## Explained vs Caused

"Percentage of Variation in $Y$ explained by $X$ " versus "Percentage of $Y$ Caused by $X$ " $R$-squared is the percentage of variation in $Y$ explained by $X$ (in a linear model)

Thesis: $\quad$ Variation explained by $X$ " is radically different from "result caused by $X$ ". Problem: If $X$ causes $Y$, there is generally no measure of how much influence $X$ had on $Y$.
Solution: In the following case, that influence is easily understood and calculated.
Result: $\quad$ The "variation in $Y$ explained by $X$ " can be smaller thatn -- or bigger than -"the percentage of $Y$ caused by $X$ "

Consider a garment selling for \$100 on average in Minnesota: no sales tax on clothing Consider the same product selling in Wisconsin which has a sales tax on clothing.

Suppose the prices in both states is normally distributed with the same StdDev Suppose the sales tax in Wisconsin is a fixed dollar amount on this item.

What percentage of the total price on a $\$ 100$ item is caused by the sales tax?

| Sales Tax\$ | $\$ 1$ | $\$ 2$ | $\$ 4$ | $\$ 8$ | $\$ 16$ | $\$ 32$ | $\$ 64$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | \%Price Caused by $\times$ | $1.0 \%$ | $2.0 \%$ | $3.8 \%$ | $7.4 \%$ | $13.8 \%$ | $24.2 \%$ |
| Math explained | $=1 / 101$ | $=2 / 102$ | $=4 / 104$ | $=8 / 108$ | $=16 / 116$ | $=32 / 132$ | $=64 / 164$ |


| Regress total price (Product Price + State Sales Tax) on State (0, 1) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R-squared |  | Wiscon | tate Sa | Tax (ST) |  |  |  |
| StdDev (SD) | \$1 | \$2 | \$4 | \$8 | \$16 | \$32 | \$64 |
| \$1 | 22.2\% | 53.3\% | 82.1\% | 94.8\% | 98.7\% | 99.7\% | 99.9\% |
| \$2 | 6.7\% | 22.2\% | 53.3\% | 82.1\% | 94.8\% | 98.7\% | 99.7\% |
| \$4 | 1.8\% | 6.7\% | 22.2\% | 53.3\% | 82.1\% | 94.8\% | 98.7\% |
| \$8 | 0.4\% | 1.8\% | 6.7\% | 22.2\% | 53.3\% | 82.1\% | 94.8\% |
| \$16 | 0.1\% | 0.4\% | 1.8\% | 6.7\% | 22.2\% | 53.3\% | 82.1\% |
| \$32 | 0.0\% | 0.1\% | 0.4\% | 1.8\% | 6.7\% | 22.2\% | 53.3\% |

Regress total price (Product Price + State Sales Tax) on State (0, 1)

| Wisconsin State Sales Tax (ST) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| StdDev (SD) | \$1.25 | \$2.50 | \$5 | \$10 | \$20 | \$40 | \$80 |
| \$1.25 | 22.2\% | 53.3\% | 82.1\% | 94.8\% | 98.7\% | 99.7\% | 99.9\% |
| \$2.50 | 6.7\% | 22.2\% | 53.3\% | 82.1\% | 94.8\% | 98.7\% | 99.7\% |
| \$5 | 1.8\% | 6.7\% | 22.2\% | 53.3\% | 82.1\% | 94.8\% | 98.7\% |
| \$10 | 0.4\% | 1.8\% | 6.7\% | 22.2\% | 53.3\% | 82.1\% | 94.8\% |
| \$20 | 0.1\% | 0.4\% | 1.8\% | 6.7\% | 22.2\% | 53.3\% | 82.1\% |
| \$40 | 0.0\% | 0.1\% | 0.4\% | 1.8\% | 6.7\% | 22.2\% | 53.3\% |

## Explained vs Caused

|  |  |
| :---: | :---: |
| Code | State |
| 0 | MN |
| 1 | WI |

## SALES DATA

| State | \$Price |
| :---: | :---: |
| 0 | 84 |
| 0 | 88 |
| 0 | 88 |
| 0 | 88 |
| 0 | 92 |

2 1. Distribution of product prices (same in each state)

| 13 | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: |
| 14 | Total Price Total Price |  |  |  |
| 15 | Z | MN (Tax=0) | WI | \# Sold |
| 16 | -2.5 | 80 | 112 | 0 |
| 17 | -2 | 84 | 116 | 1 |
| 18 | -1.5 | 88 | 120 | 3 |
| 19 | -1 | 92 | 124 | 5 |
| 20 | -0.5 | 96 | 128 | 7 |
| 21 | 0 | 100 | 132 | 8 |
| 22 | 0.5 | 104 | 136 | 7 |
| 23 | 1 | 108 | 140 | 5 |
| 24 | 1.5 | 112 | 144 | 3 |
| 25 | 2 | 116 | 148 | 1 |
| 26 | 2.5 | 120 | 152 | 0 |

3. Dynamic graph of data and the associated linear model (with slope and R-squared)



| Row | L | M | N | O | P | Q | R | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 48 |  | $\$ 1$ | $\$ 2$ | $\$ 4$ | $\$ 8$ | $\$ 16$ | $\$ 32$ | $\$ 64$ |
| 49 | $\$ 1$ | $33.33 \%$ | $50.00 \%$ | $66.67 \%$ | $80.00 \%$ | $88.89 \%$ | $94.12 \%$ | $96.97 \%$ |
| 50 | $\$ 2$ | $0.00 \%$ | $20.00 \%$ | $42.86 \%$ | $63.64 \%$ | $78.95 \%$ | $88.57 \%$ | $94.03 \%$ |
| 51 | $\$ 4$ | $-33.33 \%$ | $-14.29 \%$ | $11.11 \%$ | $38.46 \%$ | $61.90 \%$ | $78.38 \%$ | $88.41 \%$ |
| 52 | $\$ 8$ | $-60.00 \%$ | $-45.45 \%$ | $-23.08 \%$ | $5.88 \%$ | $36.00 \%$ | $60.98 \%$ | $78.08 \%$ |
| 33 | $\$ 16$ | $-77.78 \%$ | $-68.42 \%$ | $-52.38 \%$ | $-28.00 \%$ | $3.03 \%$ | $34.69 \%$ | $60.49 \%$ |

