Ch 1. Review

Statistics are generally used as evidence to support an argument. The influences on a statistic are of four kinds: Context, Assembly, Randomness or Error.

Randomness

1. How do random excesses correct? Offset or dilution?
2. How can we use chance to estimate and adjust?
   - Truthful Answers on Sensitive Issues using Chance
   - Adjusting for Guessing using Chance
   - Estimating Population Size Using Chance
3. How to estimate survey sampling error @ 95% confidence.
   - 95% Margin of Error for subgroups
   - Sample size needed to achieve desired accuracy.

Overcoming Excess

A fair coin has been giving an excess of ‘heads’. Should you bet ‘tails’? Must coin offset to get 50-50?
A roulette wheel has been giving an excess of ‘Red’. Should you bet ‘black’? Must wheel offset the ‘red’?
No! A coin and a wheel have no memory.
They have no power to offset. Offset is impossible!
So how do they return to 50-50?
By dilution. Just doing 50-50 will dilute any excess.

Truthful Answers on Sensitive Issues

Sensitive question: Have you done something bad?
Assign people to two groups: Truthful and “Yes-only”

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<th>B</th>
<th>C</th>
<th>D</th>
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<td>Randomly assigned</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>+00</td>
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<tr>
<td>3</td>
<td>Answered “Yes”</td>
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<td>+00</td>
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<tr>
<td>4</td>
<td>ALL</td>
<td>+04</td>
<td>+04</td>
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Fraction of truthful who say Yes: (Yes – N/2) / (N/2)
This assumes random assignment gives a 50-50 split.
Never check on this. Doing so breaks anonymity.

Adjust for Guessing

N multiple-choice questions with k possible answers.
Decision: Mark answer or leave blank.
If one guesses: 1 chance in k of correct answer.
Pure chance: N/k right; N-N/k = N[(k-1)/k] wrong.
To minimize guessing, make net score equal to zero
Subtract 1/(k-1) points for each wrong answer.
N/k right. Subtract \{N[(k-1)/k] [1/(k-1)] = N/k.
Rule: Guess only if chance of right answer > 1/(k-1)
Estimate hard-to-count population

Examples: Fish in a lake or uncounted in a census. This method of estimating the population (N), involves capture-recapture: taking two random samples at different times.

1. Count and mark those in the first sample. Call the count n.
2. At a later time, take a second random sample of size s.
   Find that m of these s are marked from the first sample.
3. Results:
   - Proportional reasoning: If \( \frac{m}{s} = \frac{n}{N} \) then \( \frac{N}{n} = \frac{s}{m} \).
   - Fractional method: If \( \frac{m}{n} = \frac{s}{N} = p \), then \( N = n/p \).

Example: 100 tagged in 1st group. 2% of 2nd random catch had tags. Estimated population = \( N = \frac{100}{0.02} = 5,000 \).

Sampling Error

Sample means are distributed around population mean.
Sample error: Sample mean minus population mean.

Margin of error:
1) is expected sample error with 95% confidence.
2) decreases as sample size (n) increases.
3) is proportional to \( 1/\text{square root of } n \).
4) is independent of size of population.

Counter-intuitive. Key is randomness: well-mixed.
To see if coffee is too weak/strong, need just one sip.
2 is expected; 3 is a small surprise; 4 is a big surprise.

Margin of Error Proportions

The exact 95% margin of error for a proportion is:
\[ 1.96 \times \sqrt{\frac{p(1-p)}{n}} \] where \( p \) is sample proportion.

The conservative 95% margin of error is:
\[ \frac{1}{\sqrt{n}} \] where \( n \) is the sample size.

This conservative value is typically reported in surveys.
It always includes the entire group and it always assumes \( p=50\% \) so it gives the largest interval.

Confidence interval of a proportion = \( p \pm \text{ME} \)

Survey Margin of Error For Subgroups

Most polls show the Survey 95% margin of error.
This is the most-conservative margin or Error for the entire survey. Assumes \( p \sim 50\% \).

Let \( N \) be size of random sample; \( n = \) size of subgroup.
Let \( F = \) fraction of group that is in a subgroup = \( \frac{n}{N} \).

Result:
Subgroup ME = Survey ME \( \sqrt{\frac{N}{n}} \) = Survey ME/\( \sqrt{n/N} \).

If \( F = 9\% \), then 95% subgroup ME = 95% Survey ME/0.3

Confidence Intervals: Measurements

Conf. Interval: Sample Mean ± ME.
95% Margin of Error = \( 2s/\sqrt{n} \)
Randomly select 100 students (n). Suppose they average 8 hours working per week with a standard deviation (s) of 5 hours.

1) What is the estimated population mean? Answer: 8 hrs.
2) What is the 95% margin of error for the average time working/week? A: 1 hr. 95% ME = \( 2s/\sqrt{n} \) = 2*5/\( \sqrt{100} \).
3) What is the upper limit of 95% confidence interval for average time working? Answer: 9 hrs: 8 hrs + 1 hr
4) What is the lower limit of 95% confidence interval for average time working? Answer: 7 hrs: 8 hrs - 1 hr

Required Sample Sizes [Not in this text yet]

Suppose allowable Margin of Error = \( E \)

1) Proportions (most conservative): \( \text{ME} = 1/\sqrt{n} = E \)  
   So \( n = (1/E)^2 \) Doubling \( E \) quadruples \( n \)  
   Example: If \( E = 0.02 \) then \( n = (1/0.02)^2 = 50^2 = 2,500 \).

2) Proportions (Exact). \( \text{ME} = 2\sqrt{p(1-p)/n} = E \), \( n = [p(1-p)](2/E)^2 \).  
   If \( E = 0.02 \), \( p = 0.2 \); \( n = 1,600 \).

3) Measures: \( \text{ME} = 2s/\sqrt{n} = E \), \( n = [E/(2s)]^2 \).  
   If \( E = 100 \) and \( s = 1 \), then \( n = 2,500 \).
Statistical Significance: Unlikely

Statistical significance (statistically significant):
Unlikely (< 1 chance in 20) if due just to chance
- Coins: 5 heads in a row (1 chance in 32)
- Cards: Dealt two Aces ((4/52)(3/51) = 0.038 < 1/20
Statistically significant => More likely due to non-chance!

Statistically insignificant (not statistically significant)
Likely (> 1 chance in 20) if due to chance
- Coins: 4 heads in a row (1 chance in 16).
- Cards: Dealt one Ace (4/52 or 1 chance in 13)

Statistical Significance: Importance and Cause

Being statistically-significant
- Does NOT mean “important” or “note-worthy”!
- Does not mean “unlikely TO BE due to chance”

Being statistically significant
- Means “unlikely IF due just to chance”
- Does support the claim that the outcome is more likely
to be due to something other than chance.

Statistical Significance: Not Due to Chance?

Q. Does statistical-significance ever mean unlikely to be due to chance?
A. Depends on whether one allows these statements
Yes! If the chance the alternate is true is more than 50%.
No! If the chance the alternate is true is less than 50%.

Example: In an ESP experiment, a subject’s choices
compared to chance were statistically-significant.
Analysis. Since the chance that ESP is real is much less
than 1%, this result is still likely to be due to chance.

Statistical Insignificance: Explanations

Q. What explains statistical insignificance?
A. Two kinds of explanations:
1. Nothing real; no real difference; just coincidence.
2. Real difference but small so it is indistinguishable from
chance/noise. Might be seen in a larger sample.

Cannot conclude there is no real difference (#1)!!!
“No difference between samples” does not mean
“no difference between populations.”
Summary: Randomness

Sampling error is often overlooked as an influence on statistics or statistical associations.

A 95% confidence interval includes the population parameter (is right) 95% of the time.

A statistically-significant association is not necessarily an important association.

A statistically In-significant association may be pure coincidence or it may be a real association.