a list of "Principles of Best Practice" and we have adapted them for this book:

> *High quality mathematical tasks are authentic, intricate, interesting, and powerful*
>
> *Authentic*: they portray common contexts and honest problems; employ realistic data, often incomplete or inconsistent; meet expectations of users of mathematics; use realistic input and output; and, above all, reflect the integrity of both mathematics and the domain of application.
>
> *Intricate*: they expect students to identify the right questions to ask; require more than substitution into formulas; employ multi-step procedures and chains of reasoning; stimulate thinking that is cognitively complex; confront students with incomplete (or inconsistent) information; and demonstrate the value of teamwork.

Interesting: touch on areas of interest to students; appeal to a large number of students; they offer multiple means of approach; invite many variations and extensions; and provide horizontal linkages to diverse areas of life and work.

Powerful: they encourage and connect graphical, numerical, symbolic, verbal, and technological approaches; offer vertical integration from elementary ideas to advanced topics; propel students to more advanced mathematics; expand students' views of mathematics, its value and uses; demonstrate the importance of mathematics in the modern high performance work place, and in everyday life.

This book contains many examples of tasks that qualify as ML.

3. PATHWAYS TO MATHEMATICAL LITERACY

How do we recognise progress in ML? Essentially, students tackle more complex problems, in contexts less familiar to them, using more powerful mathematics – but only where it pays off in greater insight and/or more effective action. What is the micro-structure of this development?

PISA has investigated and described growth in mathematical literacy by focussing on a set of mathematical competencies that are based on the KOM framework. In conformity with the approach taken by PISA to report levels of proficiency in reading following the first round of assessment in 2000 (OECD 2001), the PISA project has developed and published six described levels of mathematical literacy (OECD 2004). A clear progression through these levels is apparent in the way in which the individual mathematical competencies specified in the PISA mathematics framework (OECD 2003) play out as mathematical literacy levels increase. They describe the *stages of development*, in increasing order, of the various competencies as:

- *Thinking and reasoning*: Follow direct instructions and take obvious actions; use direct reasoning and literal interpretations; make sequential decisions, interpret and reason from different information sources; employ flexible reasoning and some insight; use well developed thinking and reasoning skills; use advanced mathematical thinking and reasoning.
- C*ommunication*: follow explicit instructions; extract information and make literal interpretations; produce short communications supporting interpretations; construct and communicate explanations and argument; formulate and communicate interpretations and reasoning; formulate precise communications.

- *Modelling*: apply simple given models; recognise, apply and interpret basic given models; make use of different representational models; work with explicit models, and related constraints and assumptions; develop and work with complex models; reflect on modelling processes and outcomes; conceptualise and work with models of complex mathematical processes and relationships; reflect on, generalise and explain modelling outcomes.
- *Problem posing and solving*: handle direct and explicit problems; use direct inference; use simple problem solving strategies; work with constraints and assumptions; select, compare and evaluate appropriate problem solving strategies; investigate and model with complex problem situations.
- *Representation:* handle familiar and direct information; extract information from single representations; interpret and use different representations; select and integrate different representations and link them to real world situations; make strategic use of appropriately linked representations; link different information and representations and translate flexibly among them.
- *Using symbolic, formal and technical language and operations*: apply routine procedures; employ basic algorithms, formulae, procedures and conventions; work with symbolic representations; use symbolic and formal characterisations; mastery of symbolic and formal mathematical operations and relationships.

Like all models of problem solving, this model of stages is informative rather than definitive. For example, a given person will be at different stages, depending on the complexity, unfamiliarity, and technical demands of the problem they are tackling. Nevertheless such stages are characteristic of growth in these various competencies. They also provide a useful point from which further research may be directed to generating greater refinement in describing development in mathematical literacy.

4. CONTENTIOUS ISSUES

Chicken or egg – which comes first?

Many people believe that skills must precede applications and that once learned, mathematical skills can be applied whenever needed (in practice, for many students, in a future that never arrives). This is a false dichotomy. Considerable evidence about the associative nature of learning suggests that the skills-first approach works imperfectly, at best. For many students, skills learned free of context are skills devoid of meaning and utility. To be useful, skills must be taught and learned in settings that are both meaningful and memorable. One may observe that Pure Mathematics:

- grows vertically;
- climbs the ladder of abstraction to reveal, from sufficient height, common patterns in seemingly different things – abstraction is what gives mathematics its power, enabling methods derived in one context to be applied in others.

Mathematical literacy, on the other hand:

- grows horizontally;
- makes multiple connections, the core of understanding;
- clings to specifics in each context;
- marshals all relevant aspects of setting and context in order to reach conclusions that are reliable in practice.

Across contexts, *ML shows the pay-off of abstraction* – that the same mathematical tools can be powerful in a wide range of different areas.

Will ML undermine "real mathematics"?

Sceptics fear that modelling, if encouraged, will replace rigour and proof in mathematics classrooms. There are many legitimate reasons to ensure that reasoning and proof do not disappear from school mathematics. Students need to learn that justification is a distinctive part of mathematics; that proof is more than plausibility or confirmation; that among the levels of convincing argument, mathematical proof alone yields certainty; and that the rigor of mathematical proof makes lengthy chains of logical argument reliable. Although mathematical modelling rarely emphasizes formal proof, it does emphasise the value of:

- accuracy at the end of a long chains of inference and calculation;
- justifying findings, especially their applicability in relation to the problem context;
- explaining reasoning to team-mates and teachers;
- presenting conclusions coherently.

Through these means, modelling both demonstrates and rehearses the importance of rigorous logical argument**.**

Who should teach ML?

Many argue that mathematical literacy must be learned in context, while others believe that only mathematics teachers have the preparation and incentive to focus on it. The issue is complex. Since you can only model situations you already understand at least qualitatively, everyday life contexts are the obvious place for students to *learn active modelling with mathematics*. Teachers of the students' first language (English teachers in the Anglophone world) have long used everyday life problems as contexts for student work in their classrooms; ML asks mathematics teachers to do the same. For many this is a new challenge, needing good teaching materials and professional development support.

Ideally, students would also develop ML in other subjects – in history and geography, in economics and biology, in agriculture and culinary arts, and in social studies. Because contexts needing ML are ubiquitous, opportunities abound to teach it across the curriculum — in reading maps, designing art projects, understanding rules of grammar, analyzing scientific data, and interpreting legal evidence. Only by repeatedly using diverse aspects of ML in real contexts will students develop the habits of mind of a numerate citizen. Thus mathematics teachers should not, and can not, bear the entire burden of helping students become numerate. Like literacy, mathematical literacy is everyone's responsibility.

But do students understand other school subject areas well enough to model them autonomously? It is usually enough of a challenge to *learn models* in physics or economics – which is an important but very different process. Experience suggests that *mathematical literacy across the curriculum* will only happen if students first learn to model familiar practical problems in mathematics lessons. Cross-curricular teaching is an ideal, often tried but rarely sustained in schools; it only works when approached 'from both sides'.

If ML is to become a reality, it will probably depend on mathematics teachers carrying prime responsibility, with other subjects building on the foundations so laid. Perhaps, following a suggestion in the report *Making Mathematics Count* (Smith, 2004), mathematics should be seen as two subjects: *mathematical literacy* as the gatekeeper subject[4] for all students, and additional[5] *specialist mathematics* for those who want it for their future interests as scientists or traditional engineers – or simply find mathematics interesting enough to want further study. In some countries, *English Language* and *English Literature* are two subjects related in much the same way.

Is this mathematics?

ML is neither an expanded list of topics to be added to the mathematics curriculum nor is it just the basic skills part of a traditional mathematics program. Many basic mathematical skills (e.g., number sense and operations, proportional reasoning, estimation, logic, data analysis) are essential for ML – but so too are other concepts not much emphasized in school mathematics curricula (e.g., computer tools, statistical inference, mathematical communication). The open-ended thinking required to diagnose problems or to make decisions relies heavily on "newly useful" areas such as combinatorics, statistics, and geometry. In contrast, algebra and calculus, dominant features of today's curriculum, are used outside of school more as tools for calculation than as tools for reasoning, and only by specialists in certain fields.

Some worry that modelling problems are a time-consuming distraction that typically use relatively routine mathematical tools. While this may be an accurate description of some weaker programs, good modelling problems require a sophistication and precision that can push even the best students to attain mathematical results well beyond those achieved by most students in today's classrooms. As they secure a broad foundation of examples and concrete mathematics, students engaged in modelling build lasting connections between mathematics and the world in which they live. This grounding in specifics will lead naturally to subsequent generalizations and abstractions. Modelling parallels good pedagogy by moving from the specific to the general and from the concrete to the abstract.

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[1] In some contexts and some nations, these terms are used narrowly to mean just "basic skills" – arithmetic plus a bit more. This is a corruption of the terms, just as *literacy* means much more than spelling, grammar and syntax. These skills are necessary but far from sufficient.

[2] The original definition, now often distorted, of the term *numeracy* – in the 1959 Crowther Report.

[3] The *classroom contract* is the agreement, usually unspoken and implicit, between teacher and students as to what each will do, what roles they will play, in the classroom. (The French, who first articulated the idea (Brousseau, 2003,p.24), call it the 'didactic contract' but in English 'didactic' is used to describe a specific, teaching style – lecturing that brooks no argument. That is the reverse of what we need here, or in any classroom focussed on learning.)

[4] Much more justifiable, as a universal requirement meeting a universal need, than current secondary mathematics which few adults can use in their lives beyond education.

[5] If it is offered as an *alternative*, it will surely remain the prestige track, with ML becoming a 'sink' subject, taken only by weak students, while the well-qualified adult population remains innumerate.